

Exercise Sheet: Week 8

- (1) The VERTEX-COVER problem: Given a graph $G = (V, E)$, a vertex cover is a subset $V' \subseteq V$ of the vertices such that every edge has at least one end in the cover V' . VERTEX-COVER is, given a graph G and an integer k , does G have a vertex cover of size k ?

Give reductions from CLIQUE to VERTEX-COVER, and the other way round. (*Hint*: complementary graph.)

- (2) The following properties were mentioned in the lectures (with only brief explanations):
- $\text{PSPACE} \supseteq \text{PTIME}$
 - $\text{PSPACE} \supseteq \text{NPTIME}$
 - $\text{PSPACE} \subseteq \text{EXPTIME}$

Write out explanations in enough detail to show how a proof would work.

- (3) P^{NP} obviously includes all of NP and co-NP. So how does NP^{NP} differ from it – what else is there? (Assuming, that is, that $\text{P} \neq \text{NP}$.)