Computational modelling of Human Behavioural Data

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CCN Lecture 14

What is comp. modelling of behavioural data ?

Goal: use precise mathematical models to make better sense of behavioural data.

- **Why behaviour?** Modelling behaviour offers the possibility for linking neurobiological hypotheses with cognitive function or dysfunction.

- **What Data?** choices, reaction times, but can also be eye movements or other observable data

- **Why models?** Precisely specify assumptions about mechanisms, “algorithmic hypotheses”.
  Models are used for simulations, parameter estimation, model comparison, latent variable inference.

A Road Map of Good Practice

(This lecture is based on this paper)

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REVIEW ARTICLE

Ten simple rules for the computational modeling of behavioral data

Robert C Wilson[1,2†*], **Anne GE Collins**[3,4†*]

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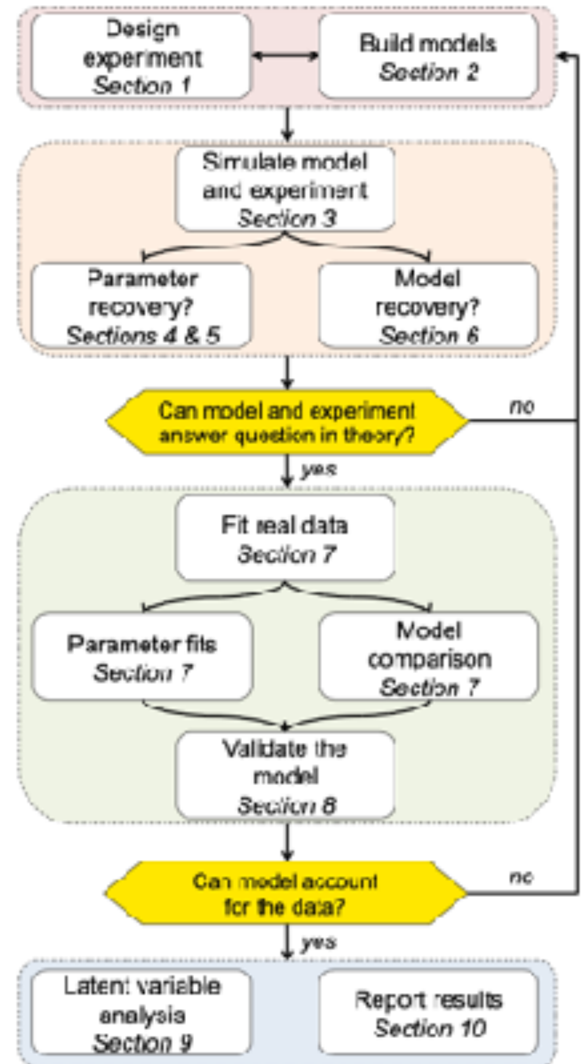
Abstract Computational modeling of behavior has revolutionized psychology and neuroscience. By fitting models to experimental data we can probe the algorithms underlying behavior, find neural correlates of computational variables and better understand the effects of drugs, illness and interventions. But with great power comes great responsibility. Here, we offer ten simple rules to ensure that computational modeling is used with care and yields meaningful insights. In particular,

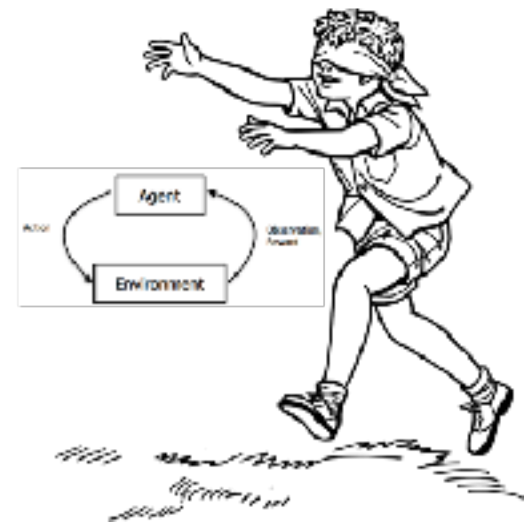


Wilson and Collins. eLife 2019;8:e49547.

1. Design a good experiment

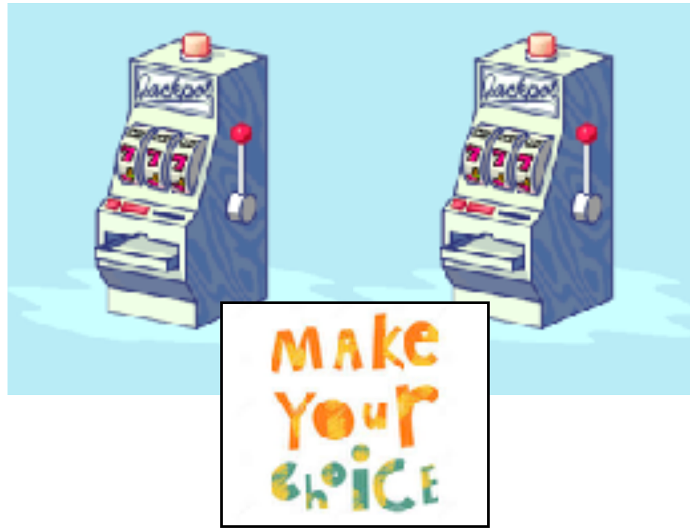
- What are the **questions** we want to answer?

  (e.g. How do individuals form an internal
  model of the world? Are depressed individuals impaired
  in reward learning?)



- Formalise the question.
- **Do we need models** to help answer these questions?
- Do you expect signature of the targeted process to be evident from simple statistics of the data? (else modelling might be pointless)
- Even the best models cannot salvage bad experiments

Example: multi-armed bandit



Question: Are depressed individuals impaired in reward learning? How do they learn to maximize their rewards in a case where the most rewarding choice is initially unknown?

e.g. $K=2$ slot machines, binary reward. $T=1000$ choices for each, $P(r|A)=0.2$, $P(r|B)=0.8$.

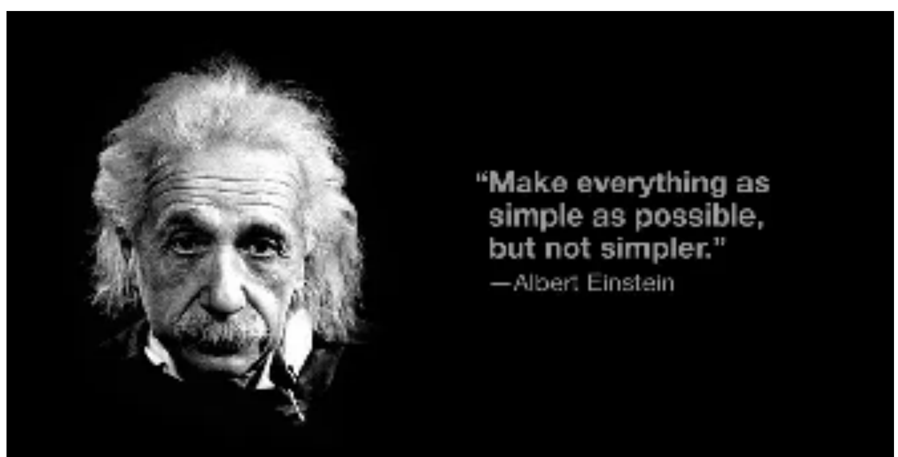
2 groups: 1 healthy controls vs Major Depressive Disorder patients.

Are patients slower to learn? worse? if so, can we dissect what makes them so?

2. Design Good Models/ Define Model Space

- Is the model's aim to be **descriptive**/ a summary of the data? **explanatory**/ mechanistic, **optimal**/normative?

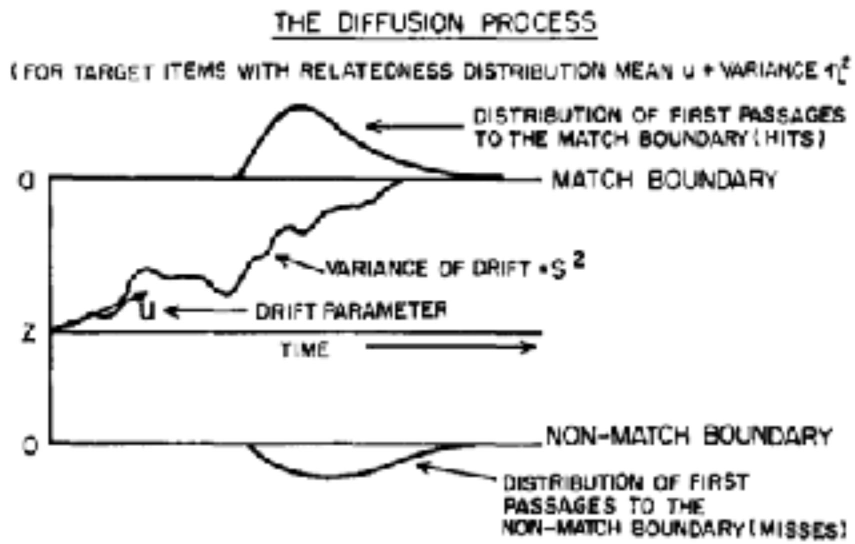
- A computational model should be **interpretable** (as much as possible)
- Models should capture all the hypotheses you want to test. **Inference is conditional on model space!**
- Beware of **bias** in how you choose/assess your model.



Reactions times are of interest:

Drift diffusion model (DDM)

Allows joint modelling of reaction times and accuracy

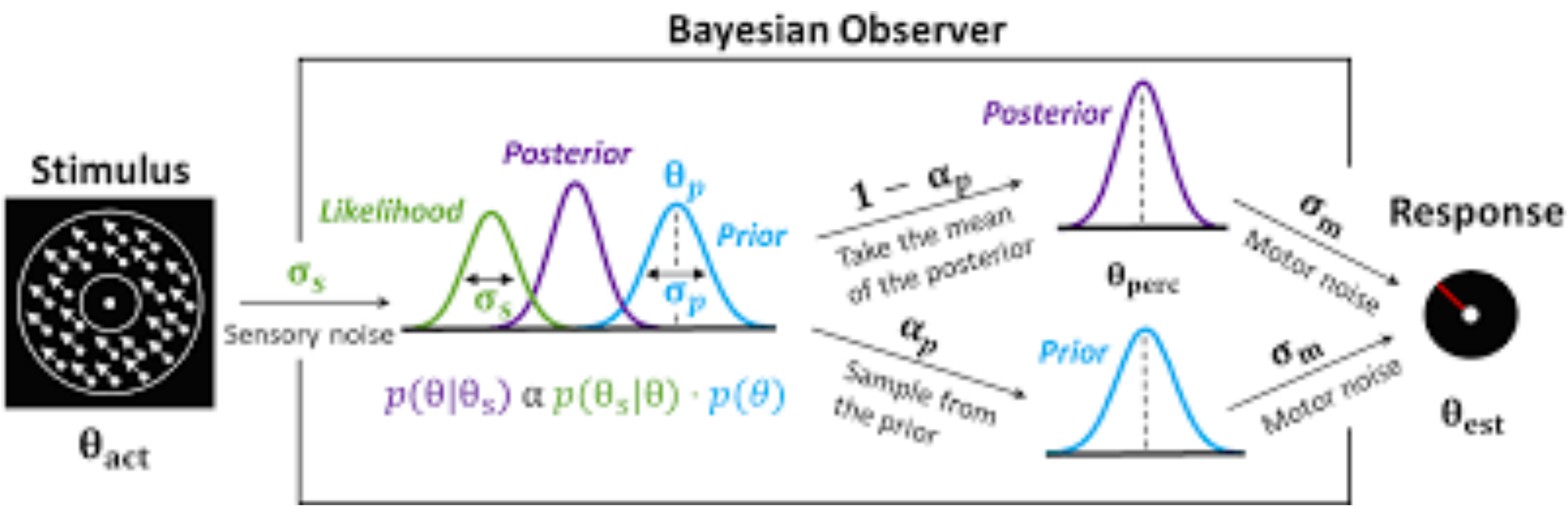


Ratcliff 1978, Psy. Rev.

(a family of models)

Comparison with benchmark performance or notion of prior is of interest:

Bayesian Models



(a family of models)

Learning is interest:

Reinforcement learning (RL)

- Make a choice
  - Based on "internal values"
- Observe an outcome (reinforcer)
  - Often probabilistic
  - Generates prediction error
- Update internal values

V_a V_b

$$V(t+1) = V(t) + \varepsilon \times (r(t) - V(t))$$

[Rescorla-Wagner]

- Decide between two options

$$p(a \mid V, \theta) = \frac{1}{1 + \exp(-\beta \times (V_a - V_b))}$$

Probability of choosing machine a [Softmax]

(a family of models)

2. Define Model Space (Example)

- **Model 1**: random responding with some bias for one or the other (one (bias) parameter);

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- **Model 3:** “vanilla” Rescorla-Wagner model:

$$Q_{t+1}^k = Q_t^k + \alpha(r_t - Q_t^k)$$

learn the values for each machine based on reward observed at each trial

[Rescorla-Wagner]

$$p_t^k = \frac{\exp(\beta Q_t^k)}{\sum_{i=1}^K \exp(\beta Q_t^i)}$$

compute probability of making a choice for one machine at each trial, based on the learned values; beta controls how deterministic/noisy this choice is.

2 free parameters, learning rate alpha and softmax parameter beta

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2 free parameters, learning rate alpha and softmax parameter beta

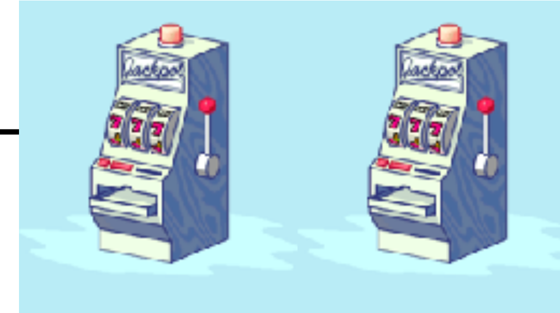
- **Model 4+:** variants on Rescorla-Wagner model that account to test for different hypotheses, e.g. role of sensitivity to reward? memory? different learning rates for reward vs punishment etc..

3. Simulate, Simulate, Simulate

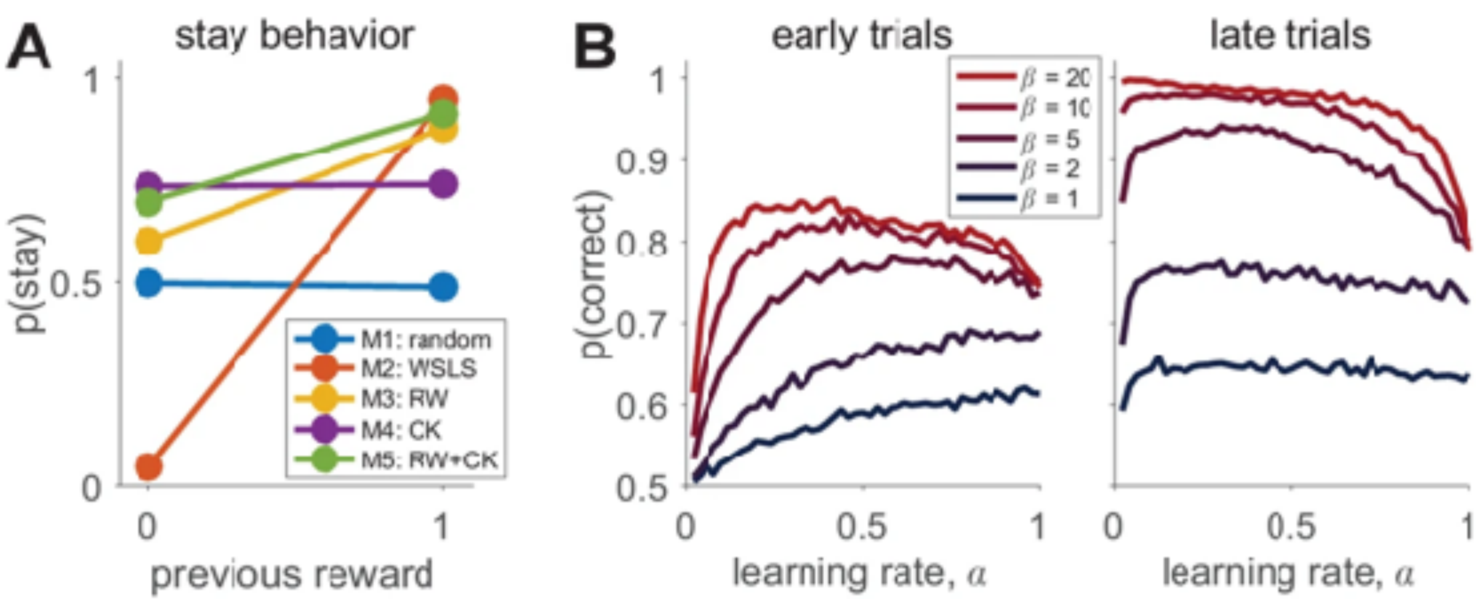
- Once you have a model,
  you can **create fake/surrogate/artificial data** and set up all your fitting pipeline.

- Define model-independent **measures** that capture key aspects of the processes you are trying to model (e.g. % correct, % stay after reward)
- Simulate the model across a range of **parameter values;**
- Visualise the simulated behaviour of **different models**, hopefully they show different patterns/make different predictions.

3. Simulate, Simulate, Simulate (Example)



Example: visualising the **win-stay-lose shift behaviour** for the different models and **% correct**, as a function of learning rate.
—> different patterns of p(stay) for different models, and shows that learning rate affects both the speed of learning and asymptotic performance



4. What model parameters fit the data best?

The simple way to estimate the value of the parameters that best describe our data is using a **maximum-likelihood approach**.

Our goal: find the parameter values $\hat{\theta}_m^{MLE}$ of model m, that maximize the likelihood of the observed data, d, given the parameters θ_m

$$LL = \log p(d_{1:T} | \theta_m, m) = \sum_{t=1}^{T} \log p(c_t | d_{1:t-1}, s_t, \theta_m, m)$$

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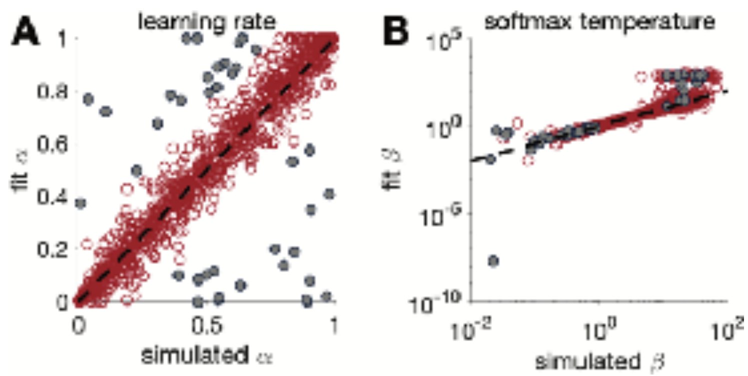
- In practice, find minimum of -LL using **gradient descent**, e.g. using Matlab `fmincon` or Python `scipy.optimize` package or the `optim` function in R
- <u>Tips:</u> Be sure your initial conditions give finite LogL. Beware rounding errors, zeros and infinities. Be careful with constraints on parameters.
- Beware local minima: run fitting procedure multiple times with random initial conditions, recording the best fitting log-likelihood for each run

5. Check that you can recover your parameters (parameter recovery)

Simulate Fake Data with fixed parameters	→	Fit models and parameters	→	Compare fitted parameters with values that generated the data

Parameter recovery for the Rescorla Wagner model (model 3) in the bandit task with 1000 trials. Grey dots in both panels correspond to points where parameter recovery for a is bad.



Parameter Recovery

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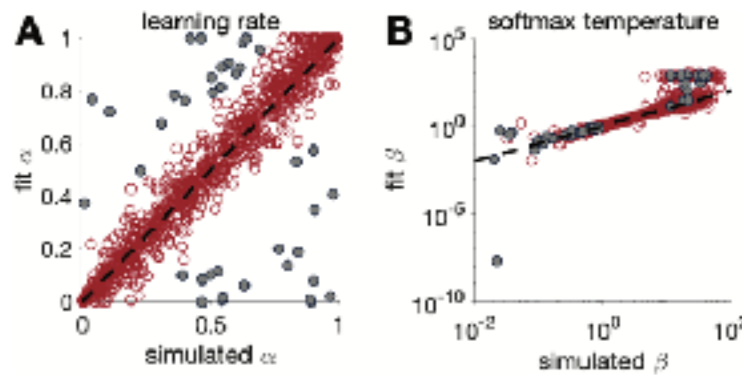

- Make sure your recovered parameters are in the right range
- Plot the **correlations** between simulated and recovered parameters
- Make sure the recovery process does not introduce correlations between parameters
- Remember that even successful parameter recovery represents a best case scenario.

6. What Model fits the data best? (Model Comparison)

- Model comparison's goal is to determine which model, out of a set of possible models, is most likely to have generated the data.

- Comparing the **log-likelihoods of each model at the best fitting parameter settings** $\hat{LL}$ doesn't work, because models with more free parameters would be favoured (overfitting).

  Many other options (e.g. AIC) but a popular/simple is the **Bayes Information Criterion, BIC**, which has an explicit penalty for **number of free parameters** (k_m).

- $$BIC = -2 \log \hat{LL} + k_m \log(T)$$

  T is the number of data points (trials).

- The model with the smallest BIC wins.

6. Model Recovery

Check that your model comparison process gives sensible results for simulated data.

Simulate Fake Data with all models and random parameters

Fit models, recover parameters, do model comparison

Compare recovered model with true model (confusion matrix)

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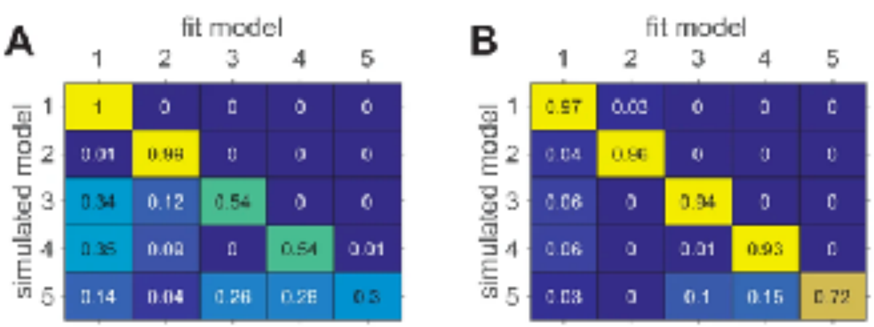
confusion matrix: p(fit model | simulated model)



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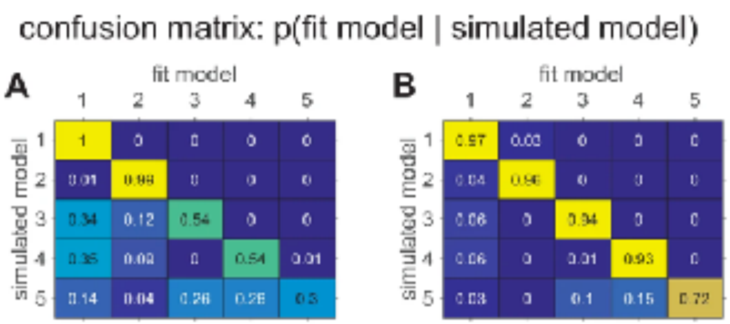

- In a perfect world, the confusion matrix will be the identity matrix, but in practice, not always the case.
- compare different measures of model comparison (e.g. AIC vs BIC)
- careful w/ choice of parameters when plotting confusion matrix
- Elephant in the room: all those models you never considered ..

7. Run the experiment; collect data and analyse

- now ready to collect data!

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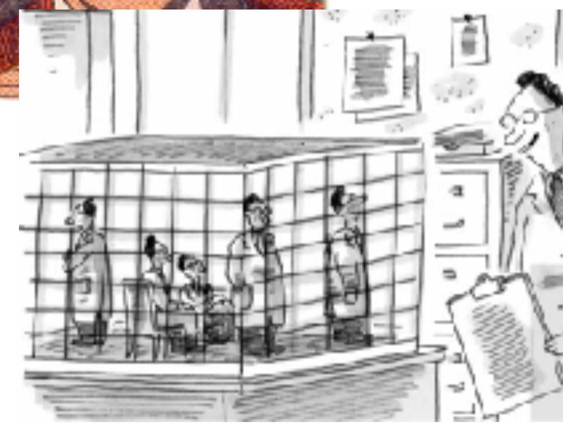
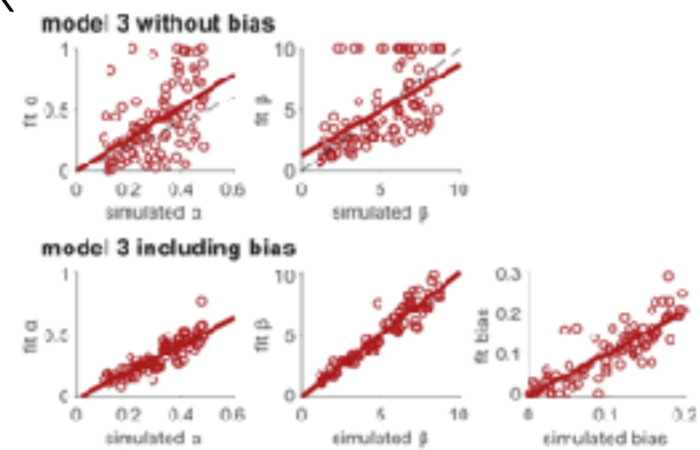


- now ready to collect data!
- **model-independent analysis**, comparison with expected patterns under different scenarios. Are the processes of interest engaged?
- **Model fitting**. How do the models fit the data?
- Might be necessary to improve/ extend model space to account for some features of the data that maybe unimportant theoretically, e.g. a systematic bias that affects fitting

8-9. Validate (at least) the winning model and Analyse

- Model comparison is **relative: the best fitting model might still be a bad model for the data!**
- Need to assess **absolute** quality of fit
- Simulate the winning model with best fitting parameters and see if it accounts for main features of the data.
- If this is not the case: back to drawing board! Possibly come up with a better model, or (disaster!) redesign the task

E.g. we find that there is a systematic bias in the data to preferring the left slot machine, independently of current p(reward)



e.g. there's a side bias in the (artificial) data but we try to fit without accounting for it

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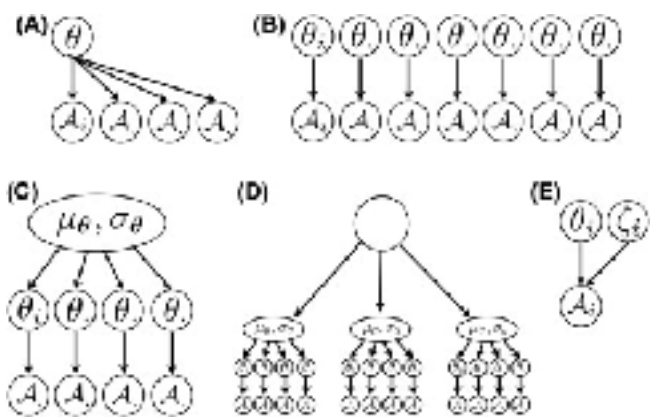
Model-dependent analyses should only be performed on the winning model, after researchers are satisfied that the model captures the behaviour.

A Rapidly Developing Field ..

- The standards are improving..
- More sophisticated methods are being developed, e.g.
- Use MAP instead of ML: include prior information on parameter values
- Hyper-parameters that are estimated at the same time as individual parameters (Hierarchical estimation)



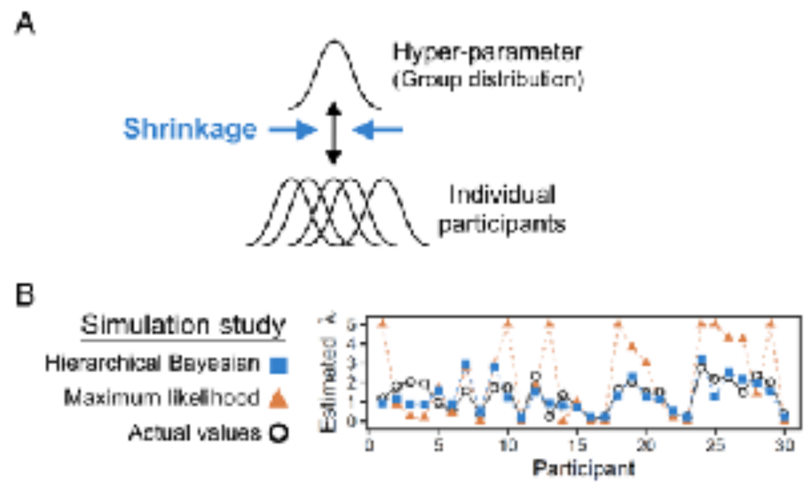


Figure 2. (A) A schematic illustration of hierarchical Bayesian analysis (HBA). In this example, the individual parameters are assumed to come from a group (hyper)parameter. (B) Results of a parameter recovery study (Ahn et al., 2011) between HBA and maximum likelihood estimation.

A Rapidly Developing Field ..

- Approximate full posterior $\log p\left(\theta_m \mid d_{1:T}, m\right)$, using sampling approaches

  (such as MCMC);
- Packages are being offered like hBayesDM (Ahn et al CP, 2017)
- Combine (latent variables of) models with other measures (e.g. fMRI).

RESEARCH

Revealing Neurocomputational Mechanisms of Reinforcement Learning and Decision-Making With the hBayesDM Package

Woo-Young Ahn[1], Nathaniel Haines[1], and Lei Zhang[2]

[1]Department of Psychology, The Ohio State University, Columbus, OH

[2][illegible]

Keywords: reinforcement learning, decision-making, hierarchical Bayesian modeling, model-based fMRI

ABSTRACT

Reinforcement learning and decision-making (RLDM) provide a quantitative framework and computational theories with which we can disentangle psychiatric conditions into the basic dimensions of neurocognitive functioning. RLDM offer a novel approach to assessing and potentially diagnosing psychiatric patients, and there is growing enthusiasm for both RLDM and computational psychiatry among clinical researchers. Such a framework can also

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Recommendations for Bayesian hierarchical model specifications for case-control studies in mental health

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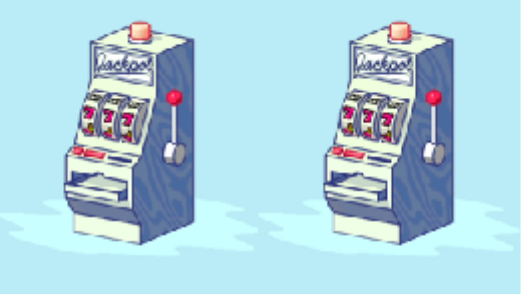
One in four people worldwide will experience a mental health disorder in their lifetime, and depressive disorders alone rank second in the leading causes of global disease burden worldwide (WHO, 2013). Mental health is one area of healthcare that is in

ences between patients and controls, leading to an increased rate of false negative findings; (2) - model both groups as deriving from separate populations at the risk of overestimating true group differences between patients and controls, leading to an increased rate of

Example code in Matlab



Example: define a model

- Assume participants keep track of value of each stimulus, update values via prediction errors
  - Note that here we include a reward sensitivity parameter (rho)

$$V_{t+1}^{(s)} = V_t^{(s)} + \varepsilon(\rho \times r_t - V_t^{(s)})$$

$$p(a = A | V^{(A)}, V^{(B)}) = \frac{1}{1 + \exp(-(V^{(A)} - V^{(B)}))}$$

Example: define simulation

```
function data = model_simulate(theta)
    eps = theta(1); rho = theta(2); V = [0, 0];
    data.choices = []; data.rewards = [];
    for t = 1:100
        p_choose_B = 1 / (1 + exp(-(V(2) - V(1))));
        c = 1 + (p_choose_B > rand);
        if c==1, r = (0.8 > rand); end
        if c==2, r = (0.2 > rand); end
        V(c) = V(c) + eps * (rho * r - V(c));
        data.choices(t) = c; data.rewards(t) = r;
    end
end
```

% simulate data for parameters theta

% Values initialised at 0

% for all trials

% compute prob choice given values

% draw a choice

% if choice 1 draw reward with prob 80%

% if choice 2 draw reward with prob 20%

% update values based on reward

% record choices and reward

Example: define likelihood

```
function nll = model_neg_log_likelihood(data, theta)
    eps = theta(1); rho = theta(2); V = [0, 0];
    rho = exp(rho); num_trials = numel(data.choices);
    choice_probabilities = nan(num_trials, 1);
    for t = 1:num_trials
        c = data.choices(t); c_alt = 1 + (c==1);
        p_choose_c = 1 / (1 + exp(-(V(c) - V(c_alt))));
        choice_probabilities(t) = p_choose_c;
        V(c) = V(c) + eps * (rho * data.rewards(t) - V(c));
    end
    nll = -sum(log(choice_probabilities));
end
```

% for all trials

% c= which target was chosen
% compute probability of choosing c
% update V

% sum all the log of choice probability

Example: simulation and model-fitting

```
1 data = model_simulate([0.4, 5.5]);
2 f = @(theta)(model_neg_log_likelihood(data, theta));
3 theta_start = [0, 0];
4 theta_opt = fminunc(f, theta_start);
5 theta_opt(2) = exp(theta_opt(2));
```

%1 - simulate the data with fixed parameters
%2 - define NLL
%3 - initialise parameters to fit
%4 - find parameters (eps, rho) that best fit the data
%5 sometimes useful to transform some parameters by taking ln or exp