Compiling Techniques

Lecture 10: Type Analysis

Types and Type System

A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute.

Benjamin Pierce Types and Programming Languages

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We operate on the AST

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Benjamin Pierce Types and Programming Languages

That's what we call ***Types***

ChocoPy Type System

- ChocoPy is a *statically typed* language
  - We verify that programs are well-typed by analyzing the program's syntax without executing it
- ChocoPy is a *strongly typed* language
  - Programs with type errors are rejected and there are no implicit type conversions
- ChocoPy has *subtyping*
  - Types form a hierarchy and a value of a subtype can safely be used in a context where a value of the supertype is expected

Types of ChocoPy

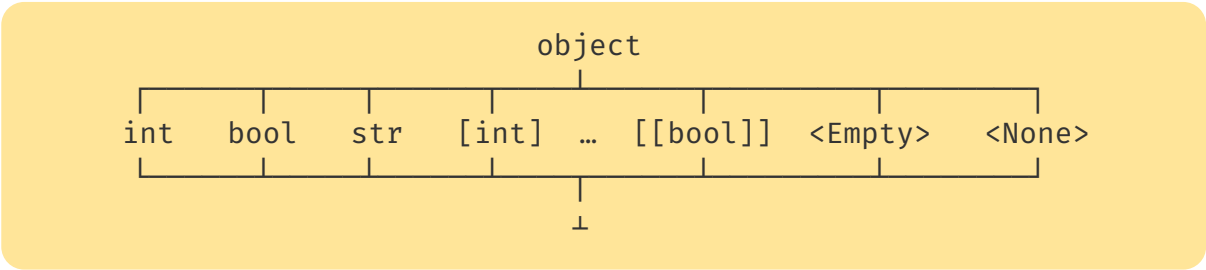
```
…
type := type_name | `[` type `]`
type_name := int | bool | str | object
…
```

- The grammar of ChocoPy contains the syntax of the following types:
  - int - representing integer values
  - `bool` - representing the two values `True` and `False`
  - `str` - representing strings
  - `object` - the *top type*, i.e., every value has this type
  - [T] - representing a list with elements of type T, where T is itself a type

- ChocoPy defines three more types that cannot be written by the user:
  - `<Empty>` - representing an empty list
  - `<None>` - representing the value `None`
  - ⊥ - the *bottom type*, i.e., the type that has no value

Type Hierarchy of ChocoPy

- The types form a hierarchy:



- The type hierarchy is precisely defining by a *subtyping relationship* (≤), where:
  - `T ≤ T` for all types `T`
  - `T ≤ object` for all types `T`
  - `⊥ ≤ T` for all types `T`
  - If none of the three cases above apply, then the types are not related by subtyping, for example: `[int]` and `[bool]` are not related by subtyping

Type Checking

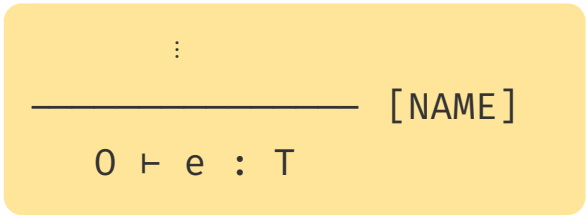
- The type checking process verifies and enforces the type system

- The type system is defined by a set of formal typing rules that describe under what conditions a syntactic construct is well-typed (*“has a valid type”*)

- To perform type checking, we process a syntactically well-formed program and apply the typing rules to check if we can justify that every definition, statement, and expression is well-typed

Typing rules are inference rules

- A typing rule is a form of a logical inference rule
- We write a typing rule like this:



- Each typing rule contains:
  - a `[NAME]`,
  - zero, one, or multiple *premises* above the line,
  - a *conclusion* below the line.

- The rule states, that if the premises are true, then the conclusion is true as well.
  In other words: to check the conclusion, we must check that all premises are true

Typing Judgement

- `O ⊢ e : T` is a *typing judgement*, where the turnstile symbol (⊢) separates the *typing environment* on the left from the proposition on the right.

- This judgement should be read as:

  "In the type environment `O` the expression `e` is well typed and has type `T`"

- Why do we need a typing environment?
- Why can't we just say: *"The expression `e` is well typed and has type `T`"?*

Typing Environment

- Consider: `⊢ x + 4 : int` Is this a valid typing judgement?

- We cannot say without knowing the type of `x`!
  - If `x` has type `int`, then the judgement seems valid
  - If `x` has not the type `int`, then the judgement seems wrong

- The *typing environment* records the type of all variables and functions that are in scope when type checking a definition, statement, or expression

- Given a typing environment, we can always conclude if a typing judgement is valid:

```
{x: int} ⊢ x + 4 : int
```

Typing Environment of ChocoPy

- For type checking ChocoPy, we use a *local environment* O that contains:
  - The types of all variables in scope
    We write `O(v) = T` to indicate that variable `v` is in the local environment and has type `T`
  - Information about all functions in scope
    We write $O(f) = \{T_1 \times \ldots \times T_n \rightarrow T_0; x_1, \ldots, x_n; v_1: T'_1, \ldots, v_m: T'_m\}$
    to indicate that function `f` is in the local environment and
    - has a function type with
      - $T_1, \ldots, T_n$ the types of the function parameters
      - T_0 the function return type
    - has function parameters with names $x_1, \ldots, x_n$
    - has identifiers and types $v_1: T'_1, \ldots, v_m: T'_m$ of variables declared in the body of `f`
- We also record the return type R of the current function in the environment

First ChocoPy Typing Rules

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First ChocoPy Typing Rules

$$\frac{O, R \vdash e_1 : \texttt{bool} \qquad O, R \vdash e_2 : \texttt{bool}}{O, R \vdash e_1 \texttt{ and } e_2 : \texttt{bool}} \text{ [AND]}$$

"If $\mathbf{e_1}$ has type **bool** in the type environment O and R, and
if $\mathbf{e_2}$ has type **bool** in the same type environment O and R,
then we can conclude that
in the same type environment O and R
the expression $\mathbf{e_1}$ **and** $\mathbf{e_2}$ *is well typed and has type* **bool***"*

Example of Type Checking

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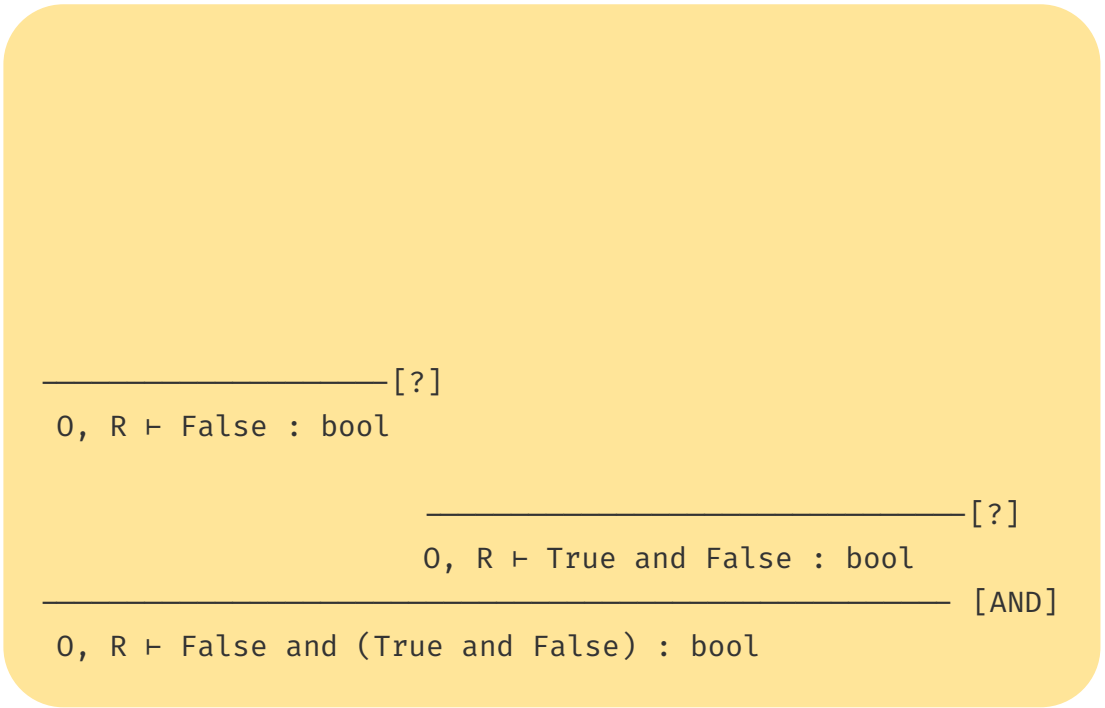

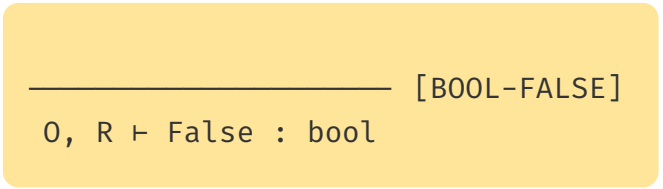

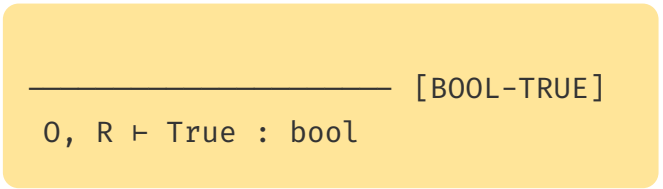

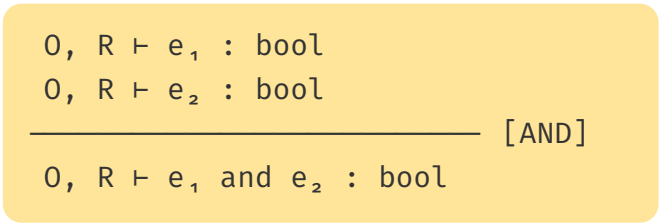

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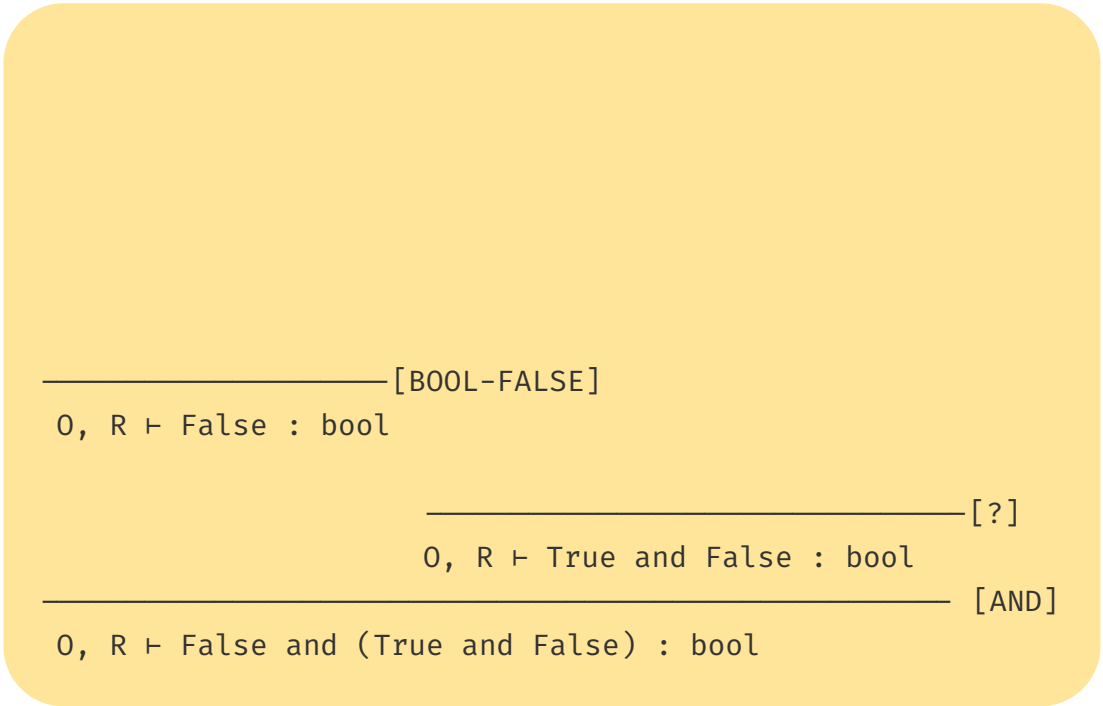

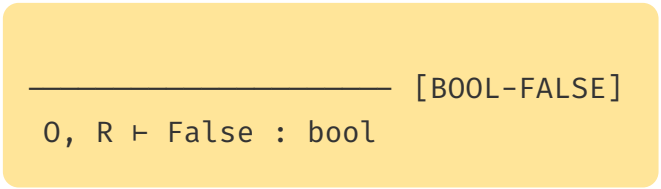

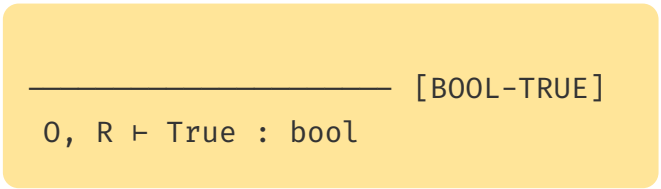

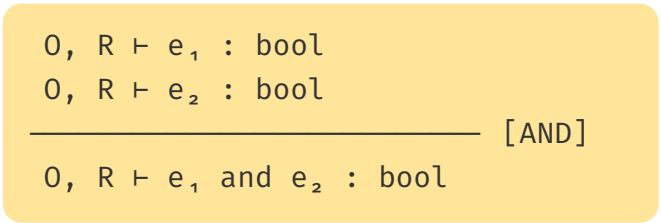

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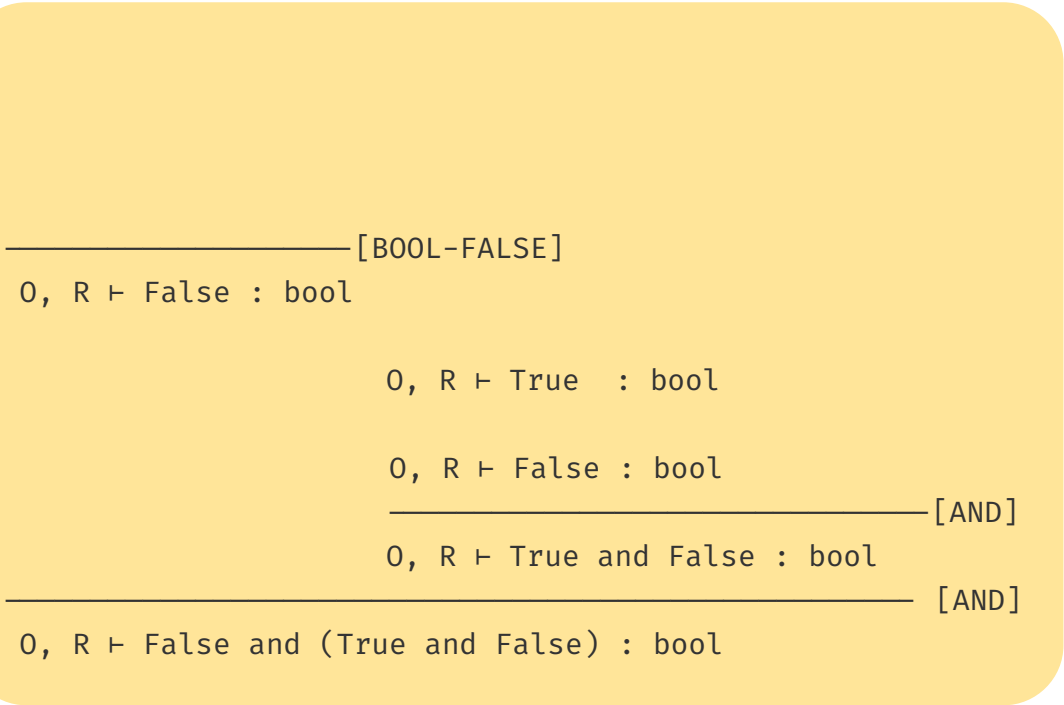

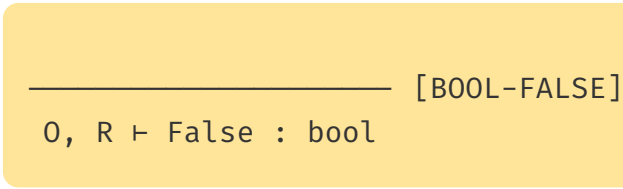

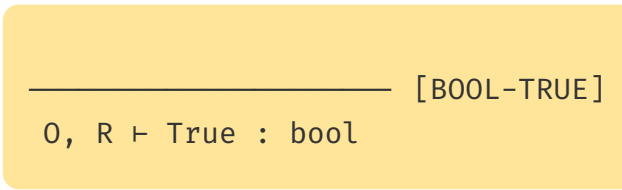

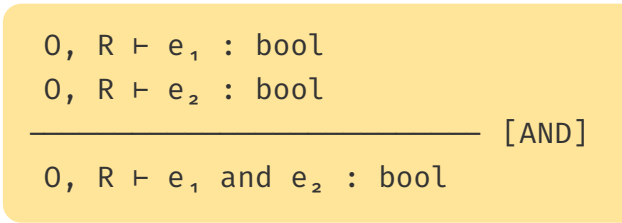

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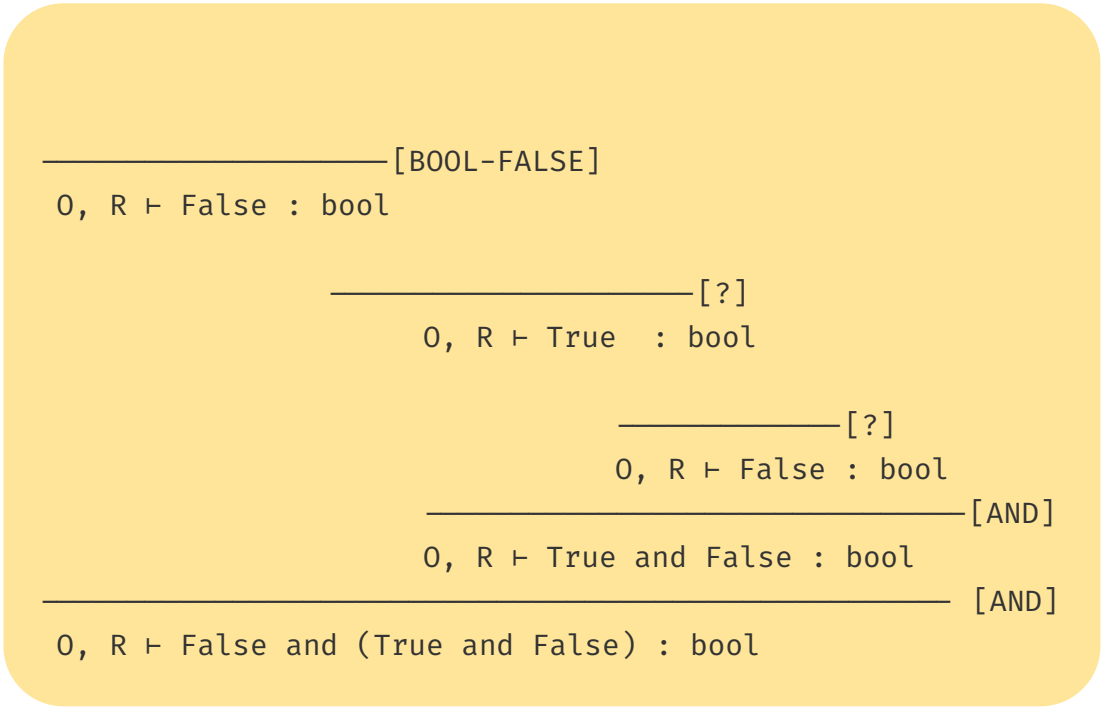

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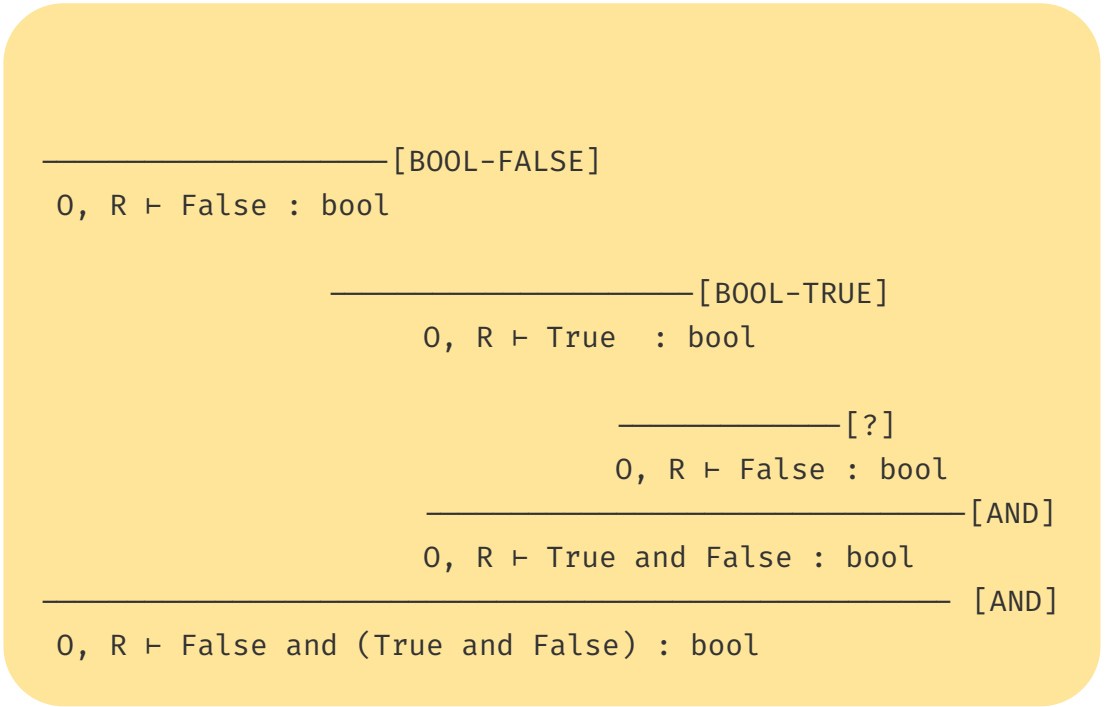

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