ADS Tutorial 8

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November 20, 2024

Problem 1

Consider the Matrix Chain Multiplication problem in which we wish to compute the product $A_1 \cdot A_2 \cdot A_3 \cdot A_4$ where A_1, A_2, A_3, A_4 are rectangular matrices with dimensions $5 \times 10, 10 \times 5, 5 \times 3, 3 \times 9$ respectively. Assume that the time required to multiply two matrices of dimensions $p \times q$ and $q \times r$ is pqr.

Apply the dynamic programming algorithm Matrix-Chain-Order to compute the optimal parenthesization on this input. Show the table M (that contains the costs of the optimal solutions to subproblems) and the table S (that contains the optimal splits) at the end of the execution of the algorithm. Your solution should include writing the recurrence relation for computing $M[i, j]$ for $i < j$.

Problem 2

A contiguous subsequence of length k a sequence S is a subsequence which consists of k consecutive elements of S. For instance, if S is $1, 2, 3, -11, 10, 6, -10, 11, -5$, then $3, -11, 10$ is a contiguous subsequence of S of length 3. Give an algorithm based on dynamic programming that, given a sequence S of n numbers as input, runs in linear time and outputs the contiguous subsequence of S of maximum sum. Assume that a subsequence of length 0 has sum 0. For the example above, the answer of the algorithm would be $10, 6. - 10, 11$ with a sum of 17.