

Introduction to Theoretical Computer Science

Exercise Sheet: Week 3

Here are a few routine exercises.

1. Consider the language $L = \{ww \mid w \in \{a, b\}^*\}$. Show that it is not regular.
2. Is the language $L' = \{ww \mid w \in \{a, b\}^* \wedge |w| < 4\}$ regular? Why or why not?
3. We saw in lectures that the language L is not context-free. Show that, by contrast, its complement *is* context-free.

The following questions/comments are intended as prompts for discussion. Of course, you can ask/discuss about anything. Some of these topics we've touched on in discussion in lectures – this is an opportunity to think about them a bit more.

4. Consider <https://www.geeksforgeeks.org/theory-of-computation/concatenation-process-in-dfa/>.

It proposes a construction for sequential concatenation of two DFAs A_1, A_2 (recognizing languages L_1 and L_2 , resp.) into a new DFA A that recognizes the language L_1L_2 . That construction is explained by example and not formally specified.

- a) Specify this construction formally.
 - b) Explain why this construction is wrong in general, and give a counterexample. (E.g., by minimally modifying their example).
 - c) Explain why this wrong construction works on their particular example.
5. Suppose we augmented a finite automaton with an additional feature: a single mutable variable. On each transition, it may read from or write to this variable.
 - a) If the variable is of type Σ (that is, it can contain one alphabet symbol), what class of languages can such automata recognize?
 - b) If the variable is of type $\mathbb{N}$, what class of languages can they recognize?
 6. A queue automaton is like a pushdown automaton, but with a queue (fifo) instead of a stack (lifo). These can recognize any Turing-recognizable language. Now suppose we instead defined a pushdown automaton with an additional stack. What class of languages can these 2-stack PDAs recognize?