ADS Tutorial 7

Instructor: Aris Filos-Ratsikas Tutor: Kat Molinet

November 13, 2025

Problem 1

Consider again the following linear program (LP) from Tutorial 6.

$$
\begin{array}{ll}
\text{maximise} & 6x_1 + 6x_2 + 5x_3 + 9x_4 \\
\\
\text{subject to} & 2x_1 + x_2 + x_3 + 3x_4 \leq 5 \\
& x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{array}
$$

In Tutorial 6 you solved this LP using the Simplex Method.

A. Write the dual LP to this LP.

B. Find an optimal solution to the dual LP.

Hint: You may look back at the way you computed the optimal solution to the primal LP using the Simplex Method, and extract the optimal solution to the dual LP from the final dictionary.

Problem 2

A cargo plane has three compartments for storing cargo: front, centre, and rear. These comparments have the following limits on both weight and space: the front can fit up to 10 tons and up to 6800 cubic metres of cargo, the centre can fit up to 16 tons and 8700 cubic metres of cargo, and the rear can fit up to 8 tons and 5300 cubic metres of cargo.

Additionally, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane. For example, if 8 tons are put in the centre compartment, 5 tons must be put in the front compartment, since $8/16 = 5/10$.

The following four cargoes are available for shippment on the next flight: Any proportion of these cargoes

Cargo	Weight (tons)	Volume (cubic meters/ton)	Profit (GBP/ton)
C_1	18	480	310
C_2	15	650	380
C_3	23	580	350
C_4	12	390	28

can be accepted. The objective is to determine how much (if any) of each cargo C_1, C_2, C_3, and C_4 should be accepted and how to distribute each among the compartments of the plane so that the total profit for the flight is maximised.

Formulate the problem above as a linear program.

Problem 3

Consider a convex polygon P in $\mathbb{R}^2$, given by an unordered list of its corner points $p_1, p_2, \ldots, p_m$, with $p_j = (a_j, b_j)$. Construct a linear program with the following properties:

A. If the origin $(0, 0)$ is in the interior of P, then the program is feasible, and the optimal value of its objective function is strictly positive.

B. If the origin $(0, 0)$ is not in the interior of P, then the program is either infeasible, or it is feasible and the optimal value of its objective function is at most 0.

Recall that a point is said to be in the interior of P if it is in P but not on the boundary of P, that is it is neither a corner nor on an edge of P. You may also use the fact that a point is in the interior of P if and only if it is a *weighted average* of the corner points of P, with all weights being strictly positive.

Problem 4

Let $G = (V, E)$ be an undirected graph with weights on the nodes (w_v for each node $v \in V$). A *vertex cover* of G is a set of nodes $S \subseteq V$ such that every edge $e \in E$ is incident to some node in S. A *minimum weight vertex cover* is a vertex cover with minimum total weight $\sum_{v \in S} w_v$.

Formulate the problem of finding a minimum weight vertex cover of G as an integer linear program (ILP). Explain the role of your variables and your constraints, and why an optimal solution to the ILP corresponds to a vertex cover of minimum weight. Write the LP-relaxation of your ILP.

Problem 5

Consider the following problem. There are n indivisible items of weights $w_1, \ldots, w_n$ to be distributed to m bags. Our goal is to minimise the weight of the heaviest bag. Formulate this problem as an integer linear program. Explain the role of your variables and your constraints, and why an optimal solution to the ILP corresponds to a vertex cover of minimum weight. Write the LP-relaxation of your ILP.

Problem 6

A tutor in Algorithms and Data Structures has decided that the tutoring salary is not enough, and has decided to offer private lessons to the students. These will be 1-to-1 lessons, with a duration of 1 hour. The tutor has divided the week into 1-hour slots which are offered to the students. Every student i specifies the following parameters:

- The availability for slot j: this is a parameter a_{ij} which is 1 if the student is availble to take that slot, and 0 otherwise.
- The amount the student is willing to pay for slot j, denoted by p_{ij}.
- The number of lessons q_i that the student wishes to take. The student is willing to either take q_i lessons or none, more or fewer lessons that q_i are not acceptable.

The tutor would like to assign students to slots in a way that maximises their profit. Help the tutor by figure out the optimal assignment by formulating the above problem as an integer linear program. Describe the objective function, the variables, and the constraints.