Introduction to Quantum Computing
Lecture 26: Universal Blind Quantum Computing

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This Lecture

1. Blind Quantum Computing: What & Why
2. Tools for MBQC-based Universal Blind Quantum Computing
3. UBQC protocol and Verification

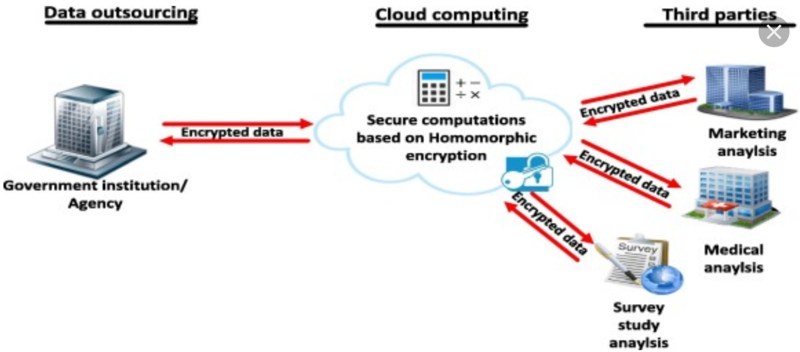
Blind Quantum Computing: What & Why

Secure Cloud Computing

Modern Cyber Security goes beyond encryption

(e.g. Privacy-preserving Data Mining)

Delegated Private Computation (e.g. sensitive medical data)



Secure Cloud Computing

Modern Cyber Security goes beyond encryption
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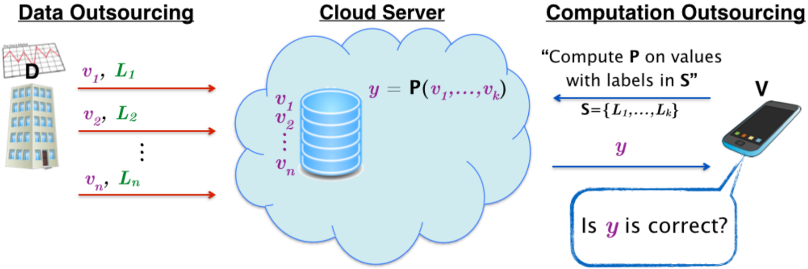
Verified Delegated Private Computation



Secure Cloud Computing

Modern Cyber Security goes beyond encryption

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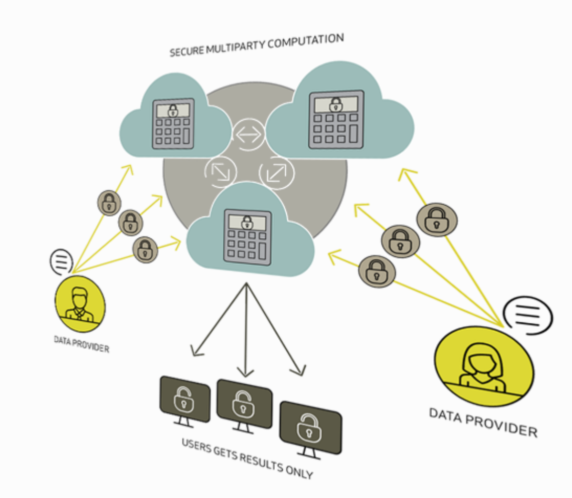
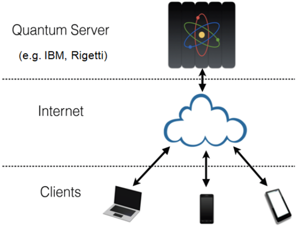
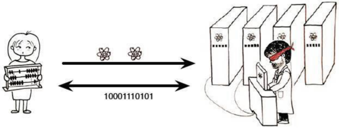
Secure Multiparty Computation (e.g. e-voting, auctions)



The Secure Quantum Cloud



- Clients wants to maintain **privacy**, **accuracy** and **reliability**
- Clients wants to use the **extra power of quantum computing**



- Universal Blind Quantum Computation (Broadbent, Fitzsimons, Kashefi 2009)
- Basis for numerous extra functionalities
- Client sends **random single qubits** to Server

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- Realistic setting (few large quantum computers)
- Active area to obtain efficient quantum analogues (e.g.):

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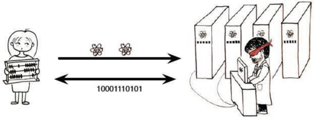

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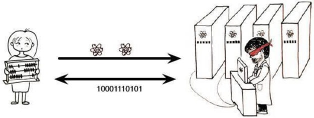
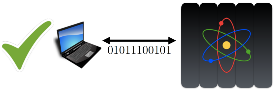
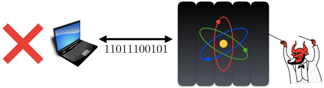


Verifiable Secure Quantum Cloud





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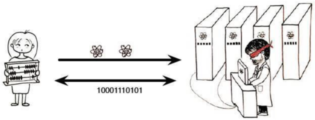
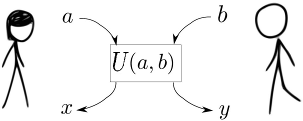


Verifiable Secure Quantum Cloud

Secure Two-Party Quantum Computation



Part II

Tools for MBQC-based Universal Blind Quantum Computing

Blind Computation Setting

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Blind Computation $\Rightarrow$ Bob cannot determine Alice's:

- Input
- Intended output
- Computation (not required for fully homomorphic encryption)

Alice must encrypt everything (input, computation, output)

General Idea

- **Use of MBQC** (possible otherwise)

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  1. **Can** prepare single qubits
  2. **Cannot** measure, store, prepare entangled qubits, apply unitary gates

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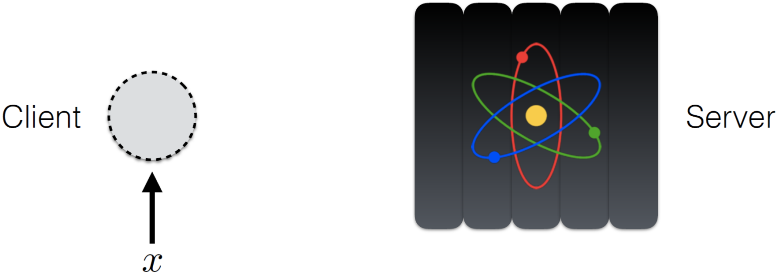
- Bob:
  1. Does **not** know what states Alice sends him
  2. Follows instructions; returns measurement outcomes to Alice

Blindness w.r.t. the "true" default angles $\{\phi_i\}_i$ and the shape of the "true" resource $|G\rangle$

Universal Blind Quantum Computation (UBQC)

Keep x and $f(x)$ hidden from server



Client $\in$ BPP

Server $\in$ BQP

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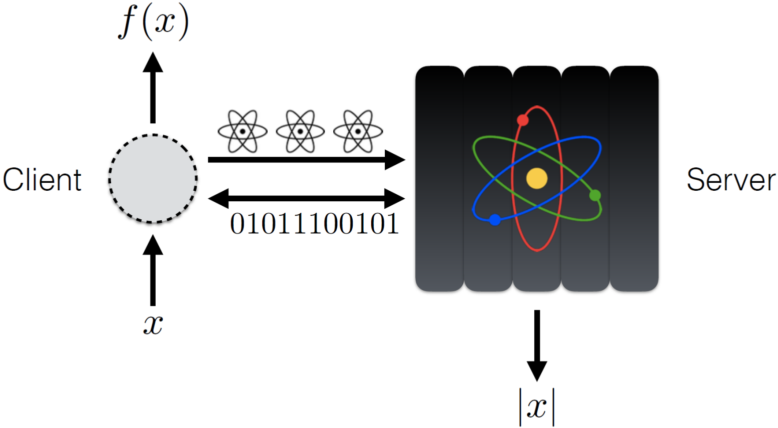

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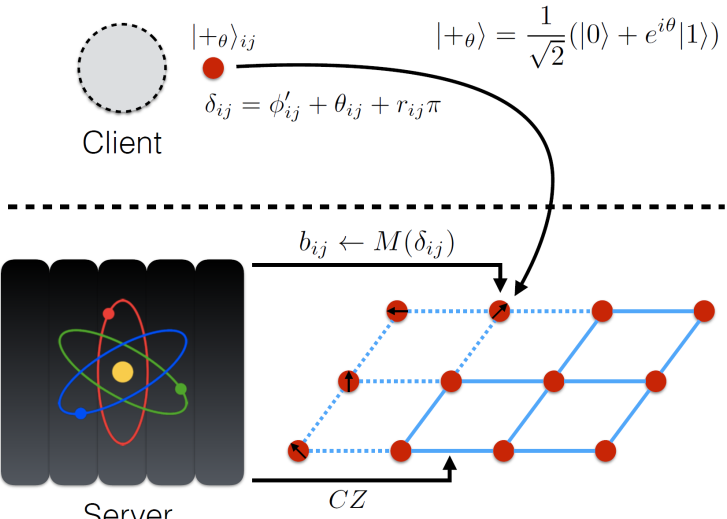

Broadbent, Fitzsimons, Kashefi - FOCS 2009

Trick 1: Hiding the measurement angle

- **Properties:**

(a) $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2) = R(\theta_2)R(\theta_1)$.

Rotations (on same axis), commute and act additively

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Preparing "$+_\theta$" states from "0".

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(c) $M^\alpha = M^Z HR(-\alpha)$.

Measuring at an angle is equivalent with applying the inverse circuit that prepares $|\pm_\alpha\rangle$ and then measure in comp. basis.

Trick 1: Hiding the measurement angle

Consider Two Scenarios:

- **Scenario 1** (normal MBQC)

$$M_1^{\phi} \wedge Z_{12} \left|+\right\rangle_1 \left|+\right\rangle_2 \rightarrow \left|s_1\right\rangle_1 X^{s_1} J(-\phi) \left|+\right\rangle_2 = \left|s_1\right\rangle_1 X^{s_1} HR(-\phi) \left|+\right\rangle_2$$

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- Two scenarios have **same** effect
- If θ is unknown to Bob, when he measures $(\phi+\theta)$ he is ignorant of the "true" angle of the J-gate he implements.

Trick 2: Hiding the measurement outcome

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- **Scenario 2** (outcome hidden by $r \leftarrow \{0,1\}$)

  $$M_1^{\phi+r\pi} \wedge Z_{12} \left|+\right\rangle_1 \left|+\right\rangle_2 \rightarrow \left|b_1\right\rangle_1 X^{b_1} J(-\phi - r\pi) \left|+\right\rangle_2$$

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- Two scenarios have **same** effect on qubit 2
- If Bob doesn't know r, when he measures $\phi + r\pi$ he is ignorant of the "true" measurement outcome s_1 and how to correct in the future angles.

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- Define angle:

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- Measuring ϕ'_i angle on state $|+\rangle$ is the same as measuring $\phi'_i + \theta_i$ angle on state $|+_{\theta_i}\rangle$
- Adding $r_i\pi$ does **not** change the measurement, only flips the outcome ($s_i = 0$ goes to $s_i = 1$ and visa-versa)

Hiding measurement angle & outcome

Summary of Instructions

- Alice sends $|+_{\theta_i}\rangle$ and instructs Bob to measure in δ_i
- Bob returns outcome b_i
- Alice computes $s_i = b_i \oplus r_i$ and uses this for $\phi'_j|\ j \in \{\text{ future of } i\}$

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The pre-rotation θ_i one-time-pads the true measurement angle ϕ'_i, and r_i one-time-pads the true measurement outcome s_i

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Note: Interaction is required so that Alice can compute the corrected measurement angle ϕ'_i which depends on the (corrected) measurement outcomes (and thus cannot be computed by Bob).

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Solution:

- A general graph (e.g. 2-dim lattice) where the actual graph used can be embedded
- A trick to "break" the graph at a vertex (remove vertex and break connectivity of the graph)
- Alternatively, could consider certain graph states that are universal with only $|\pm_\phi\rangle$ measurements (e.g. "brickwork state)

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- $\wedge Z_{12}\,|0\rangle_1\,|\psi\rangle_2 = |0\rangle_1\,|\psi\rangle_2\ ;\ \wedge Z_{12}\,|1\rangle_1\,|\psi\rangle_2 = |1\rangle_1\,(Z\,|\psi\rangle_2)$

  Computational basis qubits do **not** get entangled with $\wedge Z$

  $\wedge Z_{12}\,|d\rangle_1\,|\psi\rangle_2 = |d\rangle_1\,\left(Z^d\,|\psi\rangle_2\right)$

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- **Scenario 1:** Alice sends randomly $\{|+\rangle, |-\rangle\}$ state, with equal probability

  Bob's view: $\rho = \frac{1}{2}(|+\rangle\langle+| + |-\rangle\langle-|) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- **Scenario 2:** Alice sends randomly $\{|0\rangle, |1\rangle\}$ state, with equal probability

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- The dummy qubits produce a Z^d **correction** to all neighbouring qubits.

  These corrections are:

  (i) known to Alice only, (ii) know from the start

  Alice takes them into account when computing the angle that she asks Bob to measure

  (Adds $d\pi$ to the angle of qubits neighbouring with $|d\rangle$ qubit)

Part III

UBQC protocol and Verification of Quantum Computing

A first UBQC protocol

We assume only M^{α} measurements (breaking to the desired resource state happens as described)

We assume classical input/output (can generalise)

Input:

- Graph G of m qubits sufficient for given computation
- m "default" measurement angles ϕ_i performing desired computation
- m random variables $\theta_i \leftarrow \{0, \pi/4, \cdots, 7\pi/4\}$, m random variables $r_i \leftarrow \{0, 1\}$ chosen secretly by Alice

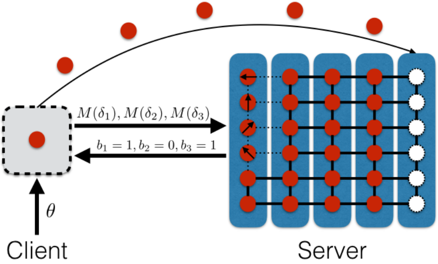
Initial Step:

- Alice sends to Bob m qubits of the form $|+_{\theta_i}\rangle$
- Bob applies $\wedge Z_{ij}$ according to the graph and generates the "secretly rotated" resource state

A first UBQC protocol

Step i: $1 \leq i \leq m$

- Alice computes the angle $\delta_i = \phi'_i + \theta_i + r_i\pi$ and instruct Bob to measure qubit i at this angle
- Bob measures qubit i and returns outcome b_i to Alice
- Alice sets the value $s_i = b_i \oplus r_i$
- Alice moves to step $i + 1$ until $i = m$ where the protocol terminates
- The outcome is obtained from the last "layer" of measurements



Towards Verification of Quantum Computation

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- If measured in the $\{\left|+_{\theta_i}\right\rangle, \left|-_{\theta_i}\right\rangle\}$ basis it will (deterministically) give the outcome:

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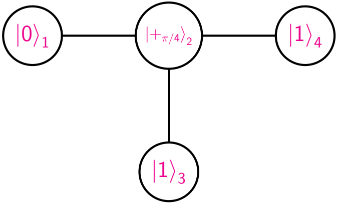
- Alice knows this result in advance, but Bod doesn't

  (Bob neither knows the d's nor the position that a trap exists)

Towards Verification of Quantum Computation

An example: with $d_1 = 0, \theta_2 = \pi/4, d_3 = 1, d_4 = 1$



Example trap at qubit 2

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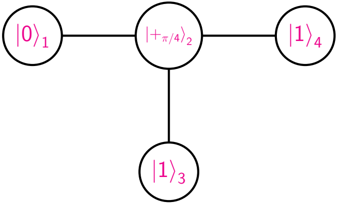


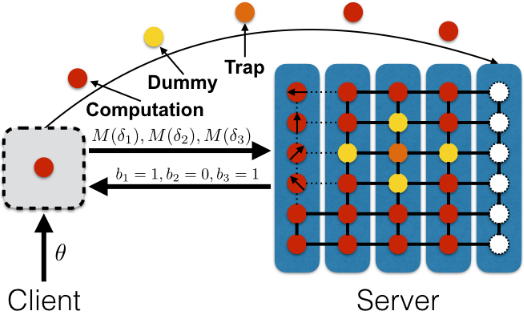
Example trap at qubit 2

- The trap qubit (after $\wedge Z$-gates) is at state:

  $Z^{d_1+d_3+d_4} |+_{\pi/4}\rangle = Z^2 |+_{\pi/4}\rangle = |+_{\pi/4}\rangle$
- If measured in the $\{|+_{\pi/4}\rangle, |-_{\pi/4}\rangle\}$-basis we get (always) $b_2 = 0$

Towards Verification of Quantum Computation

- **Position** of traps and dummies is **unknown** to Bob
- **Result of measurement of trap**, if measured in M_t^θ-basis is deterministic and **known** in advance **to Alice**
- **If Bob deviates** at the protocol, he may deviate on the trap qubit and this will be **detected by Alice**!



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Method to verify/test any quantum computation device

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