Foundations of Data Science: Estimation - Confidence intervals for the mean for small samples
Small samples

\[ n \leq 40 \]
\[ n = 32 \quad \text{female squirrels} \]
\[ \bar{x} = 341.0 \text{ g} \ , \ 0.9 \bar{x} = 3.9 \text{ g} \]
\[ \hat{\mu} = \bar{x} \]

\[ t-\text{statistic} \quad T = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \]
The \( t \)-distribution

\[
T = \frac{\hat{x} - \mu}{\sigma / \sqrt{n}}
\]

t-critical value \( t_{\alpha, \nu} \): value of \( t \) at which, in a \( t \)-distribution with \( \nu \) d.f, there is area under the curve of \( \alpha \) to its right.
Using the $t$-distribution for a confidence interval

$95\% \text{ C.I. } \Rightarrow \alpha = 0.05$

Sample size $n \Rightarrow \nu = n-1$ d.f.

$t_{\alpha/2, \nu} = t_{\alpha/2, n-1}$ t-critical value

$$\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$n=32, \alpha = 0.05 \Rightarrow t_{0.025, 31} = t_{0.025, 31} = 2.040$

$$t_{0.025, 31} \frac{s}{\sqrt{n}} = 2.040 \times 3.9 = 7.956$$

$$\Rightarrow \hat{\mu} = 341.0 \pm 8.0 \text{g } (95\% \text{ C.I.})$$