ADS Tutorial 5

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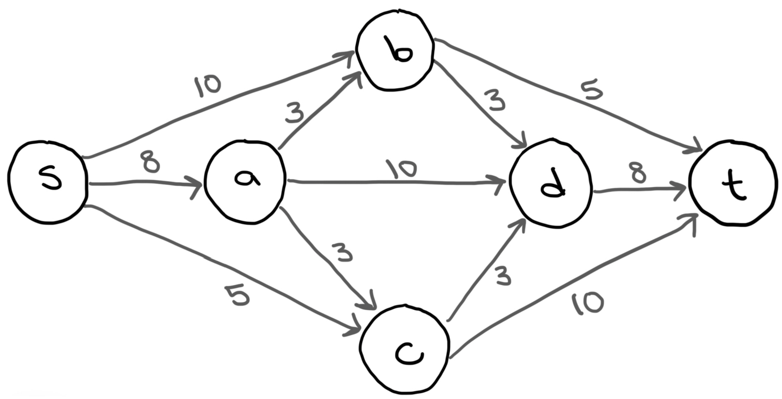
Problem 1

Consider the following flow network, with edge capacities as indicated in the figure.



Apply the Ford-Fulkerson algorithm to this flow network to compute the value of the maximum flow $f_{\max}$ from s to t. In every step of the algorithm draw the residual graph, and indicate the value of the current flow, the augmenting path chosen, and the bottleneck capacity of that path.

In the flow network above, indicate a minimum cut (as two sets of nodes S and T with $s \in S$ and $t \in T$). Provide an argument for why the cut you have indicated is minimal.

Problem 2

The final year project coordinator at a major university in Scotland would like to assign final year projects to the students. There is a set S of students, and a set P of projects which have been proposed by the supervisors, and each project j has a certain capacity q_j, meaning that it can be assigned to at most q_j students. To make the task of eliciting preferences from the students easier, the project coordinator has only asked the students to rank each project as either "acceptable" or "unacceptable". In the end, each student should be assigned to at most one project, and a student cannot be assigned to a project which is unacceptable to them. For each student that is left unassigned, the project coordinator will lose one night of sleep.

Design a polynomial-time algorithm for minimising the nights of sleep that the project coordinator will need to lose. To do that, reduce the problem described above to the problem of finding a maximal flow in a flow network. Provide an argument for the correctness of your reduction.

Problem 3

The timetabling team at a major university in Scotland would like to assign the current cohort of students to courses for the duration of their studies. The courses exhibit a pre-requisite relation, i.e., there are courses on which students can be assigned only if they have been assigned to some other courses that are prerequisites. One way to think of this is in terms of a graph, in which the nodes corresponds to the courses and a directed edge (i, j) denotes that course i is a prerequisite for course j. Luckily, the degree programme is well-designed, and thus the prerequisite relation graph is acyclic. Each course i has a capacity c_i on the number of students that can be assigned to it. To simplify the assignment process, the timetabling team has decided to do the assignment without allowing students to select courses. A student will not be assigned to any courses if it is not feasible for them to get a degree.

Design a polynomial-time algorithm for maximising the number of students that can eventually get a degree. To do that, reduce the problem described above to the problem of finding a maximal flow in a flow network. Provide an argument for the correctness of your reduction.