Introduction to Algorithms and Data Structures

Lecture 6: Representation of program data in memory

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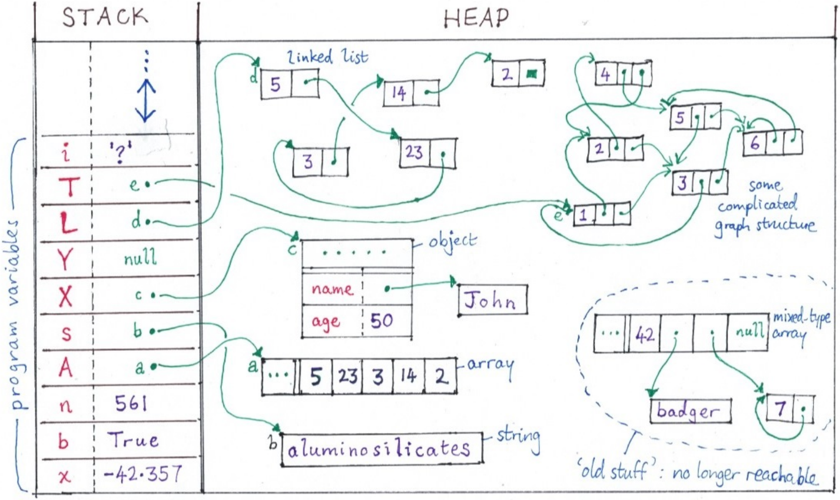
Remember: simplifying a bit, we think of memory as consisting of words each with an address (i.e. location).

Any address can be itself be stored in a single word — though there may be fewer addresses than word values.

Typical organization of program data in memory



Program memory: summary

- Contents of program variables are stored on a stack, which grows and shrinks as variables come in and out of scope.
- Stack items are contiguously arranged in memory, so must have fixed size (e.g. 1 or 2 words).
- Typically, a stack item contains either a basic value (e.g. 561, True) or a reference to something on the heap.
- Heap objects can live anywhere in memory, be of any size, and may contain references to other heap objects.
- When heap objects are created (allocated), the memory manager will decide where to put them. But references to other objects can be changed later – so we can end up with a real mess!
- As execution proceeds, some heap objects may become unreachable. In many languages (e.g. Java, Python, Haskell), a garbage collector (i.e. memory recycler) detects this and reclaims the space.

Details may vary . . .

Our picture is mostly 'Java-like' (except for the mixed-type array).

- In Java, anything of reference type (including 'objects' and 'arrays') lives on the heap.
  In C, built-in 'arrays' live on the stack, and their size is static: an array variable A has an associated size fixed throughout its lifetime.
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- Python also offers 'lists' and 'arrays', both implemented much like the heap array in our picture. (Difference: 'lists' allow mixed types.)
  In functional programming languages (Haskell, ML, Lisp), 'lists' are more typically implemented as linked lists.
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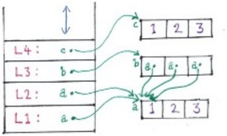

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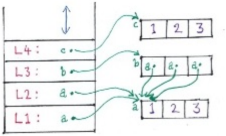

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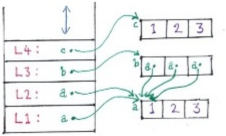

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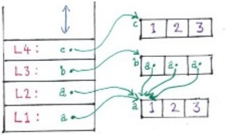

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Geek point: Technically, a small integer like 2 is itself a reference to a pre-allocated heap object. But harmless to write this ref just as '2'.

Shallow vs. deep cloning

In Python, '[:]' makes only a shallow clone: copies 'top level only'. E.g. think about lists of lists:

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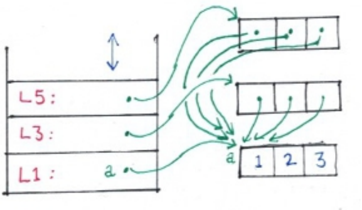

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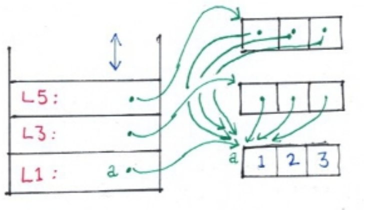

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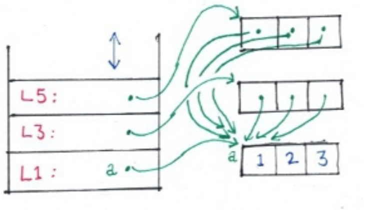

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For a deep clone (fresh copy of entire structure, with no sharing), we could in this case write `L6 = [L1[:],L1[:],L1[:]]`.

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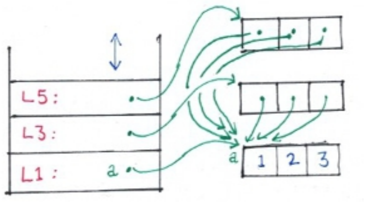

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In general, may need to write a (possibly recursive) program to deep-clone the data structures in question.

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Warning: In Java, `==` means reference equality!
For value equality, typically use an `.equals` method.

For numbers and strings, always use value equality.
(Identity is unpredictable!)

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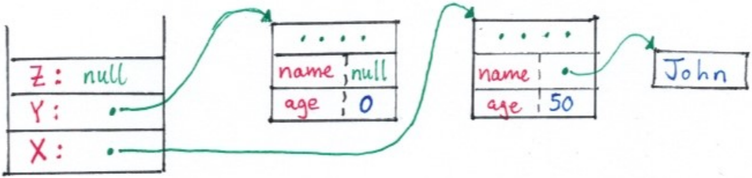

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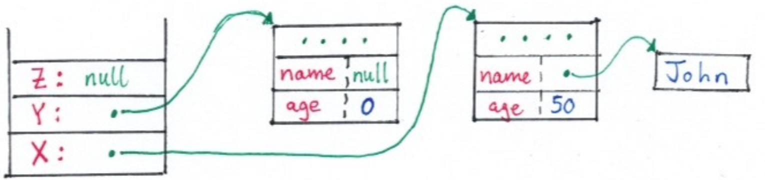

Understanding this (and drawing pictures) can go a long way towards rooting out those pesky NullPointerExceptions.

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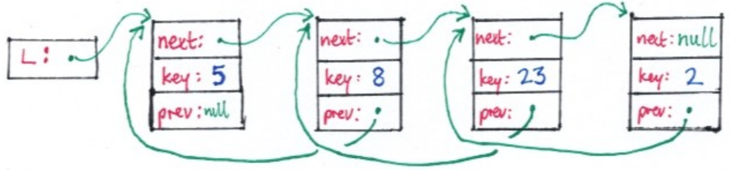

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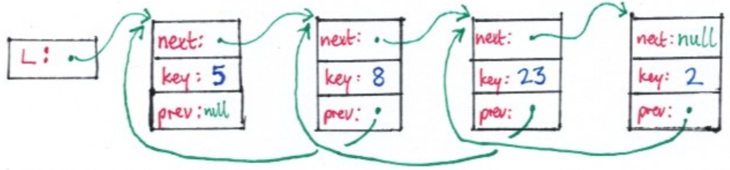

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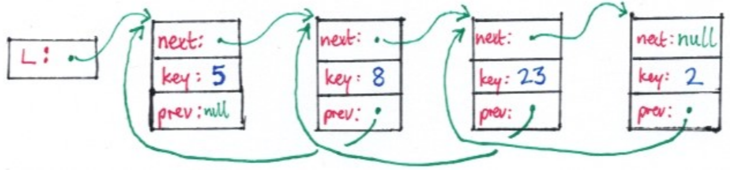

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Anyway, find nth element of a linked list L takes time $\Theta(n)$.

Reading:
https://docs.python.org/3/reference/datamodel.html
(just 3.1);
CLRS chapter 10, especially 10.2 and 10.3.

Photograph: Glen Etive from path to Lairig Gartain.
Kyriakos Kalorkoti (by kind permission)