Introduction to Algorithms and Data Structures

Tutorial 7

your tutor

University of Edinburgh

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Q1: Dynamic programming *Edit Distance*

$$d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} \begin{array}{c} \quad \mathrm{A} \quad \mathrm{T} \quad \mathrm{G} \quad \mathrm{G} \quad \mathrm{C} \quad \mathrm{T} \\ \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{bmatrix} \end{array}, a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \end{bmatrix}$$

Q1: Dynamic programming *Edit Distance*

$$
d = \begin{array}{c} \\ \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\begin{array}{c} \quad A \quad T \quad G \quad G \quad C \quad T \\
\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{bmatrix} \end{array},
a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \end{bmatrix}
$$

row 1: $s_1 =$'A' and $t_1 =$'A', so $d[1,1] \leftarrow 0$, $a[1,1] \leftarrow 0$.

For the $[1,j]$ cells for $j > 1$, *don't really need the recurrence yet*

we have the match of $s_1 = t_1$ and then $j-1$ "insertions"

$\Rightarrow d[1,j] \leftarrow (j-1)$ and $a[1,j] = 2$ for each $j > 1$

Q1: Dynamic programming *Edit Distance*

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} & A & T & G & G & C & T \end{array} \\
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} &
\left[\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & & & & & & \\
4 & & & & & & \\
5 & & & & & & \\
6 & & & & & &
\end{array}\right]
\end{array}
, a = \left[\begin{array}{ccccccc}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & & & & & & \\
3 & & & & & & \\
3 & & & & & & \\
3 & & & & & &
\end{array}\right]
$$

row 2: for cell $[2,1]$ recurrence says $d[2,1] \leftarrow 1 + \min\{d[1,1], d[2,0], d[1,0]\}$

looking up already-computed values, $d[2,1] \leftarrow 1 + \min\{\ 0,\ 2,\ 1\ \}$

So $d[2,1] \leftarrow 1$, last op was a "deletion" $(a[2,1] \leftarrow 3)$.

cell $[2,3]$: again $s_2 \neq t_3$, recurrence $d[2,3] \leftarrow 1 + \min\{d[1,3], d[2,2], d[1,2]\}$

looking up already-computed values, $d[2,1] \leftarrow 1 + \min\{\ 2,\ 1,\ 1\ \}$

$\Rightarrow d[1,j] \leftarrow 2$, last op was a substitution or a insertion $(a[2,3] \leftarrow 1/2)$

Q1: Dynamic programming *Edit Distance*

$$d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} \overset{\begin{array}{cccccc} \text{A} & \text{T} & \text{G} & \text{G} & \text{C} & \text{T} \end{array}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{bmatrix}}, a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & & & & & & \\ 3 & & & & & & \\ 3 & & & & & & \end{bmatrix}$$

Just a reminder which cells we need to check for each $[3, j]$ $j > 1$:
$d[3, j-1]$ ("left"), $d[2, j]$ ("above"), and $d[2, j-1]$ ("above left")
Also take account of whether $s_3 = t_j$ (true for $j = 2$ and $j = 6$ only)

Q1: Dynamic programming *Edit Distance*

$$
\begin{array}{c} \\ \\ \\ d = \end{array}
\begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\overset{\begin{array}{cccccc} \mathsf{A} & \mathsf{T} & \mathsf{G} & \mathsf{G} & \mathsf{C} & \mathsf{T} \end{array}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & 3 & 2 & 1 & 2 & 3 & 4 \\ 5 & & & & & & \\ 6 & & & & & & \end{bmatrix}}
, a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\ 3 & & & & & & \\ 3 & & & & & & \end{bmatrix}
$$

Note that we write 1/2 in $a[i, j]$ when there are *competing* options of the recurrence which result in an optimum alignment for those prefixes.

The pseudocode will only enter one entry into $a[i, j]$, of course.

Q1: Dynamic programming *Edit Distance*

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} & A & T & G & G & C & T \end{array} \\
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} &
\left[\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & & & & & &
\end{array}\right]
\end{array}
, a = \left[\begin{array}{ccccccc}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & 0 & 2 & 2 \\
3 & & & & & &
\end{array}\right]
$$

Q1: Dynamic programming *Edit Distance*

$$
\begin{array}{ccccccccc}
 & & & \text{A} & \text{T} & \text{G} & \text{G} & \text{C} & \text{T}
\end{array}
$$

$$
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & 5 & 4 & 3 & 2 & 2 & 2
\end{bmatrix},
a = \begin{bmatrix}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & 0 & 2 & 2 \\
3 & 3 & 0 & 3 & 3 & 1 & 0
\end{bmatrix}
$$

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ *values:*

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} & \mathsf{A} & \mathsf{T} & \mathsf{G} & \mathsf{G} & \mathsf{C} & \mathsf{T} \end{array} \\
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} & \left[\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & 5 & 4 & 3 & 2 & 2 & 2
\end{array}\right]
\end{array}
, a = \left[\begin{array}{ccccccc}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & 0 & 2 & 2 \\
3 & 3 & 0 & 3 & 3 & 1 & 0
\end{array}\right]
$$

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ values:

$$d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} \overset{\begin{array}{cccccc} \text{A} & \text{T} & \text{G} & \text{G} & \text{C} & \text{T} \end{array}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & 3 & 2 & 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 & 1 & 2 & 3 \\ 6 & 5 & 4 & 3 & 2 & 2 & 2 \end{bmatrix}}, a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\ 3 & 3 & 3 & 0 & 0 & 2 & 2 \\ 3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0} \end{bmatrix}$$

match (so next we will check $a[5,5]$)

‘T’
‘T’

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i, j]$ *values:*

$$
\begin{array}{cc}
 & \begin{array}{cccccc} A & T & G & G & C & T \end{array} \\
\end{array}
$$

$$
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & 5 & 4 & 3 & 2 & 2 & 2
\end{bmatrix},
a = \begin{bmatrix}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & 0 & \mathbf{2} & 2 \\
3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0}
\end{bmatrix}
$$

insertion (and next we check $a[5, 4]$)

-	‘T’
’C’	‘T’

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ *values:*

$$
\begin{array}{cc}
 & \begin{array}{cccccc} A & T & G & G & C & T \end{array}
\end{array}
$$

$$
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & 5 & 4 & 3 & 2 & 2 & 2
\end{bmatrix},
a = \begin{bmatrix}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & 0 & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & \mathbf{0} & \mathbf{2} & 2 \\
3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0}
\end{bmatrix}
$$

match (so next we will check $a[4,3]$)

'G'	-	'T'
'G'	'C'	'T'

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ values:

$$d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} \overset{\begin{array}{cccccc} A & T & G & G & C & T \end{array}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & 3 & 2 & 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 & 1 & 2 & 3 \\ 6 & 5 & 4 & 3 & 2 & 2 & 2 \end{bmatrix}}, \quad a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & 0 & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & 3 & 3 & \mathbf{0} & 0 & 2 & 2/3 \\ 3 & 3 & 3 & 0 & \mathbf{0} & \mathbf{2} & 2 \\ 3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0} \end{bmatrix}$$

match (so next we will check $a[3,2]$)

‘G’	‘G’	-	‘T’
‘G’	‘G’	’C’	‘T’

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ values:

$$
\begin{array}{c}
\quad\quad\quad\quad\quad \text{A} \quad \text{T} \quad \text{G} \quad \text{G} \quad \text{C} \quad \text{T}
\end{array}
$$

$$
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array}
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
2 & 1 & 1 & 2 & 3 & 3 & 4 \\
3 & 2 & 1 & 2 & 3 & 4 & 3 \\
4 & 3 & 2 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 2 & 1 & 2 & 3 \\
6 & 5 & 4 & 3 & 2 & 2 & 2
\end{bmatrix},
a = \begin{bmatrix}
- & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 0 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 1 & 1/2 & 1/2 & 0 & 2 \\
3 & 3 & \mathbf{0} & 1/2 & 1/2 & 1/2/3 & 0 \\
3 & 3 & 3 & \mathbf{0} & 0 & 2 & 2/3 \\
3 & 3 & 3 & 0 & \mathbf{0} & \mathbf{2} & 2 \\
3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0}
\end{bmatrix}
$$

match (so next we will check $a[2,1]$)

‘T’	‘G’	‘G’	-	‘T’
‘T’	‘G’	‘G’	’C’	‘T’

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i, j]$ values:

$$d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} \overset{\begin{array}{cccccc} A & T & G & G & C & T \end{array}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & 3 & 2 & 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 & 1 & 2 & 3 \\ 6 & 5 & 4 & 3 & 2 & 2 & 2 \end{bmatrix}}, \quad a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 0 & 2 & 2 & 2 & 2 & 2 \\ 3 & \mathbf{3} & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & \mathbf{0} & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & 3 & 3 & \mathbf{0} & 0 & 2 & 2/3 \\ 3 & 3 & 3 & 0 & \mathbf{0} & \mathbf{2} & 2 \\ 3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0} \end{bmatrix}$$

deletion (and next we check $a[1, 1]$)

'C'	'T'	'G'	'G'	-	'T'
-	'T'	'G'	'G'	'C'	'T'

Q1: Dynamic programming *Edit Distance*

(b) Find the optimum alignment using the $a[i,j]$ values:

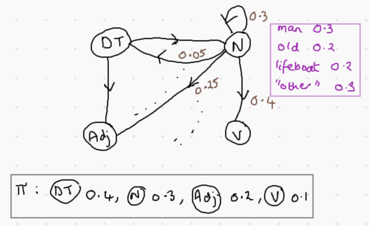
$$
\begin{array}{cc}
 & \begin{array}{cccccc} \mathsf{A} & \mathsf{T} & \mathsf{G} & \mathsf{G} & \mathsf{C} & \mathsf{T} \end{array} \\
d = \begin{array}{c} \\ A \\ C \\ T \\ G \\ G \\ T \end{array} & \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 1 & 2 & 3 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 & 4 & 3 \\ 4 & 3 & 2 & 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 & 1 & 2 & 3 \\ 6 & 5 & 4 & 3 & 2 & 2 & 2 \end{bmatrix}
\end{array}
, a = \begin{bmatrix} - & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & \mathbf{0} & 2 & 2 & 2 & 2 & 2 \\ 3 & \mathbf{3} & 1 & 1/2 & 1/2 & 0 & 2 \\ 3 & 3 & \mathbf{0} & 1/2 & 1/2 & 1/2/3 & 0 \\ 3 & 3 & 3 & \mathbf{0} & 0 & 2 & 2/3 \\ 3 & 3 & 3 & 0 & \mathbf{0} & \mathbf{2} & 2 \\ 3 & 3 & 0 & 3 & 3 & 1 & \mathbf{0} \end{bmatrix}
$$

match

'A'	'C'	‘T’	‘G’	‘G’	-	‘T’
‘A’	-	‘T’	‘G’	‘G’	'C'	‘T’

Q2: Viterbi Algorithm example

We have an HMM to model sentences in English.



Full transition matrix p, "emissions probabilities" (b_q for every state q) are:

	DT	N	V	Adj
DT	0	0.6	0	0.4
N	0.05	0.3	0.4	0.25
V	0.4	0.3	0.1	0.2
Adj	0.1	0.5	0.2	0.2

Transitions

	lifeboats	man	old	the	"other"
DT	0	0	0	0.5	0.5
N	0.2	0.3	0.2	0	0.3
V	0	0.1	0	0	0.9
Adj	0	0	0.4	0	0.6

Emissions

Q2: Viterbi Algorithm example

The "start state" distribution π assigns probabilities as follows:
$\pi(\mathrm{DT}) = 0.4, \pi(\mathrm{N}) = 0.3, \pi(\mathrm{Adj}) = 0.2, \pi(\mathrm{V}) = 0.1$.

The old man the lifeboats.

$s_1 = \textit{the}$, $s_1 s_2 = \textit{the old}$, $s_1 \dots s_3 = \textit{the old man}$, $s_1 \dots s_4 = \textit{the old man the}$.

"bottom-up":

- consider $s_1 = \textit{the}$ to find $mlp[1, q]$ for every state $q \in Q$ ...

  $\pi(q) \times b_{q,the}$ for every $q \in Q$
- then consider $mlp[2, q]$ for every state $q \in Q$
- ... then consider $mlp[3, q]$ for every state $q \in Q$
- ...

$$mlp[i, q] = \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q* \in Q}\{mlp[i-1, q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

Q2: Viterbi Algorithm example

row 1: Initialisation for the base case, where *the* might be emitted from a specific q. Probability $\pi(q) \times b_{q,the}$ for every $q \in Q$

$b_{q,the}$ is 0 for every q except DT.

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old				
man				
the				
lifeboats				

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] = \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q* \in Q}\{mlp[i-1,q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old				
man				
the				
lifeboats				

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i=1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i>1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0		0	
man				
the				
lifeboats				

the old: old only has positive emission probability at N and Adj

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \; = \; \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q* \in Q}\{mlp[i-1,q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man				
the				
lifeboats				

the old: the only positive probability option for *the* is DT, so consider $mlp[1, \mathrm{DT}] = 0.2$ followed by the options DT $\rightarrow$ N, DT $\rightarrow$ Adj to emit *old*

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i=1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i>1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man				
the				
lifeboats				

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i=1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i>1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0			0
the				
lifeboats				

the old man: notice *man* only possible at states N and V

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048		0
the				
lifeboats				

the old man: notice *man* only possible at states N and V

Ending at N: $b_{\mathrm{N},man} = 0.3$. Take max of $mlp[2,\mathrm{Adj}] \times p_{\mathrm{Adj},\mathrm{N}} = 0.032 \times 0.5 = 0.016$ against $mlp[2,\mathrm{N}] \times p_{\mathrm{N},\mathrm{N}} = 0.024 \times 0.3 = 0.0072$, multiply the 0.016 by $b_{\mathrm{N},man} = 0.3$

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i=1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i>1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048	.1x.024x.4 = .00096	0
the				
lifeboats				

the old man: notice *man* only possible at states N and V

Ending at V: $b_{\mathrm{V},man} = 0.1$. Take max of $mlp[2,\mathrm{Adj}] \times p_{\mathrm{Adj,V}} = 0.032 \times 0.2 = 0.0064$ against $mlp[2,\mathrm{N}] \times p_{\mathrm{N,V}} = 0.024 \times 0.4 = 0.0096$, multiply the 0.0096 by $b_{\mathrm{N},man} = 0.1$

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q* \in Q}\{mlp[i-1,q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048	.1x.024x.4 = .00096	0
the				
lifeboats				

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i=1 \\ \max_{q*\in Q}\{mlp[i-1,q*]\cdot p_{q*,q}\cdot b_{q,s_i}\} & i>1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048	.1x.024x.4 = .00096	0
the		0	0	0
lifeboats				

the old man the: notice final item *the* is only possible at state DT

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] = \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q* \in Q}\{mlp[i-1,q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048	.1x.024x.4 = .00096	0
the	.5x.00096x.4 = 0.000192	0	0	0
lifeboats				

the old man the: notice final item *the* is only possible at state DT

$b_{\mathrm{DT},the} = 0.5$. Take max of $mlp[3,\mathrm{N}] \times p_{\mathrm{N},\mathrm{DT}} = 0.0048 \times 0.05$ against $mlp[3,\mathrm{V}] \times p_{\mathrm{V},\mathrm{DT}} = 0.00096 \times 0.4 = 0.000384$, multiply the 0.000384 by $b_{\mathrm{N},the} = 0.5$

Q2: Viterbi Algorithm example

For subsequent rows, use the recurrence

$$mlp[i,q] \;=\; \begin{cases} \pi_q \cdot b_{q,s_1} & i = 1 \\ \max_{q*\in Q}\{mlp[i-1,q*] \cdot p_{q*,q} \cdot b_{q,s_i}\} & i > 1 \end{cases}$$

	DT	N	V	Adj
the	.4x.5 = .2	0	0	0
old	0	.2x.6x.2 = .024	0	.2x.4x.4 = .032
man	0	.3x.032x.5 = .0048	.1x.024x.4 = .00096	0
the	.5x.00096x.4 = 0.000192	0	0	0
lifeboats	0	.00002304	0	0

lifeboats can only be output at N, *the old man the* can only end at DT $\Rightarrow 0.000192 \times 0.6 \times 0.2$

Q2: Viterbi Algorithm example

Thus the most probable tagging is:

The/DT old/N man/V the/DT lifeboats/N

(The backtrace pointers can be read off from the above matrix in an ad hoc fashion: e.g. in the cell for (man, N), the carried factor is .032 which comes from the cell for (old, Adj) ... though actually our "most likely" path doesn't label *man* with N)

Or we could have built the `prev` array as in the pseudocode for Viterbi.

Q2: Viterbi Algorithm example

`prev` array for students to verify their own.

	DT	N	V	Adj
the	-	-	-	-
old	-	DT	-	DT
man	-	Adj	N	-
the	V	-	-	-
lifeboats	-	DT	-	-

Q3: The Travelling Salesman Path

TSP problem: find a minimum cost path from the source to destination visiting all vertices **exactly once**.

- undirected graph $G = (V, E)$, start node s, end node t
- weight function $w : E \longrightarrow \mathbb{Q}^+$
- ordering π s.t. $\pi_1 = s, \pi_n = t$ and $(\pi_i, \pi_{i+1} \in E)$ for all i.

$$TSP((V,E),s,t) = \begin{cases} w(s,t) \ \text{ if } V = \{s,t\} \\ \min_{\substack{u \in V \setminus \{s,t\}, \\ (u,t) \in E}} \left\{ w(u,t) + TSP((V \setminus \{t\}, E_{\setminus \{t\}}), s, u) \right\} \text{ if } |V| \geq 3 \end{cases}$$

- Notice the recurrence in the above formula
- Can we build a dynamic programming algorithm for this problem?
- What are the four properties of a dynamic programming algorithm?