AGTA Tutorial 8

Exercise 1. Consider a single-item auction with n bidders with values drawn independently from the uniform distribution over $[0, 1]$.

A. Prove that, when $n = 2$, the expected revenue of the second price auction in this case is $1/3$.

B. Prove that, when $n = 2$, the expected revenue of the second price auction with reserve price at $1/2$ in this case is $5/12$.

C. What is the maximum possible expected revenue that can be extracted for this auction when $n = 2$?

D. Prove that the expected revenue of the second-price auction is maximum among all truthful auctions that *always* allocate the item to some bidder.

E. Using D above, prove that the expected revenue of the second-price auction with $n + 1$ bidders is at least as high as the expected revenue of the optimal (revenue-maximising) auction with n bidders (the Bulow-Klemperer Theorem).

Exercise 2. Consider a single-item auction with n bidders, with values drawn independently from distributions $F_1, \ldots, F_n$. Show by example that, in an optimal auction, the bidder with the highest bid need not win, even if the bidder has positive virtual valuation. Give an intuitive explanation why this property might be beneficial to the expected revenue of the auction.

Exercise 3. Consider an all *All-Pay Auction* in which the values of the n bidders are drawn independently from the uniform distribution over $[0, 1]$. In this auction format, the winner is the bidder with the highest bid, and the payment of each bidder is her bid, regardless of whether the bidder is a winner or not.

A. Show that, for the case of $n = 2$, the stategy profile in which every bidder $i \in \{1, 2\}$ bids $b_i = v_i^2/2$ is a Bayes-Nash equilibrium of the auction.

B. For general n, the (unique) symmetric equilibrium of the auction is given by $b_i = \frac{n-1}{n} v_i^n$, for each bidder i. Show that the expected revenue of the auction in this case is $\frac{n-1}{n+1}$. How does that compare to the expected revenue of the First-Price auction with n bidders whose values are drawn independently from the uniform distribution over $[0, 1]$?