Introduction to Algorithms and Data Structures

Tutorial 3

tutor name

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Q1: List implementation via *Extensible Array*

Initial array capacity of 1000. Increase by 100 each time an expansion is needed.

Append operator: adds one single element at the right hand of the array.

Copying operator: copies a single element from one array to another.

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(a) Number of **copyings** *for* **5000** *appends.*

- When do we apply **append**?
- What about **copying**?

Array size: $1000 \rightarrow 1100 \rightarrow 1200 \rightarrow \ldots \rightarrow 4900$

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$$100(\sum_{i=1}^{49} i - \sum_{i=1}^{9} i) = 100((49 \times 50)/2 - (9 \times 10)/2)$$

$$100(1225 - 45) = 11800 \text{ copyings}$$

Q1: List implementation via *Extensible Array*

(b) Number of copying for n appends.

We perform an expansion on the $(1000 + 100j + 1)$th append.

For each $j = 0, 1, ...$, we copy the $1000 + 100j$ elements already present.

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Summing them all up:

$$\sum_{i=10}^{\lfloor (n-1)/100 \rfloor} 100i = 50\lfloor (n-1)/100 \rfloor \lfloor (n-1)/100 + 1 \rfloor - 4500$$

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Asymptotically: $\Theta(n^2)$

Amortized cost: $\Theta(n)$

Q1: List Implementation via *Extensible Array*

(c) Contraction policy: if the current array capacity is **2000** *or more, and the proportion of the array in use dips below* **0.5**, *move the contents to an array of half the size.*

What bad behaviour might result from this policy?

Suggest a better contraction policy.

- What happens when we append **1001** elements?
- What if we follow up with two deletes and then two appends?

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Improvements: use dips < 0.25 before contracting by 0.5.
At least $n/2 - 1$ list operations before the next expansion/contraction.
Amortized $\Theta(1)$ cost per operation.

Q2: Bucket-style Hash Tables

(a) Perform the following sequence of set operations in a hash table or size 10 using the hash function: $h(n) = n \mod 10$*:*

insert(47), insert(93), insert(17), insert(143), insert(777), contains(93), contains(7)

- How many non-empty buckets do we have by the end ?
- How does the bucket list look like after 5 operators?

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Bucket 1:	[ ]	Bucket 6:	[ ]
Bucket 2:	[ ]	Bucket 7:	[47]
Bucket 3:	[ ]	Bucket 8:	[ ]
Bucket 4:	[ ]	Bucket 9:	[ ]
Bucket 5:	[ ]	Bucket 0:	[ ]

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Bucket 2:	[ ]	Bucket 7:	[777,17, 47]
Bucket 3:	[143,93]	Bucket 8:	[ ]
Bucket 4:	[ ]	Bucket 9:	[ ]
Bucket 5:	[ ]	Bucket 0:	[ ]

contains(93): in Bucket 1,2,3 $\rightarrow$ *True*

contains(7): in Bucket 1,2,...,10 $\rightarrow$ *False*

Q2: Probing in Hash Tables

We can represent such sets by storing the elements within the hash table itself, using **probing** *to resolve collisions. We may use* -1 *to indicate a blank entry in the table. We use the* **hash-probe** *function:*

$$g(n, i) = (n + (i \times F(n)) \mod 10$$

where F is some magical function such that:

$F(47) = 1, F(93) = 3, F(17) = 3, F(143) = 7, F(777) = 3, F(7) = 1$

(b) What happens with the given hash/probe scheme?

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	47	-1	-1

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	93	-1	-1	-1	47	-1	-1

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free
- **insert(17):** 7 is taken. $\mathrm{F}(17) = 3$ so $g(17, 1) = 0$

0	1	2	3	4	5	6	7	8	9
17	-1	-1	93	-1	-1	-1	47	-1	-1

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free
- **insert(17):** 7 is taken. F(17) $= 3$ so $g(17, 1) = 0$
- **insert(143):** 3 is taken. F(143) $= 7$, the successive probe values are 3,0,7,4,1,... g(143, 3) $= 4$ is free.

0	1	2	3	4	5	6	7	8	9
17	-1	-1	93	143	-1	-1	47	-1	-1

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free
- **insert(17):** 7 is taken. F(17) = 3 so $g(17, 1) = 0$
- **insert(143):** 3 is taken. F(143) = 7, the successive probe values are 3,0,7,4,1,... g(143, 3) = 4 is free.
- **insert(777):** 7 is taken. Have F(777) = C(1) = 3, so probes are 7,0,3,6,9,... . Of these, 6 is the first free one.

0	1	2	3	4	5	6	7	8	9
17	-1	-1	93	143	-1	777	47	-1	-1

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- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free
- **insert(17):** 7 is taken. F(17) = 3 so $g(17, 1) = 0$
- **insert(143):** 3 is taken. F(143) = 7, the successive probe values are 3,0,7,4,1,... g(143, 3) = 4 is free.
- **insert(777):** 7 is taken. Have F(777) = C(1) = 3, so probes are 7,0,3,6,9,... . Of these, 6 is the first free one.

- **contains(93):** first probe 3, True

0	1	2	3	4	5	6	7	8	9
17	-1	-1	93	143	-1	777	47	-1	-1

Q2: Probing in Hash Tables

- **insert(47):** table empty, successful probe
- **insert(93):** slot 3 is free
- **insert(17):** 7 is taken. F(17) = 3 so $g(17, 1) = 0$
- **insert(143):** 3 is taken. F(143) = 7, the successive probe values are 3,0,7,4,1,... g(143, 3) = 4 is free.
- **insert(777):** 7 is taken. Have F(777) = C(1) = 3, so probes are 7,0,3,6,9,... . Of these, 6 is the first free one.

- **contains(93):** first probe 3, True
- **contains(7):** $C_7 = C(0) = 1$ False

0	1	2	3	4	5	6	7	8	9
17	-1	-1	93	143	-1	777	47	-1	-1

Q2: Probing in Hash Tables

(c) Why not F(-) = 5?*

Discuss!

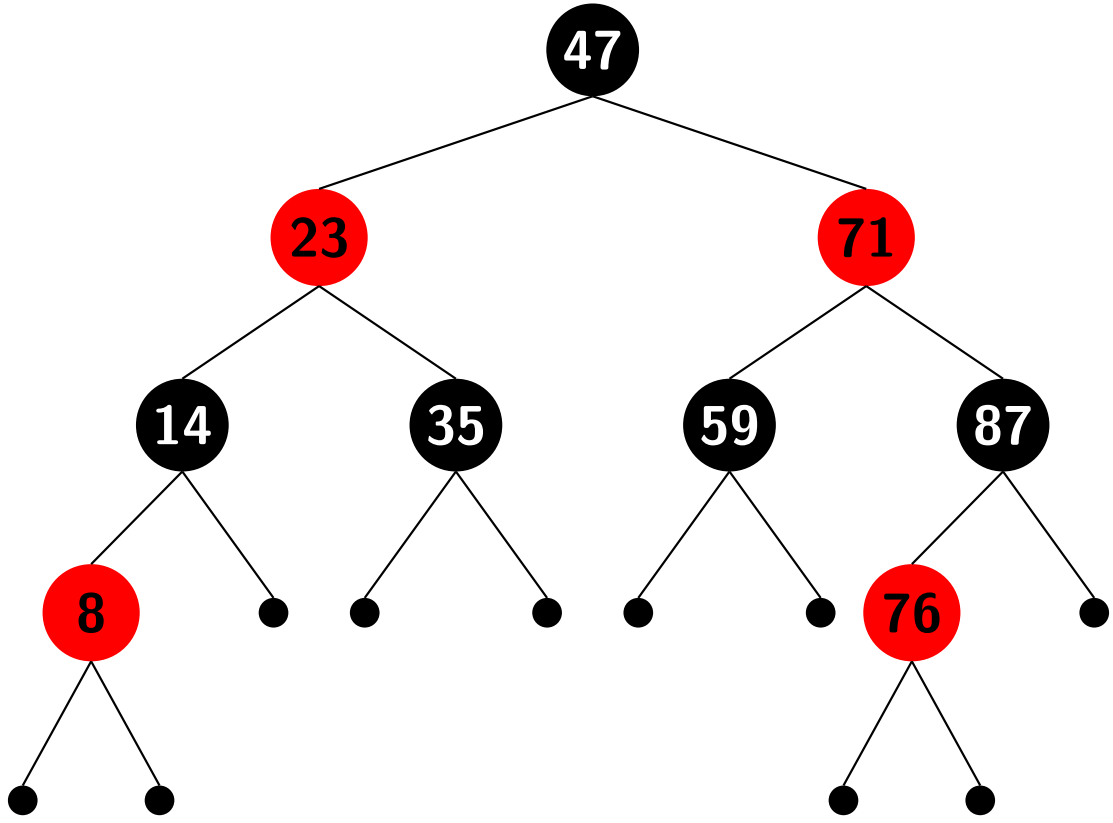
Q3: Red-Black Tree Operations

Red-Black Tree Rules:

- Every single node in the tree must be either **red** or **black**.
- The root node of the tree must always be **black**.
- Two **red** nodes can never appear consecutively, one after another.
- Every branch path — the path from a root node to a leaf node — must pass through the exact same number of **black** nodes.

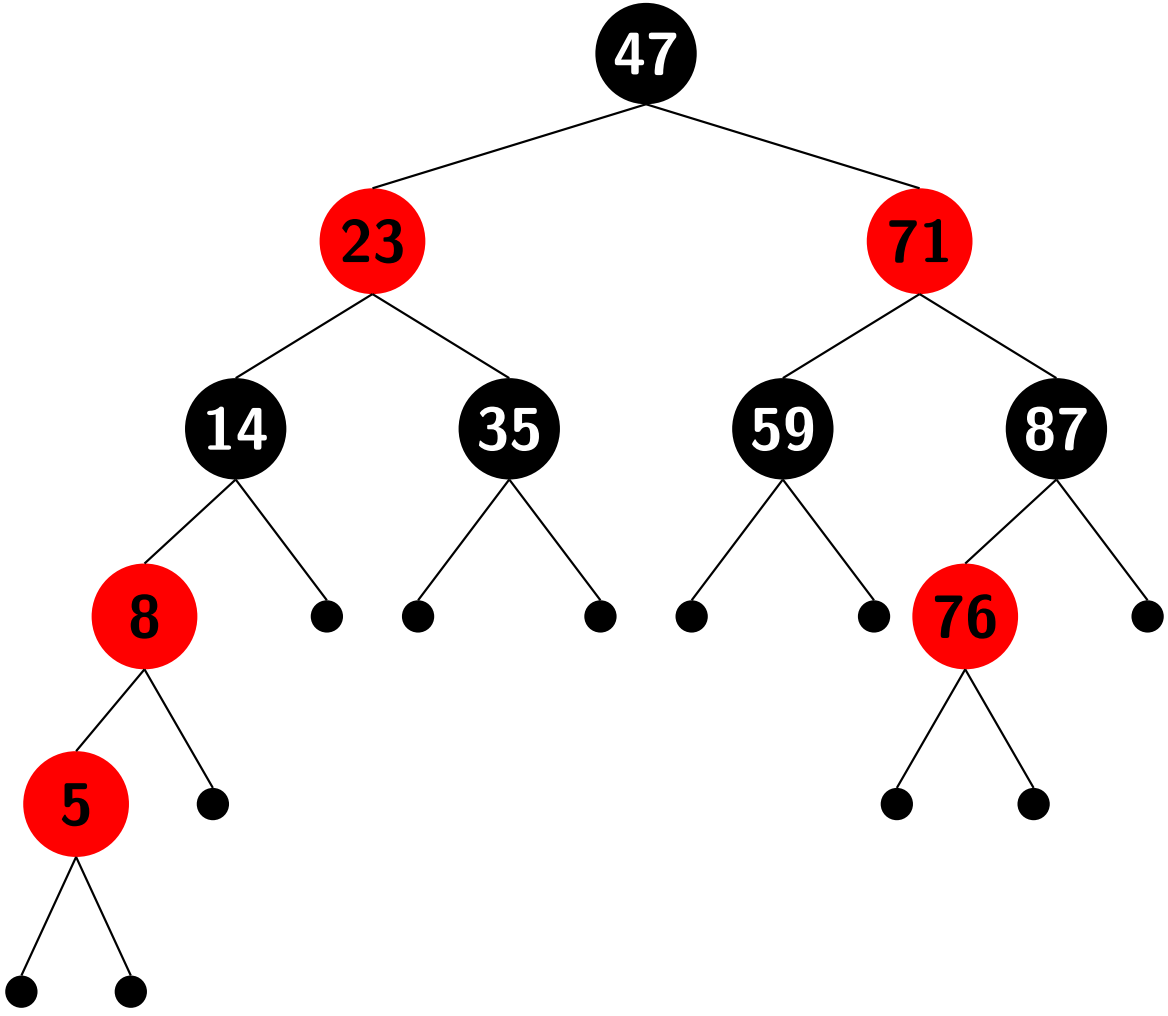
Q3: Red-Black Tree Operations

(a) What happens when the following are performed in sequence: insert(5), insert(17)?



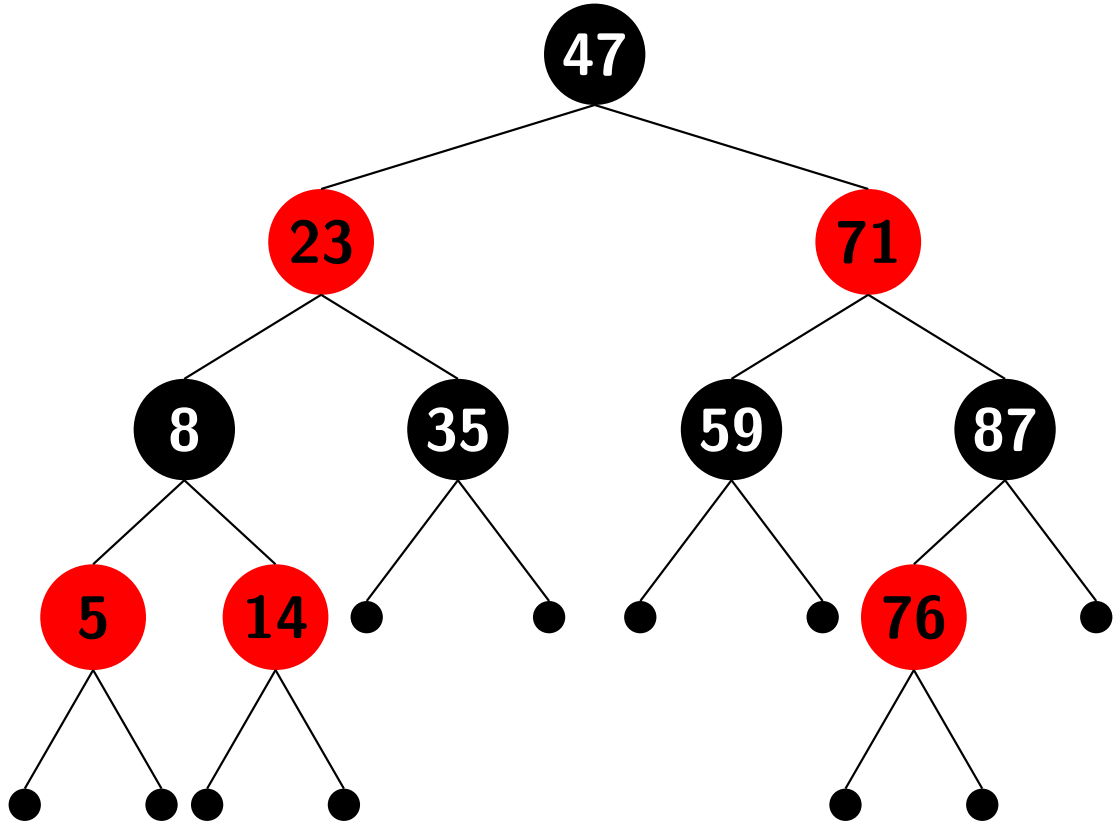
Q3: Red-Black Tree Operations

insert(5)



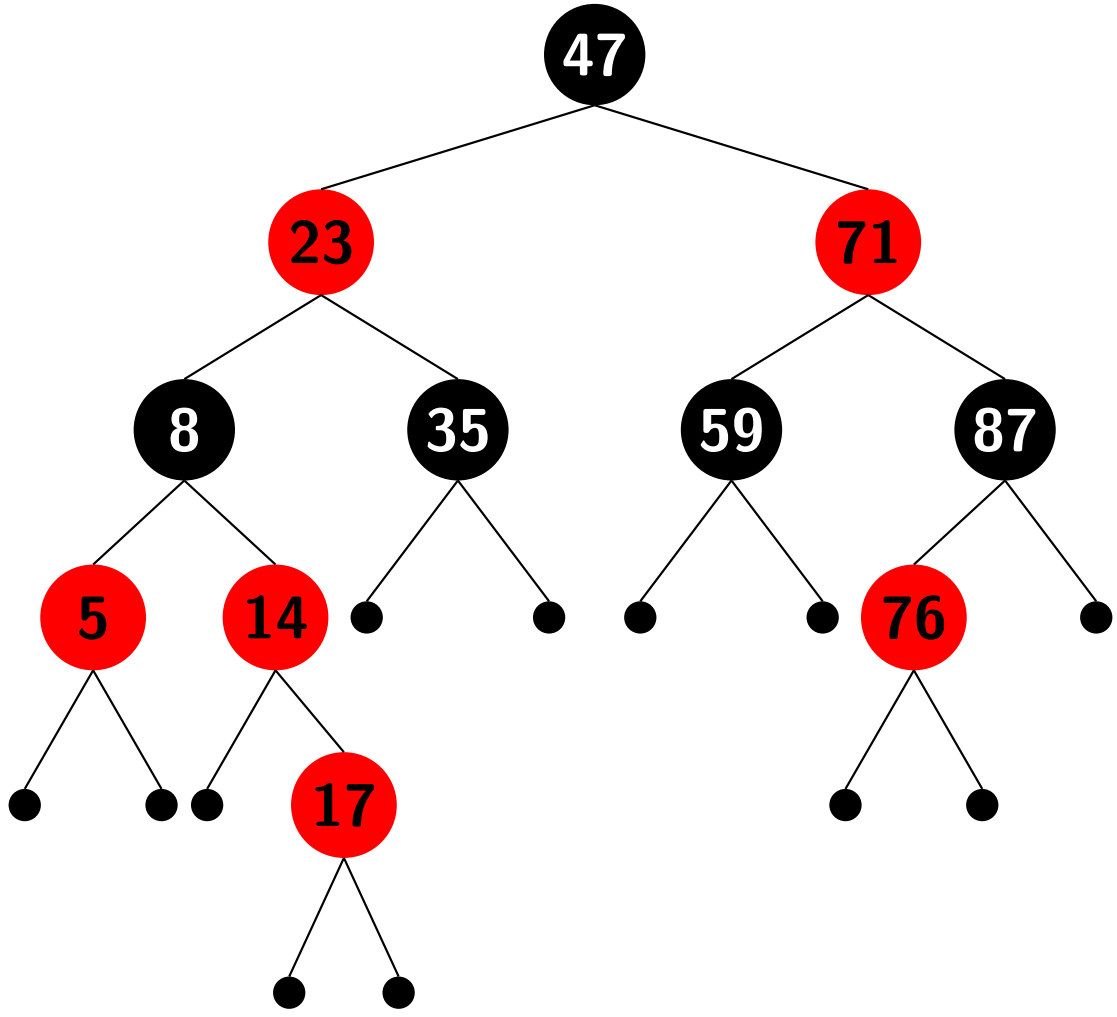
Q3: Red-Black Tree Operations

Rebalanced tree!



Q3: Red-Black Tree Operations

insert(17)



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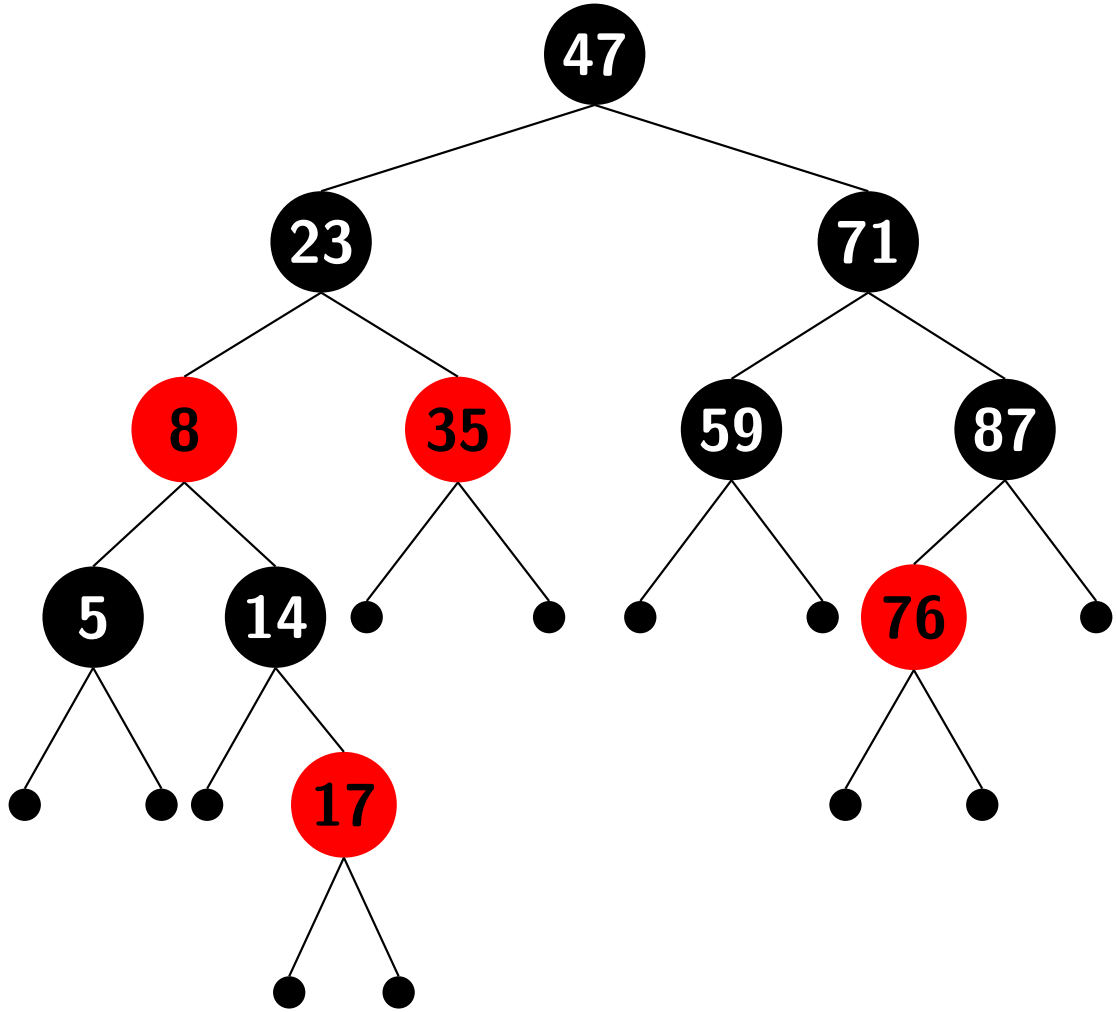

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(b) How would we use red-black trees to implement lists?

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Idea: each internal node should store, in addition to its key value, the number of list elements in the sub-tree rooted at that node. It does require a tree update for each operation.

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(b) How would we use red-black trees to implement lists?

Idea: each internal node should store, in addition to its key value, the number of list elements in the sub-tree rooted at that node. It does require a tree update for each operation.

- How would **get(i)** operation work?
- Using the graph below explain how **get(8)** would work.