

Exercise Sheet: Week 4

- (1) Write down the code for an RM macro ‘if $R_1 > R_2$ then goto I_j ’. The macro must leave all registers unchanged after its execution. Assume a predefined GOTO macro.
- (2) Give a simple recursive definition of a sequence coding function $\mathbb{N}^* \rightarrow \mathbb{N}$, based on the pairing function in the slides.

- (3) Our register machines have a finite number of registers, each holding an unbounded number. Turing machines have an unbounded number of cells, each holding one of a finite set of symbols.

Suppose we allow register machine to have an unbounded number of registers, but each register is finite (e.g. 32 bits) – like current computer memory. With no changes to the instruction set, are these machines still Turing powerful? Why or why not?

Suppose now that we add a form of indirect addressing. For example, we might say that the register operand of an instruction can now be either i , as before, meaning R_i , or (i) , meaning R_{R_i} . Does that help?

Why aren’t Turing machines affected by this issue? Can you adapt ideas from TMs to solve this issue for RM?

- (4) The proof of the Halting Problem relies on the lethal combination of *self-reference* (when the machine is run on itself) and *negation* (when we flip the result of the halting analyser). Here are some other famous contradictions/paradoxes. Discuss what they show or how they might be resolved.
 - (a) ‘The barber shaves all and only the men who do not shave themselves.’
 - (b) ‘The set of sets that are not members of themselves.’
 - (c) ‘The smallest natural number not definable in under eleven words.’