AGTA Tutorial 1

Lecturer: Aris Filos-Ratsikas Tutor: Charalampos Kokkalis

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Exercise 1 (Dominant and Dominated Strategies)**.** Consider the following simple game. Alice and Bob have ten different game theory books to divide between them and each one has a strict preference over them, i.e. they are not indifferent between any two books. They come up with the following solution. They will both state their complete preferences over the books and then Alice will select her five favourite books first and Bob will get the rest.

A. What are the strategy spaces S_A and S_B of Alice and Bob respectively?

B. How many strictly dominant and how many weakly dominant strategies does Alice have?

C. How many strictly dominant and how many weakly dominant strategies does Bob have?

D. Does Alice have any strictly dominated strategies? What about Bob?

E. Does Alice have any weakly dominated strategies? What about Bob?

Exercise 2 (Second price auction)**.** Consider the following auction scenario. There is an item for sale and n interested bidders; the *valuation* of bidder i for the item is v_i, which represents how many pounds the bidder would be willing to spend on buying the item.

In a *second price auction*, the auctioneer asks the bidders to report their valuations v_i and then sells the item to the bidder with the highest bid at a price p equal to the second highest bid (break ties arbitrarily). All other agents (who do not receive the item) are charged 0. The payoff of any bidder i is 0 if she does not receive the item and $v_i - p$ if she does.

A. What is the (pure) strategy space S_i of each bidder i?

B. Show that for any bidder i, there is a strategy s_i that weakly dominates any strategy $s'_i > v_i$.

C. Show that for any bidder i, there is a strategy s_i that weakly dominates any strategy $s'_i < v_i$.

D. Does this game has a weak dominant strategy equilibrium? If not, explain your answer. If yes, state the equilibrium profile.

Exercise 3 (Iteratively Removing Dominated Strategies)**.** Consider the game given by the utility bi-matrix.

	C1	C2	C3	C4	C5
R1	4,-1	3,0	-3,1	-1,4	-2,0
R2	-1,1	2,2	2,3	-1,0	2,5
R3	2,1	-1,1	0,4	4,-1	0,2
R4	1,6	-3,0	-1,4	1,1	-1,4
R5	0,0	1,4	-3,1	-2,3	-1,-1

Exercise 4 (Finding all Mixed Nash Equilibria)**.** Consider the game given by the following utility bi-matrix. Reduce this game to a 2×2 game by iteratively removing strictly dominated strategies.

	C1	C2	C3	C4
R1	7,3	6,3	5,5	4,7
R2	4,2	5,7	8,6	5,8
R3	6,1	3,8	2,4	5,9

A. Consider the mixed strategies $x_1 = (1/4, 1/2, 1/4)$ and $x_2 = (2/3, 1/3, 0, 0)$ for Player 1 and Player 2, respectively. What is the expected utility $u_1(x_1, x_2)$ of Player 1 from the mixed strategy profile (x_1, x_2). Write the explicit formula for computing the utility and show the calculations in detail.

B. Find all the mixed Nash equilibria of this game. Which of those are pure Nash equilibria?

Exercise 5 (Guess Half the Average)**.** Consider the following game called "Guess Half the Average", amongst n players, with $n > 1$: Each player independently chooses (guesses) a whole number between 1 and 1000. The person that guessed the number that is closest to *half the average* of the chosen numbers wins this game (for a utility of 1), and everyone else loses the game (for a utility of 0). If there are multiple players that have the same closest to half the average guess (call those *winners*), then we use one of the following tie-breaking rules:

- Each winner gets a utility of $1/k$, where k is the number of winners.
- Each winner gets a utility of 1.

A. Discuss how you would play each of those two versions of the game if you were playing them with your classmates from the AGTA course and why.

B. For both version of the game, find all the pure Nash equilibria. Argue why there are no other pure Nash equilibria besides those that you have found.