# Compiling Techniques

## Lecture 14: Building SSA Form

# Reminder: Static Single-Assignment (SSA) Form

*"A program is defined to be in* ***Static Single-Assignment (SSA)*** *form if each variable is a target of exactly one assignment statement in the program text."*

- Each assignment statement defines a unique name.
- Each use refers to a single name.

# Representing Control Flow

```
x = 0
if (a == 42)
    x = x + 1
else
    x = 3

y = x + 5
```

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$x_1 = 0$

$\texttt{if (a == 42)}$

$\quad x_2 = x_1 + 1$

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$y = x_? + 5$

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$\Longrightarrow$

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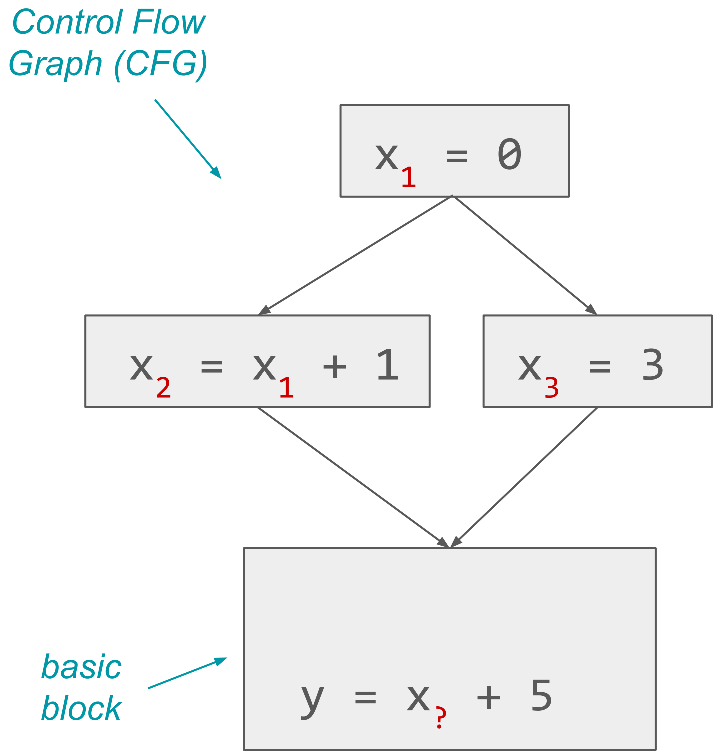

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⟹

$x_1 = 0$

$\texttt{if (a == 42)}$

$\quad x_2 = x_1 + 1$

$\texttt{else}$

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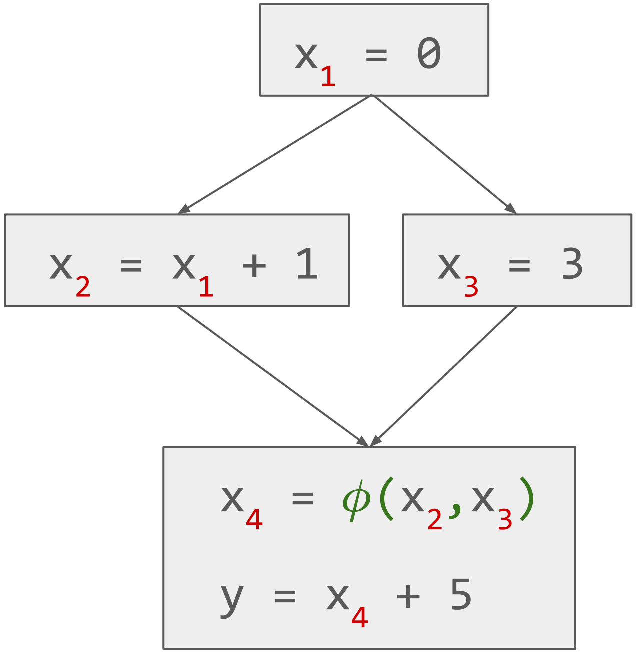
$x_4 = \phi(x_2, x_3)$

$y = x_4 + 5$

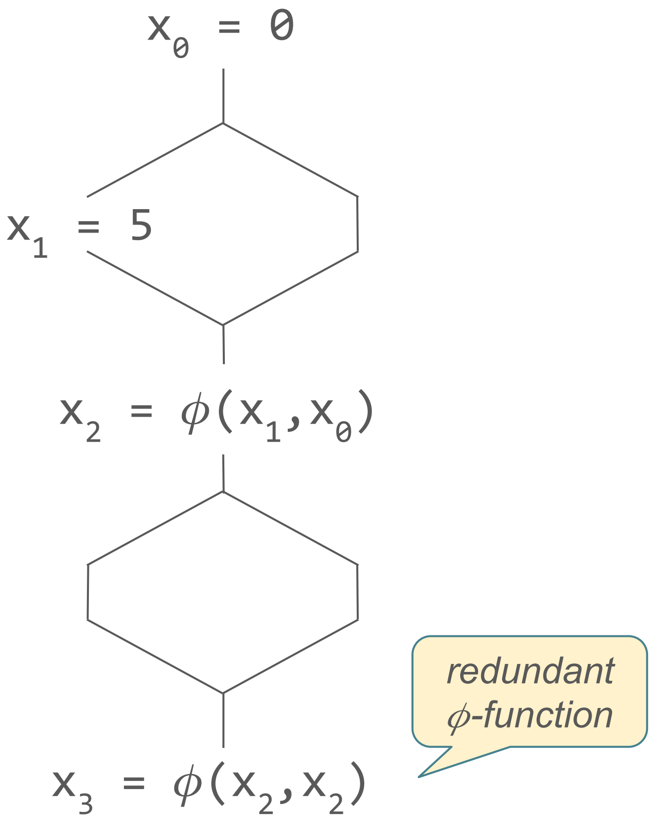


# $\phi$-function placement

**Naive approach:**

1. At each join point insert a $\phi$-function for every variable name
2. Rename adding unique subscripts

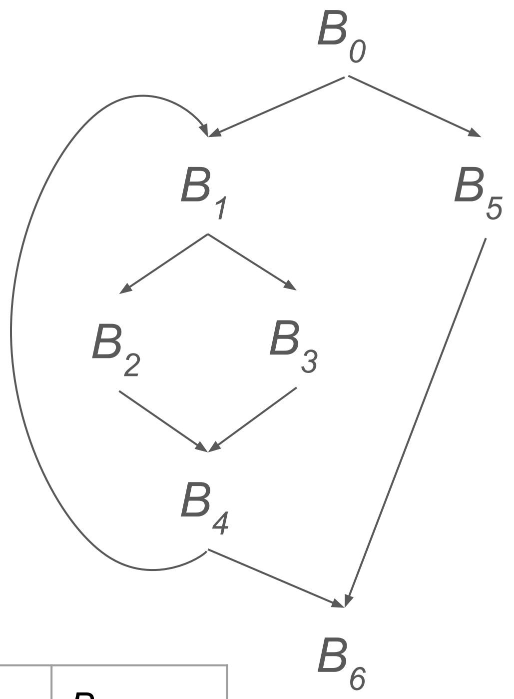
Computes *maximal SSA form*



# Dominators

*p* dominates *q* (*p* >> *q*, *p* *dom* *q*) iff
every path from the entry node $b_0$ to *q* also visits *p*.

*Dom(q)* – set of nodes that dominate *q*.




| **B** | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|---|---|---|---|---|---|---|---|
| ***DOM(B)*** | $B_0$ | | | | | | |

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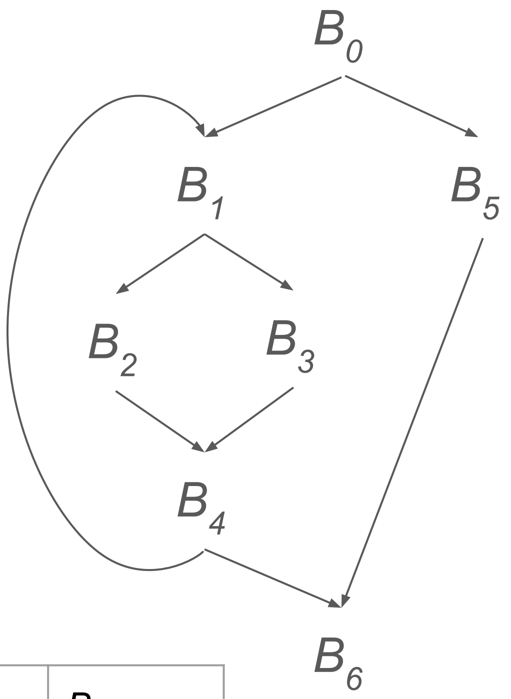


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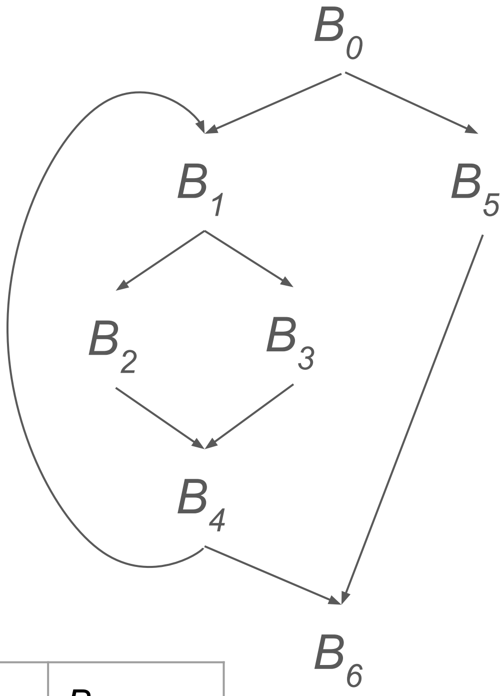


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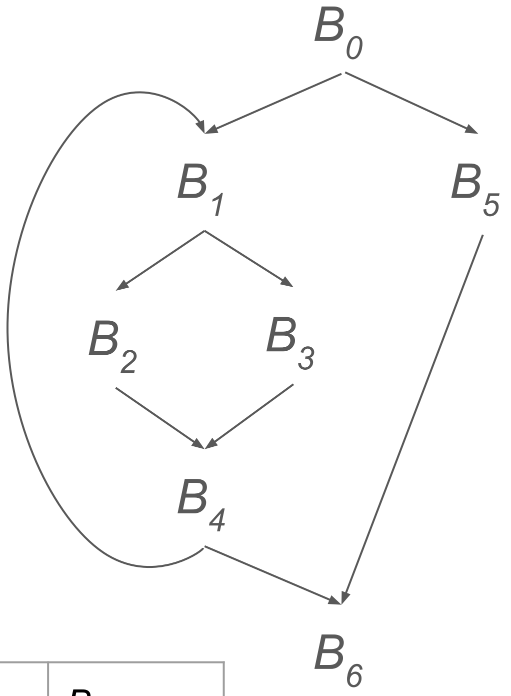


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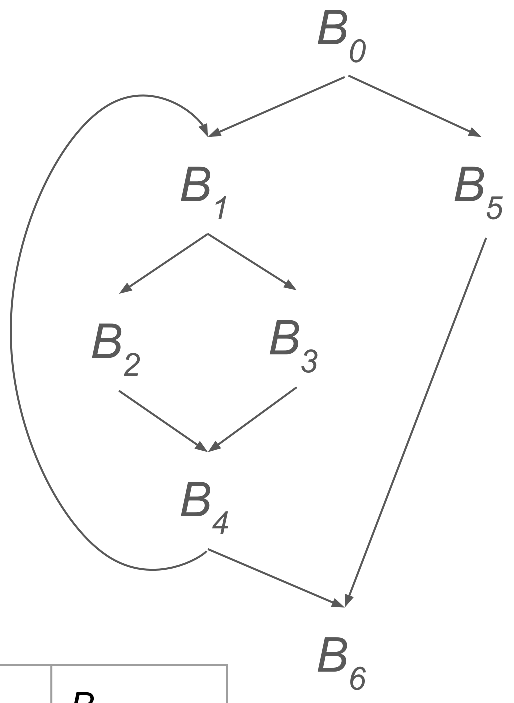


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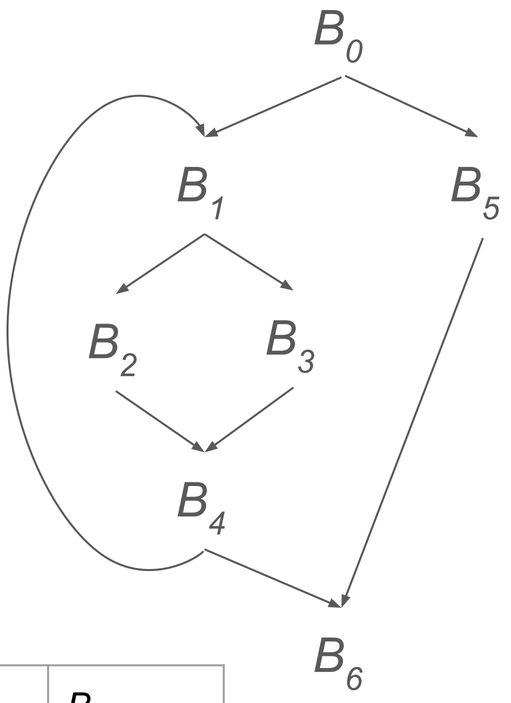


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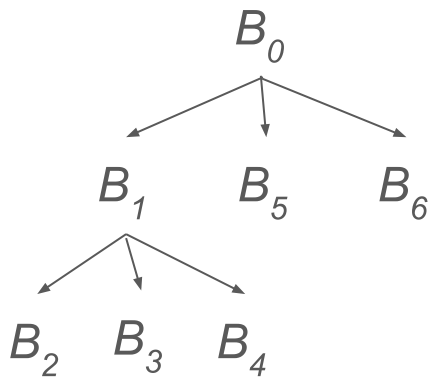
# *dom* Relation

- reflexive

  $a \text{ dom } a$
- antisymmetric

  $a \text{ dom } b \land b \text{ dom } a \Rightarrow a = b$
- transitive

  $a \text{ dom } b \land b \text{ dom } c \Rightarrow a \text{ dom } c$



# Dominance frontier

*p* *strictly dominates* *q* iff

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*q* is in *dominance frontier* of *p* iff

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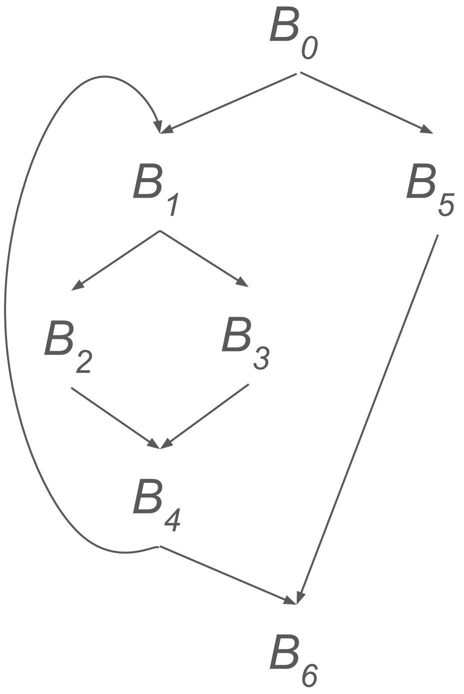


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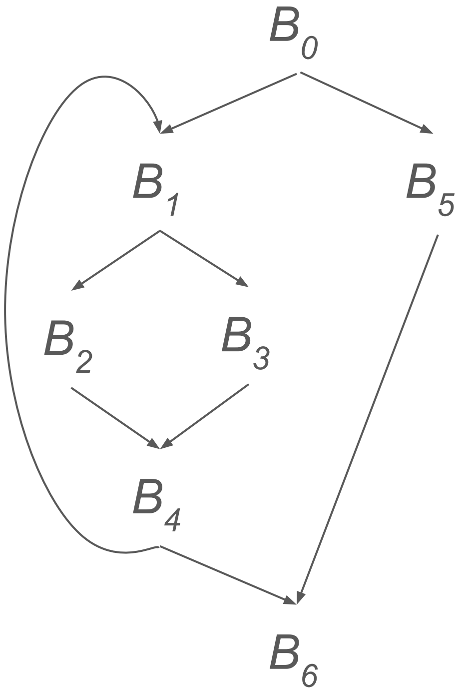


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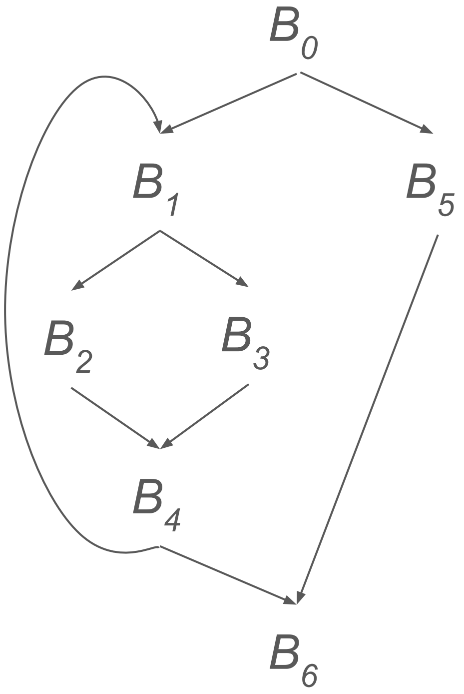


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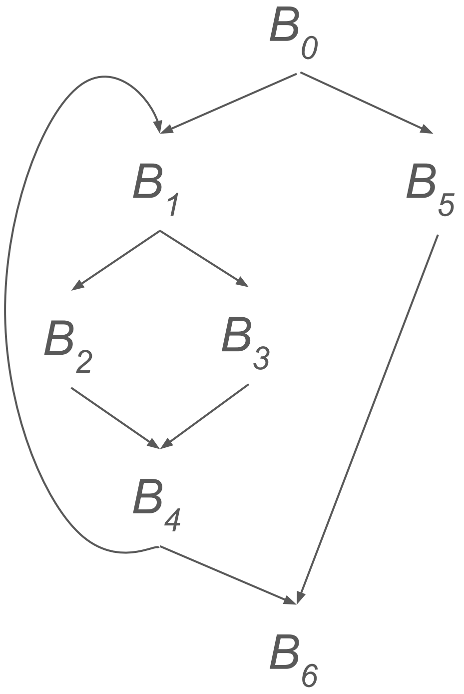


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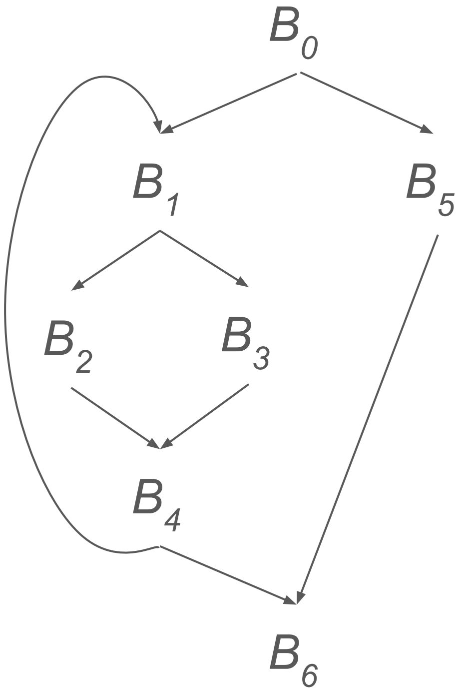


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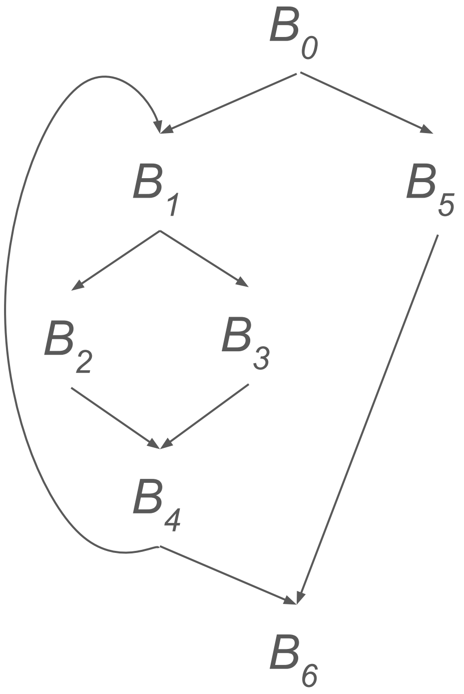


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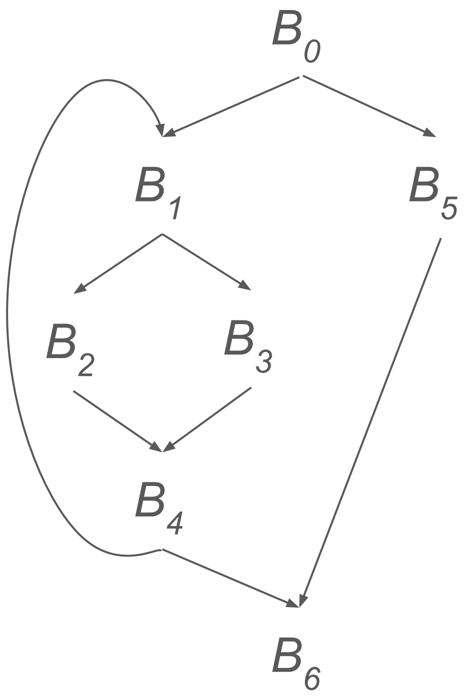


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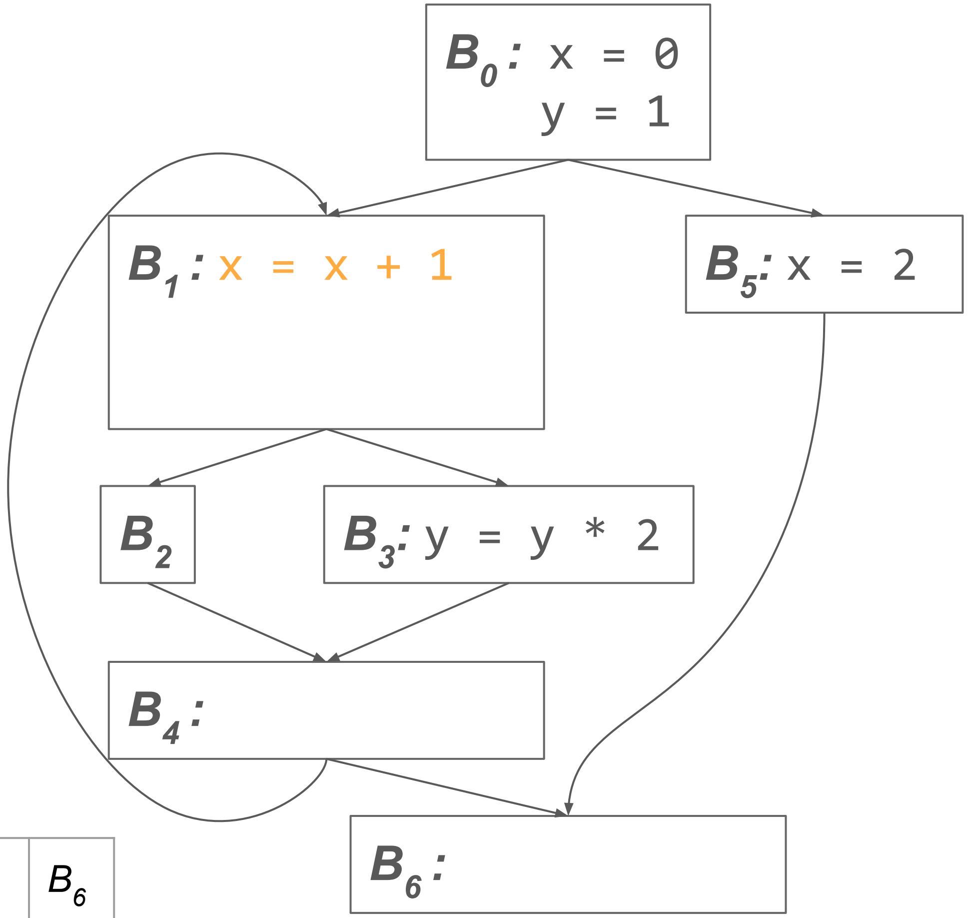
# Minimal SSA

**Idea**

an assignment to x in the node *B* introduces a $\phi$-function in every node from DF(B)

1. **$\phi$-function placement**
2. renaming

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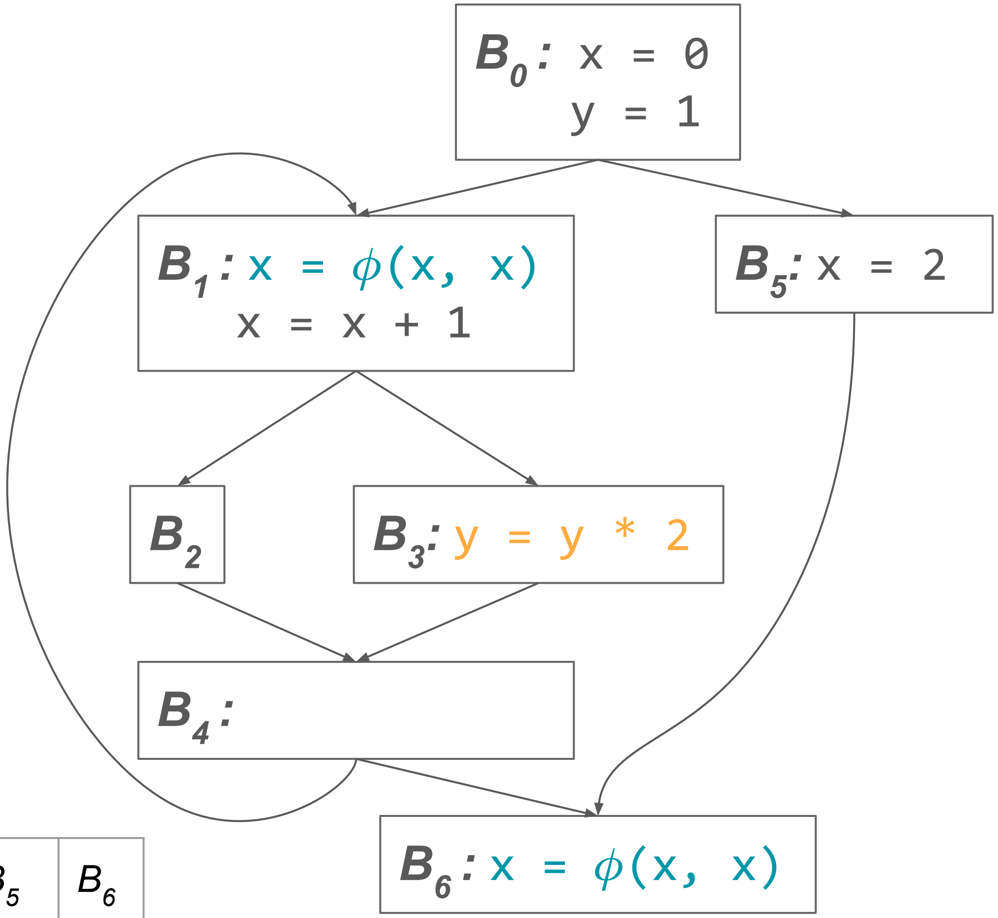


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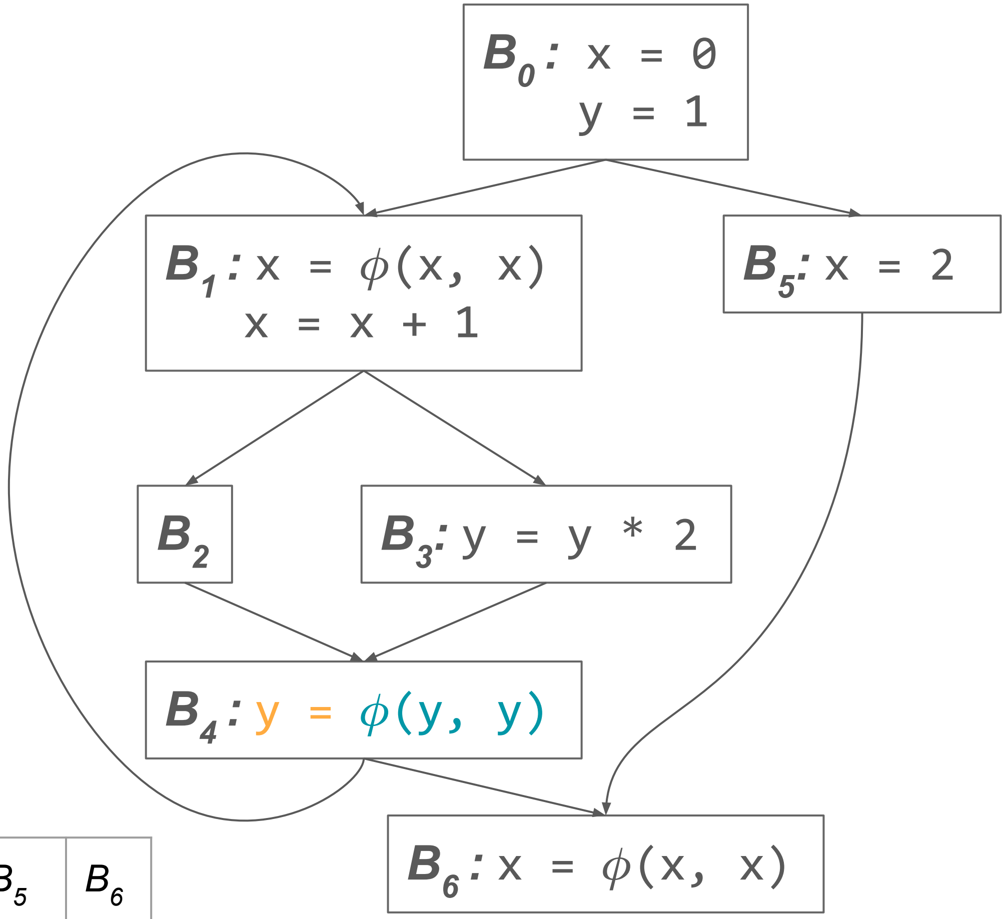


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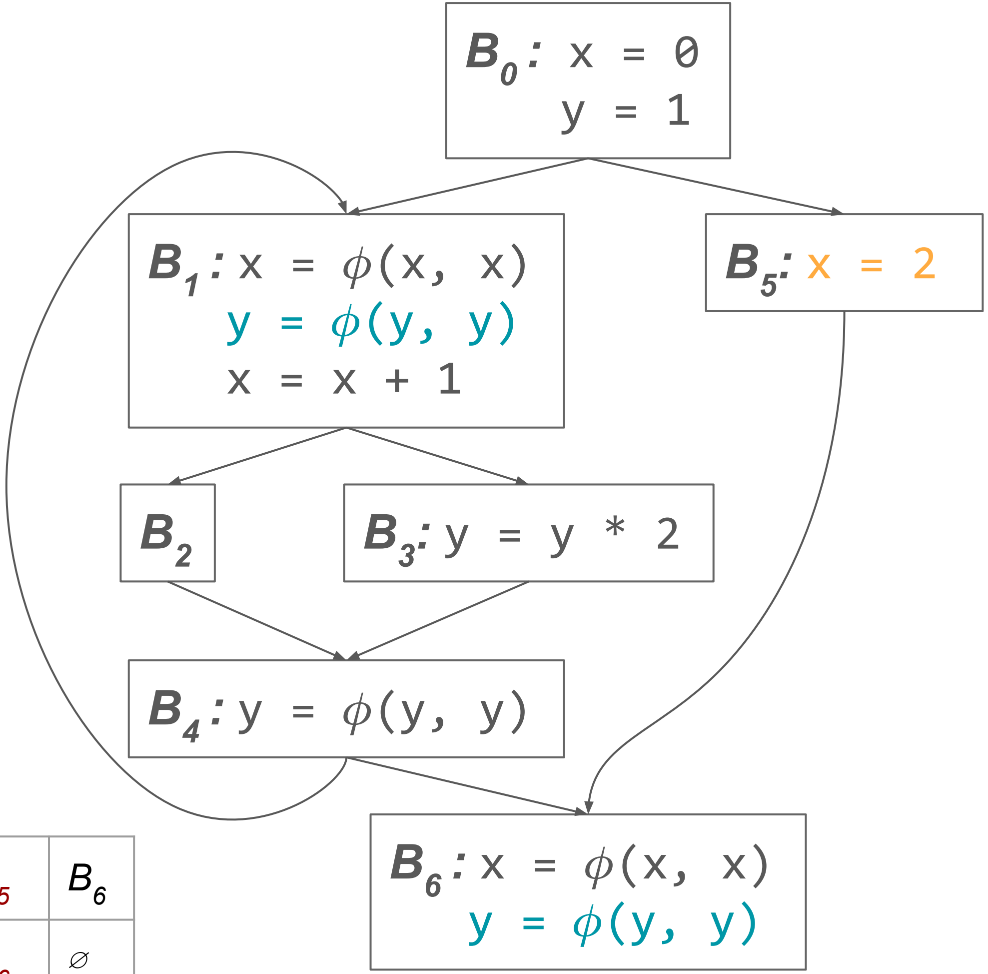


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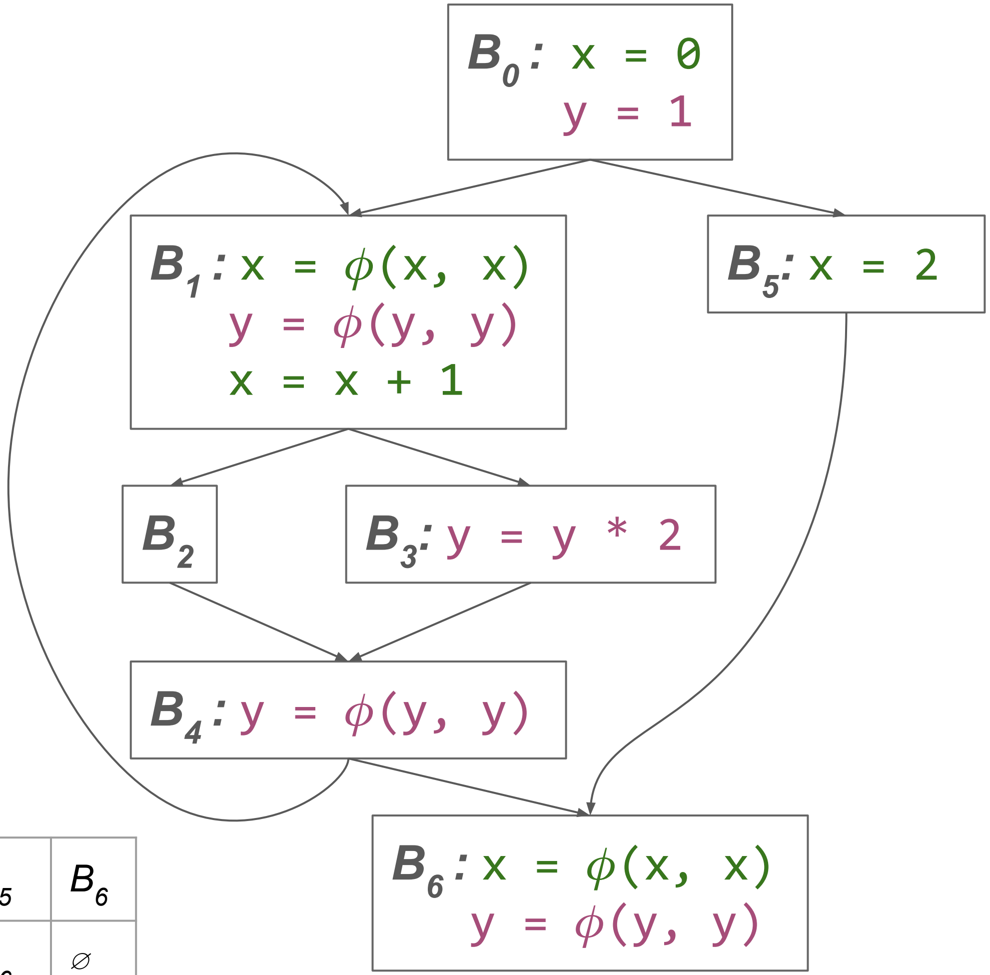


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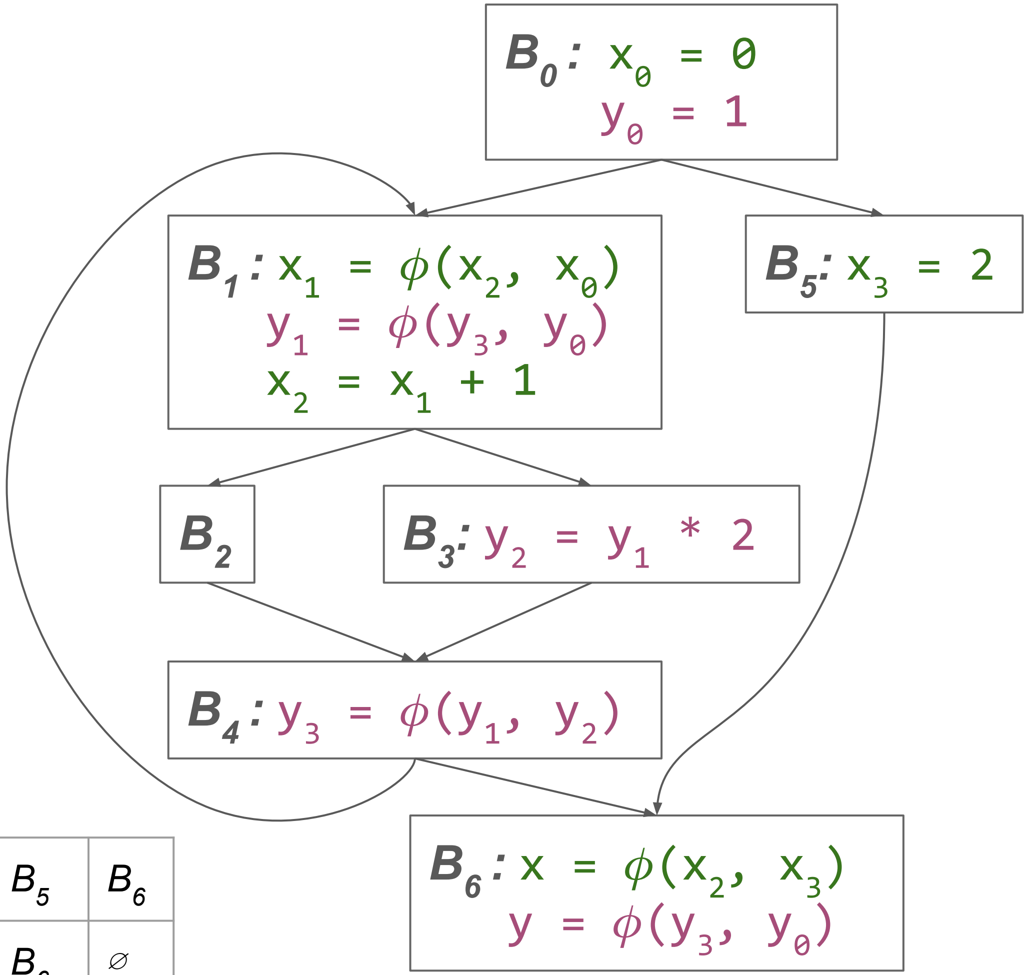


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# SSA forms

- **Maximal SSA**
  Introduce a $\phi$-function at every join node for every variable

- **Minimal SSA**
  Introduce a $\phi$-function at every join node for every variable where two distinct definitions of the same name meet

- **Pruned SSA**
  Same as minimal SSA, but don’t insert $\phi$-functions if its result is not *live*.

- **Semipruned SSA**
  Same as minimal SSA, but don’t insert $\phi$-functions for names that are not live across a block boundary

# Block Arguments

Instead of using $\phi$-nodes (like LLVM), xDSL and MLIR use block arguments to represent control flow – dependent values.

```
func.func @simple(i64, i1) -> i64 {
^bb0(%a: i64, %cond: i1): // Code dominated by ^bb0 may refer to %a
  cf.cond_br %cond, ^bb1, ^bb2

^bb1:
  cf.br ^bb3(%a: i64)    // Branch passes %a as the argument

^bb2:
  %b = arith.addi %a, %a : i64
  cf.br ^bb3(%b: i64)    // Branch passes %b as the argument

// ^bb3 receives an argument, named %c, from predecessors
// and passes it on to bb4 along with %a. %a is referenced
// directly from its defining operation and is not passed through
// an argument of ^bb3.
^bb3(%c: i64):
  cf.br ^bb4(%c, %a : i64, i64)

^bb4(%d : i64, %e : i64):
  %0 = arith.addi %d, %e : i64
  return %0 : i64   // Return is also a terminator.
}
```

https://mlir.llvm.org/docs/LangRef/#blocks