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Tutorial 7

Problem 1

Recall from the lectures the definition of the J gate as:

$$J(\theta) = HR(\theta) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}$$

Additionally, recall that any single qubit gate can be decomposed using J gates:

$$U = J(0)J(\theta_1)J(\theta_2)J(\theta_3), \tag{1}$$

for some θ_1, θ_2, θ_3.

a. Using Eq. (1) express the general single-qubit unitary U as a matrix that its elements depend on $\theta_1, \theta_2, \theta_3$.

Solution: In certain cases, we can use various heuristic techniques to find the theta parameters. These would entail using things like $J(0) = H$, $HH = I$, $HXH = Z$, $HZH = X$ etc. However, we will calculate the general expression for U from the decomposition and equate it to Z and X respectively, to determine the values of the theta parameters. As mentioned, we have that $J(0) = H$ and that $HH = I$, and so our expression for U becomes:

$$U = HHR(\theta_1)J(\theta_2)J(\theta_3) = R(\theta_1)J(\theta_2)J(\theta_3)$$

If we do the explicit matrix multiplication, we find that:

$$U = \frac{1}{2} \begin{pmatrix} 1 + e^{i\theta_2} & e^{i\theta_3} - e^{i(\theta_2+\theta_3)} \\ e^{i\theta_1} - e^{i(\theta_1+\theta_2)} & e^{i(\theta_1+\theta_3)} + e^{i(\theta_1+\theta_2+\theta_3)} \end{pmatrix}$$

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b. Using the matrix form for U derived in the first part of the question (known as the J-decomposition), express the gates Z and X in the form of Eq. (1), i.e. find the corresponding angles $\theta_1, \theta_2, \theta_3$.

Solution: It is now a simple matter of identifying the matrix elements corresponding to the Z and X operators and picking the appropriate values for θ_1, θ_2, θ_3. It is easy to notice that for Z we have $\theta_1 = \pi$, $\theta_2 = \theta_3 = 0$ and for X we have $\theta_1 = \theta_3 = 0$ and $\theta_2 = \pi$.

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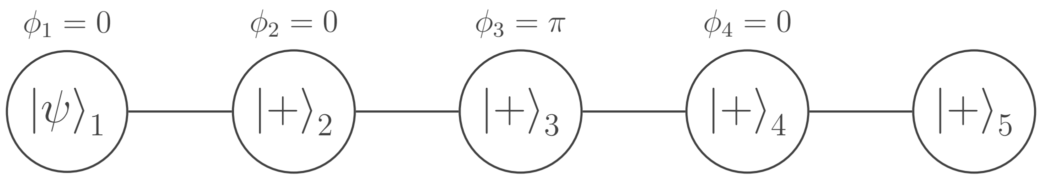
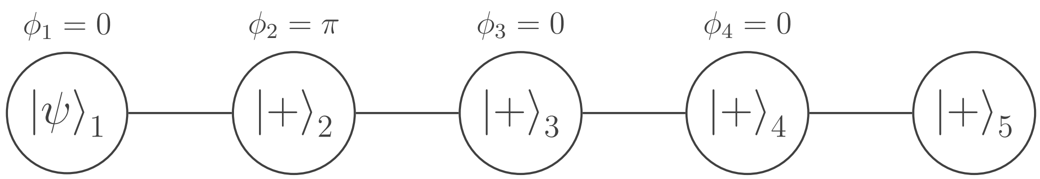
c. Using this decomposition, find a five-qubit measurement pattern that implements the gate Z and the same for the gate X.

Note: This is not the simplest way to implement X, Z with an MBQC measurement pattern. Can you guess, by inspection, simpler measurement patterns for X, Z?

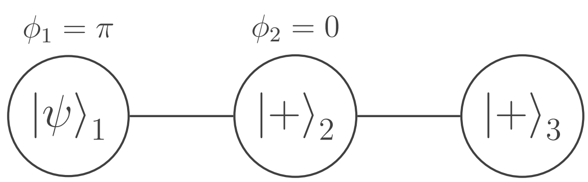
Solution: For the gate Z:



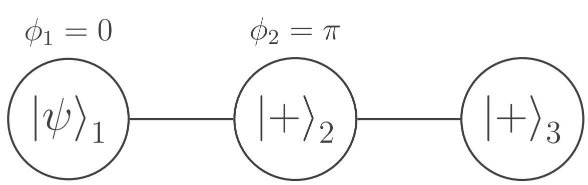
For the gate X:



By inspection, we could find simpler patterns. For Z:

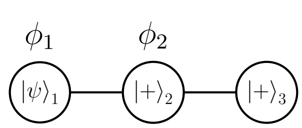


For X:

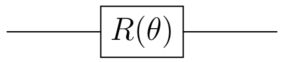


Problem 2

Consider the following MBQC graph with the input state $|\psi\rangle_1$ and the output on qubit 3.



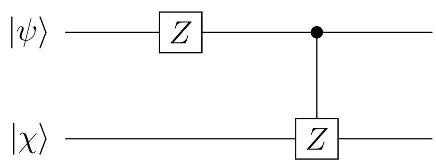
Find the angles ϕ_1 and ϕ_2 so that the MBQC graph is equivalent to an application of the rotation θ gate:



Solution: Observe that since $J(\theta) = HR(\theta)$, if we set $\phi_1 = -\theta$ and $\phi_2 = 0$, the MBQC graph implements $J(-\phi_2)J(-\phi_1) = J(0)J(\theta) = HR(0)HR(\theta) = HIHR(\theta) = R(\theta)$

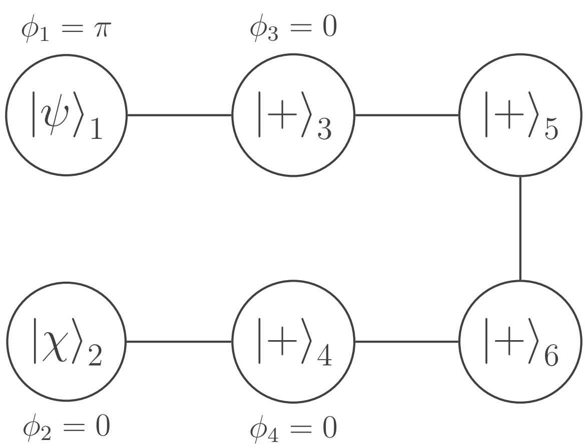
Problem 3

Find a measurement pattern that implements the following quantum circuit. You need to give the graph and default measurement angles that implement the said circuit, while you can ignore the "corrections".



Hint: You need the second qubit, while the first qubit implements the gate Z, to do nothing, i.e. implement the identity gate I.

Solution: For the above circuit, we have the following measurement pattern:

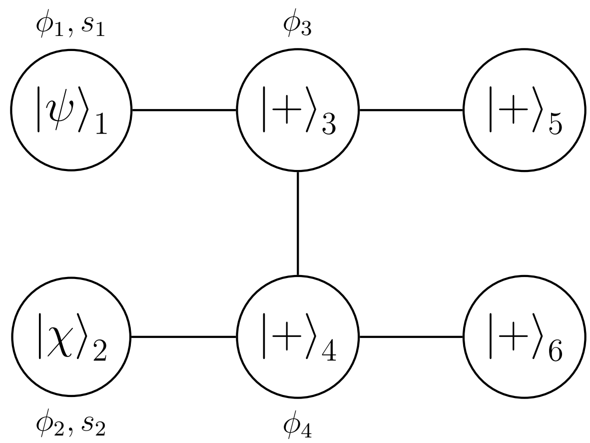


With the output:

$$\wedge Z_{56}[HR(0)HR(\pi)\,|\psi\rangle_5 \otimes HR(0)HR(0)\,|\chi\rangle_6] = \wedge Z_{56}[Z\,|\psi\rangle_5 \otimes \mathbb{1}\,|\chi\rangle_6] = (\wedge Z)_{56}(Z \otimes \mathbb{1})_{56}\,|\psi\rangle_5 \otimes |\chi\rangle_6$$

Problem 4

Consider the following MBQC graph state. Assume that the input is the following product state $|\psi\rangle_1 |\chi\rangle_2$, and that the outputs are the qubit 5 and the qubit 6. The flow of the measurement pattern is the standard one, goes horizontally from left to right, i.e. $f(i) = i + 2$. The measurement pattern is defined with the following "default" measurement angles: $\phi_1 = 0, \phi_2 = \pi, \phi_3 = -\pi/4$ and $\phi_4 = 0$.



a. What two-qubit unitary does the above measurement pattern implement?

Solution: Measuring qubits 1 and 2 in the above angles ($\phi_1 = 0, \phi_2 = \pi$), we apply the gate $H \otimes (HZ)$. Next, the $\wedge Z$ gate is applied, i.e. we now have the gate $(\wedge Z)(H \otimes (HZ))$. Finally, we measure qubits 3 and 4 that results in applying $HR(\pi/4) \otimes H$, leading to the overall unitary that we computed:

$$(HR(\pi/4) \otimes H)(\wedge Z)(H \otimes (HZ))$$

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b. Assume that we perform the above measurement pattern, and in the corresponding measurements we first get the outcomes: $s_1 = 1$, $s_2 = 1$.

Find the sets of vertices $S_z(3)$ and $S_x(3)$ of Z and X corrections for qubit 3.

Find the corrected measurement angle ϕ'_3 that the third qubit should be measured.

Solution: $S_x(3) = \{1\}$, we see that $f(1) = 3$ so this is where the X-corrections come from. $S_z(3) = \{2\}$, we look all neighbours of qubit 3, and these are: qubit 1, qubit 4 and qubit 5. Now the "past" of qubit 5 is qubit 3 itself (so it doesn't count), qubit 1 has nothing to its past, so we are left with qubit 4 that has qubit 2 at its past. This means that the only Z-correction to qubit 3, comes from qubit 2.

The corrected angle is:

$$\phi_3' = (-1)^{s_1}\phi_3 + \pi s_2 = -(-\pi/4) + \pi = 5\pi/4$$
