Introduction to Theoretical Computer Science

Exercise Sheet: Week 8

(1) The VERTEX-COVER problem: Given a graph $G = (V, E)$, a vertex cover is a subset $V' \subseteq V$ of the vertices such that every edge has at least one end in the cover V'. VERTEX-COVER is, given a graph G and an integer k, does G have a vertex cover of size k?

Give reductions from CLIQUE to VERTEX-COVER, and the other way round. (*Hint:* complementary graph.)

(2) The following properties were mentioned in the lectures (with only brief explanations):

- PSpace $\supseteq$ PTime
- PSpace $\supseteq$ NPTime
- PSpace $\subseteq$ ExpTime

Write out explanations in enough detail to show how a proof would work.

(3) P^{NP} obviously includes all of NP and co-NP. So how does $\mathrm{NP}^{\mathrm{NP}}$ differ from it – what else is there? (Assuming, that is, that $\mathrm{P} \neq \mathrm{NP}$.)