AGTA Tutorial 7

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March 18, 2025

Exercise 1.

In a VCG-based auction, four *identical* items are being auctioned simultaneously. Suppose there are three buyers (bidders), A, B, and C who provide their claimed valuation functions v_A, v_B, and v_C as follows; $v_x(j)$ denotes the value, in pounds, that bidder x has for receiving j items:

	valuation				
bidder x	$v_x(0)$	$v_x(1)$	$v_x(2)$	$v_x(3)$	$v_x(4)$
$x := A$	0	3	6	9	12
$x := B$	0	2	7	9	14
$x := C$	0	3	8	11	13

An allocation outcome for this auction is specified by three numbers $j_A, j_B, j_C \in \{0, 1, 2, 3, 4\}$, such that for $x \in \{A, B, C\}$, j_x is the number of (identical) items allocated to bidder x, and such that $j_A + j_B + j_C \leq 4$. Each bidder will also be asked to pay a certain amount (in British pounds), p_A, p_B, and p_C, respectively, for their allocation.

What are VCG allocations, and VCG payments, for this auction? In other words, how many items will each bidder get, and what price will each pay for the items they get, if the VCG mechanism is used?

Exercise 2. Answer the following questions.

A. Explain why the second-price auction is a special case of the VCG mechanism.

B. Explain how to derive the second-price auction allocation and payments from Myerson's characterisation for single-parameter domains.

C. We saw that the second price auction maximises the social welfare, so its approximation ratio with regard to this objective is 1. What about its Price of Anarchy (with regard to all pure Nash equilibria, not just dominant strategy ones)? Is it also 1? Justify your answer.

Exercise 3. Consider a single-item auction with two bidders with values drawn independently from the uniform distribution over $[0, 1]$.

A. Prove that the expected revenue of the second price auction in this case is $1/3$.

B. Prove that the expected revenue of the second price auction with reserve price at $1/2$ in this case is $5/12$.

C. What is the maximum possible expected revenue that can be extracted in this case?