Introduction to Algorithms and Data Structures

Lecture 5: Asymptotics for Insertsort and Mergesort

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Algorithms and cost models

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This lecture: Start with 1, then move towards 2.

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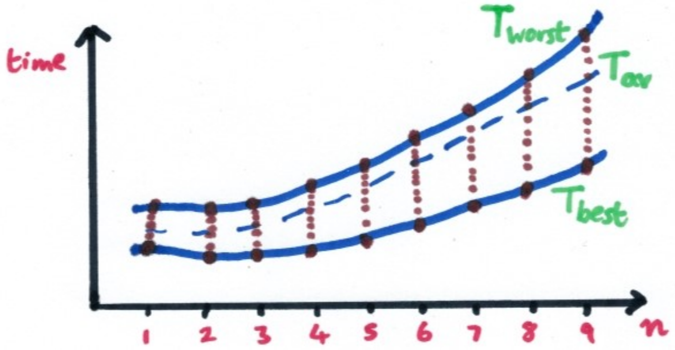
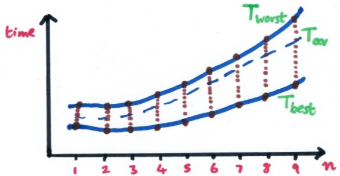
Typically, different inputs of size n give different runtimes!
However, the best case, worst case and (sometimes) average case times give well-defined functions we can talk about.



Best/worst case and asymptotic bounds



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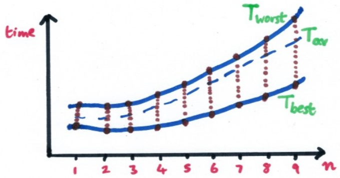

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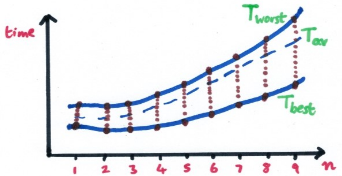

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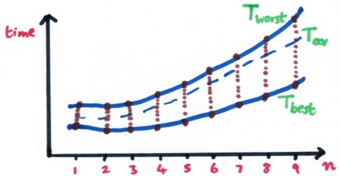

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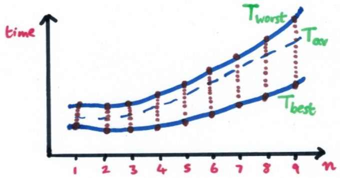

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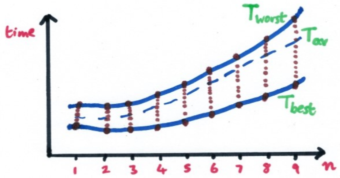

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Moral: Don't confuse '$O/\Omega/\Theta$' with 'best/worst/average'!
(See CLRS pages 48-49.)

InsertSort: ‘number of comparisons’ analysis

Pseudocode for InsertSort again (line numbers added):

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0  InsertSort(A):
1      for i = 1 to n−1        # write n for size of A
2          x = A[i]
3          j = i−1
4          while j ≥ 0 and A[j] > x
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- But i itself runs from 1 to n−1 (line 1).
- So total number of '$>$' ops is at most $\Sigma_{i=1}^{n-1} i = O(n^2)$.

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Headline is that the worst-case time is $\Omega(n^2)$, hence $\Theta(n^2)$.

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Can now say: number of comparisons performed by InsertSort on size n inputs is $O(n^2)$ and $\Omega(n)$.

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That is, if P_n is the set of $n!$ orderings of $\{x_0, x_1, \ldots, x_{n-1}\}$, then

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It can be shown that

$$T_{av}(n) = n^2/4 - O(n)$$

(compare $T_{worst}(n) = n^2/2 - O(n)$). Anyway, $T_{av}(n) = \Theta(n^2)$.

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Now what about **MergeSort** itself?

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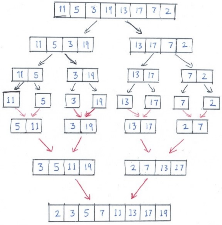
We'll reason informally. Recall our diagram:



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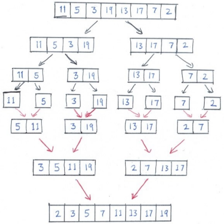

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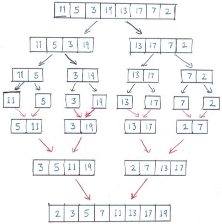

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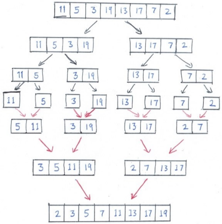

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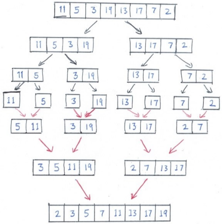

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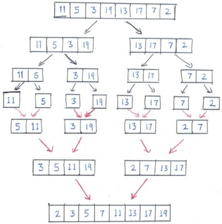

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Even if n isn't a power of 2, can show with a little care that $\sharp$comparisons is $< n\lceil \lg n \rceil$, which is certainly $O(n \lg n)$.

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Using this, can show that on lists of length n, **MergeSort** requires $\Omega(n \lg n)$ comparisons.

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So $T_{worst} = O(n \lg n)$, $T_{best} = \Omega(n \lg n)$.
Can immediately conclude that $T_{worst}, T_{best}, T_{av}$ are all $\Theta(n \lg n)$.
(Shown without deriving exact formulae for $T_{worst}, T_{best}, T_{av}$!)

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Summary: In worst and average cases, MergeSort is asymptotically better than InsertSort ($n \lg n = o(n^2)$).

But InsertSort does better in best case ($n = o(n \lg n)$).

Measuring ‘overall runtime’

Consider again InsertSort (for integer arrays):

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InsertSort(A):
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Claim: Each line execution takes $\Theta(1)$ time (i.e. between two positive constants $t < t'$). This implies that, for any such impl,

$$\text{total execution time} \;=\; \Theta(\text{number of line executions})$$

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            A[j+1] = A[j]
            j = j−1
        A[j+1] = x
```

Common to take number of line executions as measure of runtime.

Broad justification: Think about how the pseudocode would be implemented on a typical Random Access Machine (RAM).
(Think 32-bit or 64-bit computer ... except that word size/number of words may be taken as large as required for the given input.)

Claim: Each line execution takes $\Theta(1)$ time (i.e. between two positive constants $t < t'$). This implies that, for any such impl,

$$\text{total execution time} \;=\; \Theta(\text{number of line executions})$$

Warning! Applies only when each line execution does just a bounded amount of work. E.g. What if ‘$>$’ is comparison for strings?

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E.g. In **InsertSort**, for each value of i, execution of lines 2–7 takes $\leq 3i + 4$ line executions ...

In this case, this tells the same story as our previous analyses:

	Worst	Average	Best
InsertSort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$
MergeSort	$\Theta(n \lg n)$	$\Theta(n \lg n)$	$\Theta(n \lg n)$

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Let's also look briefly at the memory requirements of our algorithms on size n inputs.

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Headline: In-place InsertSort wins on space efficiency.

'End of Part 1'

That completes our introduction to the general concepts that underpin this course, and the way we'll be approaching algorithms.

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Next lecture: Start on data structures — beginning with how program data is organized in memory.

Reading

Roughgarden 1.5, 1.6
Kleinberg-Tardos 2.4 (first half), 4.1
CLRS 2.2