

AGTA Tutorial 4

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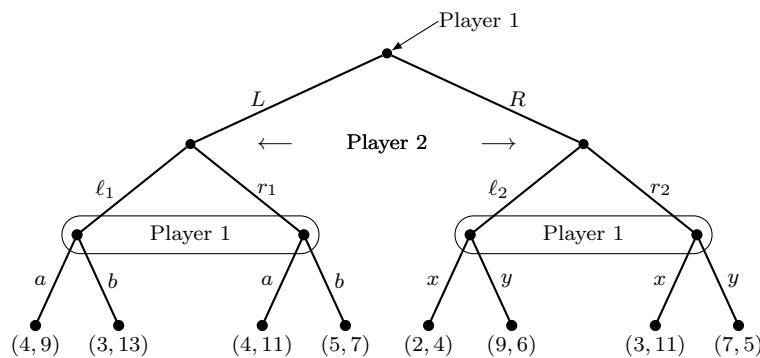
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February 27, 2025

Exercise 1. Consider the following game between Alice and Bob. First, Alice proposes to either go to the cinema or for a walk in the park and then Bob decides whether to accept the proposition or to reject it. If they agree to go to the cinema, each player gets a payoff of 2. If Alice proposes to go to the cinema and Bob rejects, Alice gets a payoff of 0 and Bob gets a payoff of 1. If Alice proposes to go for a walk in the park and Bob rejects, they each get a payoff of 1. Finally, if they agree to go for a walk in the park, Alice gets a payoff of 3 and Bob gets a payoff of 0.

- Formulate the above scenario as a two-player complete information game in extensive form, explicitly presenting the game tree.
- Convert the extensive form game to a game in normal form, explicitly writing down the utility matrix.
- Find all the pure Nash equilibria of the game.
- Explain how to use backwards induction to find a subgame perfect equilibrium of the game.

Exercise 2. Consider the 2-player extensive form game (of imperfect information) described by the game tree below. At the leaves, the left payoff is for Player 1, and the right payoff is for Player 2. Describe all subgame-perfect Nash equilibria in this game, in terms of behavioural strategies.



Exercise 3. Consider the following finite extensive form game of perfect information. There are two players. Each player receives 1 British pound at the beginning of the game. The two players then alternate moves, starting with player 1. In each move, the player whose turn it is to move either chooses **stop** or **give**. If a player chooses **give** then the referee takes 1 pound from that player and gives 2 pounds to the other player. If it chooses **stop**, then the game stops immediately, and both players keep the money they have already accumulated. In any case, the game stops immediately if we reach a state where both players have accumulated exactly 4 pounds.

- Draw the finite game tree for this game, indicating the payoffs to the two players at the leaves.

- B. Compute a subgame perfect Nash Equilibrium (SPNE) for this game.
- C. Is there more than one SPNE? Explain.
- D. Are there any other NEs? Explain.
- E. How would you play this game if you were, say, player 1? Is there a plausible game-theoretic explanation for how you would play it?

Exercise 4. Consider the following game between Alice and Bob. Out of a regular deck of 52 cards, one is selected uniformly at random. Alice goes first and decides whether to play or quit. If she quits, the game ends and no player gains or loses any money. If she decides to play, Bob guesses whether the selected card is a King or not. If he guesses correctly, Alice pays him 1000 British pounds, otherwise no exchange of money takes place.

- A. State the game above as an imperfect information extensive form game by explicitly writing down the game tree. Make sure to depict the information sets. *Hint: Assume that the card is chosen by a third player that is called nature and does not participate in the game, but is placed at the root of the tree.*
- B. Convert the game into a normal form game by explicitly writing down the payoff matrix.
- C. Find all the mixed Nash equilibria of the game. Would you say that any of these equilibria are counter-intuitive?

Exercise 5.

Consider the following extensive form game of imperfect information with chance nodes. Compute a SPNE of the game. Are there any other NEs for this game?

