A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute.

Benjamin Pierce Types and Programming Languages
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That’s what we call **Types**

We operate on the AST

Detecting of Semantic Errors
ChocoPy Type System

- ChocoPy is a *statically typed* language
  - We verify that programs are well-typed by analyzing the program’s syntax without executing it

- ChocoPy is a *strongly typed* language
  - Programs with type errors are rejected and there are no implicit type conversions

- ChocoPy has *subtyping*
  - Types form a hierarchy and a value of a subtype can safely be used in a context where a value of the supertype is expected
Types of ChocoPy

- The grammar of ChocoPy contains the syntax of the following types:
  - int - representing integer values
  - bool - representing the two values True and False
  - str - representing strings
  - object - the top type, i.e., every value has this type
  - [T] - representing a list with elements of type T, where T is itself a type

- ChocoPy defines three more types that cannot be written by the user:
  - <Empty> - representing an empty list
  - <None> - representing the value None
  - ⊥ - the bottom type, i.e., the type that has no value
The types form a hierarchy:

- The type hierarchy is precisely defining by a subtyping relationship ($\leq$), where:
  - $T \leq T$ for all types $T$
  - $T \leq \text{object}$ for all types $T$
  - $\bot \leq T$ for all types $T$
  - If none of the three cases above apply, then the types are not related by subtyping, for example: $\text{[int]}$ and $\text{[bool]}$ are not related by subtyping
Type Checking

- The type checking process verifies and enforces the type system.

- The type system is defined by a set of formal typing rules that describe under what conditions a syntactic construct is well-typed (“has a valid type”).

- To perform type checking, we process a syntactically well-formed program and apply the typing rules to check if we can justify that every definition, statement, and expression is well-typed.
Typing rules are inference rules

- A **typing rule** is a form of a logical inference rule

- We write a typing rule like this:

  \[
  0 \vdash e : T
  \]

- Each typing rule contains:
  - a **NAME**,  
  - zero, one, or multiple **premises** above the line,  
  - a **conclusion** below the line.

- The rule states, that if the premises are true, then the conclusion is true as well. In other words: to check the conclusion, we must check that all premises are true
Typing Judgement

- \( 0 \vdash e : T \) is a **typing judgement**, where the turnstile symbol (\(\vdash\)) separates the **typing environment** on the left from the proposition on the right.

- This judgement should be read as:

  "In the type environment 0 the expression e is well typed and has type T"

- Why do we need a typing environment?
- Why can’t we just say: “The expression e is well typed and has type T”? 
Typing Environment

- Consider: $\vdash x + 4 : \text{int}$ Is this a valid typing judgement?

- We cannot say without knowing the type of $x$!
  - If $x$ has type $\text{int}$, then the judgement seems valid
  - If $x$ has not the type $\text{int}$, then the judgement seems wrong

- The *typing environment* records the type of all variables and functions that are in scope when type checking a definition, statement, or expression

- Given a typing environment, we can always conclude if a typing judgement is valid: $\{x: \text{int}\} \vdash x + 4 : \text{int}$
Typing Environment of ChocoPy

- For type checking ChocoPy, we use a local environment $O$ that contains:
  - The types of all variables in scope
    We write $O(v) = T$ to indicate that variable $v$ is in the local environment and has type $T$
  - Information about all functions in scope
    We write $O(f) = \{T_1 \times \ldots \times T_m \rightarrow T_0; x_1, \ldots, x_m; v_1: T'_1, \ldots, v_m: T'_m\}$ to indicate that function $f$ is in the local environment and
    - has a function type with
      - $T_1, \ldots, T_m$ the types of the function parameters
      - $T_0$ the function return type
    - has function parameters with names $x_1, \ldots, x_m$
    - has identifiers and types $v_1: T'_1, \ldots, v_m: T'_m$ of variables declared in the body of $f$
  - We also record the return type $R$ of the current function in the environment
First ChocoPy Typing Rules

\[ O, R \vdash False : \text{bool} \]
First ChocoPy Typing Rules

\[ (0, R \vdash \text{False} : \text{bool}) \]

“\text{\textit{There is no premise that must be true, so we can directly conclude that in the type environment 0 and R the expression False is well typed and has type bool}}”
First ChocoPy Typing Rules

O, R ⊢ False : bool

“There is no premise that must be true, so we can directly conclude that in the type environment O and R the expression False is well typed and has type bool”

O, R ⊢ True : bool
First ChocoPy Typing Rules

- \[0, R \vdash False : bool\]  
  "There is no premise that must be true, so we can directly conclude that in the type environment 0 and R the expression False is well typed and has type bool”

- \[0, R \vdash True : bool\]  
  "There is no premise that must be true, so we can directly conclude that in the type environment 0 and R the expression True is well typed and has type bool”
First ChocoPy Typing Rules

\[
\begin{align*}
0, R ⊢ e_1 : \text{bool} \\
0, R ⊢ e_2 : \text{bool}
\end{align*}
\]

\[\text{[AND]}\]

\[
0, R ⊢ e_1 \text{ and } e_2 : \text{bool}
\]

“If \( e_1 \) has type \textit{bool} in the type environment 0 and R, and \( e_2 \) has type \textit{bool} in the same type environment 0 and R, then we can conclude that in the same type environment 0 and R the expression \( e_1 \text{ and } e_2 \) is well typed and has type \textit{bool}.”
Example of Type Checking

\[ O, R \vdash \text{False} : \text{bool} \]
\[ O, R \vdash \text{True} : \text{bool} \]
\[ O, R \vdash e_1 : \text{bool} \]
\[ O, R \vdash e_2 : \text{bool} \]
\[ O, R \vdash e_1 \text{ and } e_2 : \text{bool} \]

\[ O, R \vdash \text{False and (True and False)} : \text{bool} \]
Example of Type Checking

\[
\begin{align*}
O, R & \vdash \text{False} : \text{bool} \\
O, R & \vdash \text{True} : \text{bool} \\
O, R & \vdash e_1 : \text{bool} \\
O, R & \vdash e_2 : \text{bool} \\
\hline
\text{AND} \\
O, R & \vdash \text{False and (True and False)} : \text{bool}
\end{align*}
\]
Example of Type Checking

\[
\begin{align*}
O, R & \vdash \text{False} : \text{bool} \\
O, R & \vdash \text{True} : \text{bool} \\
O, R & \vdash e_1 : \text{bool} \\
O, R & \vdash e_2 : \text{bool} \\
\hline
O, R & \vdash e_1 \text{ and } e_2 : \text{bool}
\end{align*}
\]
Example of Type Checking

0, R ⊢ False : bool

0, R ⊢ True and False : bool

0, R ⊢ False and (True and False) : bool
Example of Type Checking

\[ O, R \vdash \text{False} : \text{bool} \]

\[ O, R \vdash \text{True} : \text{bool} \]

\[ O, R \vdash e_1 : \text{bool} \]

\[ O, R \vdash e_2 : \text{bool} \]

\[ O, R \vdash e_1 \text{ and } e_2 : \text{bool} \]

\[ O, R \vdash \text{False} : \text{bool} \]

\[ O, R \vdash \text{True} \text{ and False} : \text{bool} \]

\[ O, R \vdash \text{False} \text{ and (True and False)} : \text{bool} \]
Example of Type Checking

\[ O, R \vdash False : \text{bool} \]
\[ O, R \vdash True : \text{bool} \]
\[ O, R \vdash e_1 : \text{bool} \]
\[ O, R \vdash e_2 : \text{bool} \]

\[ O, R \vdash e_1 \text{ and } e_2 : \text{bool} \]
Example of Type Checking

\(\text{O, R} \vdash \text{False : bool}\)

\(\text{O, R} \vdash \text{True : bool}\)

\(\text{O, R} \vdash \text{e}_1 : \text{bool}\)

\(\text{O, R} \vdash \text{e}_2 : \text{bool}\)

\(\text{O, R} \vdash \text{e}_1 \text{ and e}_2 : \text{bool}\)

\(\text{O, R} \vdash \text{e}_1 \text{ and False : bool}\)

\(\text{O, R} \vdash \text{True and False : bool}\)

\(\text{O, R} \vdash \text{False and (True and False) : bool}\)
Example of Type Checking

\[ \text{[BOOL-FALSE]} \]
\[
0, R \vdash \text{False : bool}
\]

\[ \text{[?] ]} \]
\[
0, R \vdash \text{True : bool}
\]

\[ \text{[?] ]} \]
\[
0, R \vdash \text{False : bool}
\]

\[ \text{[AND]} \]
\[
0, R \vdash \text{True and False : bool}
\]

\[ \text{[AND]} \]
\[
0, R \vdash \text{False and (True and False) : bool}
\]
Example of Type Checking

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\text{Example of Type Checking}
\]

\[
0, R \vdash \text{False} : \text{bool}
\]

\[
0, R \vdash \text{True} : \text{bool}
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0, R \vdash \text{False} : \text{bool}
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\[
0, R \vdash \text{True} : \text{bool}
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\[
0, R \vdash \text{True and False} : \text{bool}
\]

\[
0, R \vdash \text{False and (True and False)} : \text{bool}
\]

\[
0, R \vdash \text{e}_1 : \text{bool}
\]

\[
0, R \vdash \text{e}_2 : \text{bool}
\]

\[
0, R \vdash \text{e}_1 \text{ and e}_2 : \text{bool}
\]
Example of Type Checking

\[\begin{align*}
0, R &\vdash \text{False} : \text{bool} \\
0, R &\vdash \text{True} : \text{bool} \\
0, R &\vdash \text{e}_1 : \text{bool} \\
0, R &\vdash \text{e}_2 : \text{bool} \\
0, R &\vdash \text{e}_1 \text{ and } \text{e}_2 : \text{bool}
\end{align*}\]