# AGTA Tutorial Sheet 2

Please attempt these questions before coming to the tutorial.

1. Consider the finite 2-player zero-sum game given by the following payoff matrix, $A$, for Player 1 (the row player):

$$A = \begin{bmatrix} 4 & 2 & 9 & 2 & 5 \\ 6 & 3 & 5 & 9 & 7 \\ 1 & 4 & 8 & 5 & 7 \\ 5 & 1 & 3 & 5 & 6 \end{bmatrix}$$

   Specify the linear programming problem you could use to "solve" this game, meaning to compute the minimax value of this game, and to compute a minmaximizer strategy for Player 1.

   Next, describe a different linear program whose optimal solution yields a maxminimizer strategy for player 2.

   Next, try to actually compute, by hand if you can, the minimax value, minmaximizer strategy for player 1, and maxminimizer strategy for player 2, by solving the linear programs you have constructed.

   (Hint: first try to simplify the game to the extent possible, by eliminating redundant pure strategies, then solve the linear programs by hand.)

2. Consider the 2-player finite strategic form game (i.e., bimatrix game), specified by the following *bimatrix*:

$$\begin{bmatrix} (7,3) & (6,4) & (5,5) & (4,7) \\ (4,2) & (7,9) & (8,6) & (8,8) \\ (6,1) & (9,7) & (2,4) & (6,9) \end{bmatrix}$$

   Compute all Nash equilibria in this game, and compute the expected payoff to each player in each Nash equilibrium. (Hint: first, simplify the game by eliminating redundant pure strategies. Then use the fact, proved in class, that in any Nash equilibrium, each pure strategy that is played with positive probability by any player is necessarily a best response for that player. Use this to set up linear equations for computing Nash equilibrium for the simplified game.)

3. A *symmetric* finite 2-player zero-sum game, is a game that "looks exactly the same" from the point of view of both players. (For example, rock-paper-scissors is a symmetric zero-sum game.)

   More formally, a 2-player zero-sum game is *symmetric* if and only if it is specified by a $(n \times n)$ payoff matrix, $A = (a_{i,j})$, for player 1 (the row player), such that for all $i, j \in \{1, \ldots, n\}$, we have $a_{i,j} = -a_{j,i}$, or in order words, such that $A = -A^T$.

   Show that the *minimax value* of any symmetric two player zero-sum game must be zero. (Intuitively, this should be obvious: in a symmetric game neither player can have an advantage, because the game "looks the same" to both players. But I want you to prove it. Hint: prove it by contradiction, assuming the value $v^* \neq 0$.)

4. Consider a 2-player zero-sum matrix game, given by the $(2 \times 3)$ payoff matrix:

$$A = \begin{bmatrix} 2 & 9 & 4 \\ 7 & 0 & 3 \end{bmatrix}$$

   Construct the LP, described in class, for computing the value, and minmaximizer strategy for player 1, for this game. Confirm that the LP looks like this:

   **Maximize** $v$
   **Subject to:**
   $v - 2x_1 - 7x_2 \leq 0$
   $v - 9x_1 - 0x_2 \leq 0$
   $v - 4x_1 - 3x_2 \leq 0$
   $x_1 + x_2 = 1,$
   $x_1 \geq 0,\ x_2 \geq 0.$

   Construct the dual of this LP, according to the "general recipe for LP duals" given in the lecture 7 slides (page 6). Conclude that the dual LP is precisely the LP for computing the value, and maxminimizer strategy for player 2, in the same game.