Open Pit Mining

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Consider the *Open pit mining* problem: There is a set of blocks to be mined, each with a cost c_i and a payoff p_i and in order to mine two blocks i and i', it is required to first mine the block j directly above them. The goal is to find a set S of blocks to mine in order to maximise the profit $\sum_{i \in S}(p_i - c_i)$.

Formulate the problem as a maximum flow problem and explain how to use a solution to the maximum flow problem in order to obtain a solution to the open pit mining problem.

Solution

First, we add a source s and a sink t and a vertex i for every block to be mined. Next, for any vertex i, if $p_i - c_i > 0$, then we add a directed edge (s, i) with capacity $p_i - c_i$. Likewise, for any vertex i, if $p_i - c_i \leq 0$, then we add a directed edge (i, t) with capacity $c_i - p_i$. Finally, for every two blocks i, j, such that block i is required in order to mine block j, we add a directed edge (i, j) to the network with capacity ∞. We will prove that a minimum (s-t) cut in this network will give us the optimal set of blocks to mine, in order to maximise the profit; these will be the blocks corresponding to vertices in $S - \{s\}$. With that established, we can run a flow network algorithm on our designed network and find the minimum (s-t) cut.

Let (S, T) be an (s-t) cut in the network. For (S, T) to be minimum, there can not be an edge of infinite capacity *crossing the cut* (i.e., going from an edge of S to an edge of T or vice-versa), as otherwise the capacity would be infinity. This means that all the blocks in the set $S - \{s\}$ that we will mine will statisfy the preequisite condition, meaning that if we mine a block, we will also mine every block that is required for that block to be mined.

Now, consider the capacity of the cut (S, T). We have:

$$
\begin{aligned}
c(S,T) &= \sum_{i \in T:(p_i - c_i) > 0} (p_i - c_i) + \sum_{i \in S:(p_i - c_i) \leq 0} (c_i - p_i) \\
&= \sum_{i \in T:(p_i - c_i) > 0} (p_i - c_i) - \sum_{i \in S:(p_i - c_i) \leq 0} (p_i - c_i) \\
&= \sum_{i \in T:(p_i - c_i) > 0} (p_i - c_i) + \sum_{i \in S:(p_i - c_i) > 0} (p_i - c_i) - \sum_{i \in S:(p_i - c_i) \leq 0} (p_i - c_i) - \sum_{i \in S:(p_i - c_i) > 0} (p_i - c_i) \\
&= \sum_{i \in V:(p_i - c_i) > 0} (c_i - p_i) - \sum_{i \in S} (p_i - c_i)
\end{aligned}
$$

Looking at the right-hand side of the last equation, we observe that the first sum does not depend on the cut (S, T) and is therefore a constant. The capacity of the cut is minimised when the quantity $\sum_{i \in S}(p_i - c_i)$ is maximised, and this is precisely the mining profit. Therefore, the maximum miniming profit is achieved at the minimum cut.