Computational Cognitive Science

Lecture 10: Categorization

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Overview

Models of Categorization

Prototype vs. Exemplar Model

Categorization of Real-World Stimuli

Human Behavior

Stimulus Space

Results

Reading: Battleday et al. (2020).
Background: Section 1.2 of Farrell and Lewandowsky (2018) on exemplar models.

Models of Categorization

Categorization

In the last lecture, we discussed a Bayesian model of concept learning (Tenenbaum 2000). This model combined rule-like behavior and similarity-based categorization.

Today, we will look at similarity-based categorization in more detail:

- Prototype models: the category membership of a new stimulus is calculated based on the similarity to prototypes of candidate categories.
- Exemplar models: decision rule based on the summed similarity to all members of the category.

We discuss these two model types and then ask: What is the role of the representations we use to calculate similarity?

Categorization

The experiment of Nosofsky (1987):

- In each trial, participants saw one of 12 colors and classified the color into categories A or B. They then received feedback.
- After a learning period, the classification responses were taken as data to evaluate categorization models.

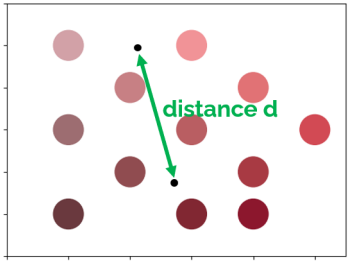


Categorization



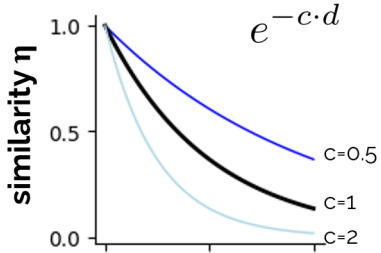
Psychological space dimension 2 (here: Brightness)

Stimuli are represented in **psychological space**



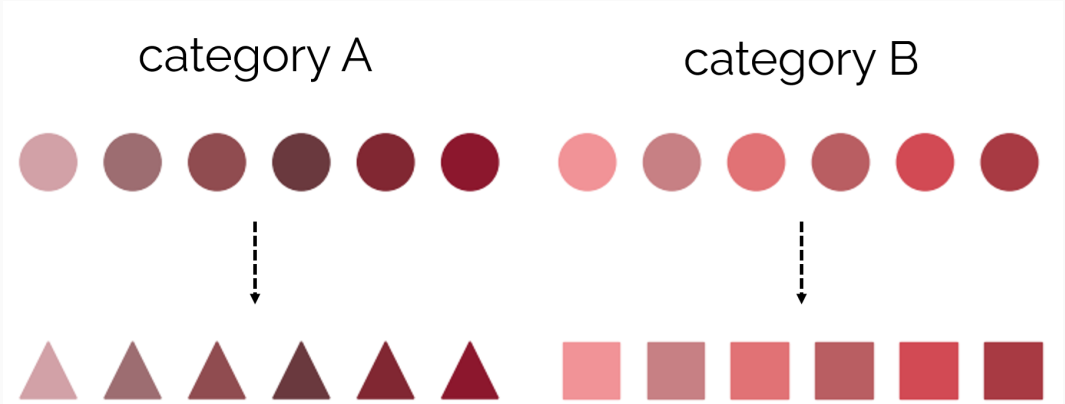
Psychological space dimension 1 (here: Saturation)

Similarity η is a function of distance in psychological space



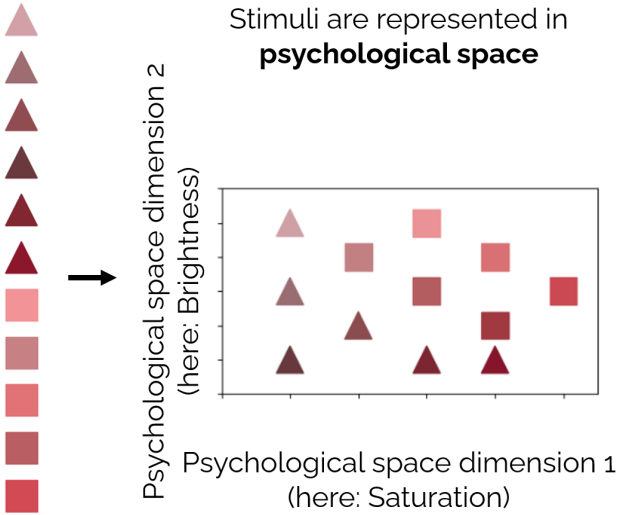
distance d in psychological space

Categorization



We can visualize the category as triangle (A) or square (B). **The participants never saw a triangle or square.**

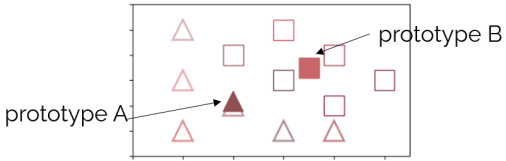
Categorization



Categorization

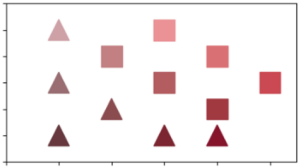
Learned representation

Prototype model



One prototype (centroid of training examples) per category in memory.

Exemplar model

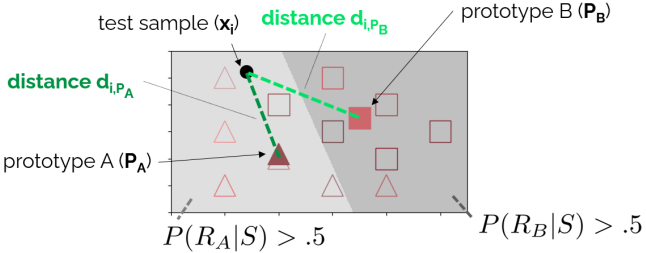


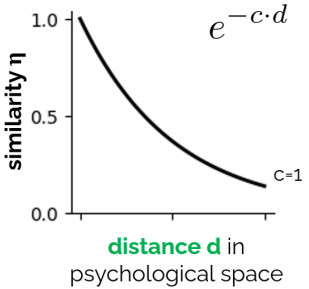
All training examples are stored in memory.

Prototype vs. Exemplar Model

Categorization: Prototype Model

psychological space





similarities

$$\eta_{i,P_A} = \exp[-c\sqrt{(x_{i,1} - P_{A,1})^2 + (x_{i,2} - P_{A,2})^2}]$$

d_{i,P_A}

similarity of x_i to prototype P_A

$$\eta_{i,P_B} = \exp[-c\sqrt{(x_{i,1} - P_{B,1})^2 + (x_{i,2} - P_{B,2})^2}]$$

d_{i,P_B}

similarity of x_i to prototype P_B

response model

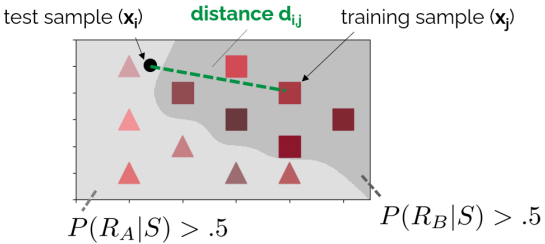
$$P(R_A|S) = \frac{B_A \eta_{i,P_A}}{B_A \eta_{i,P_A} + (1 - B_A)\eta_{i,P_B}}$$

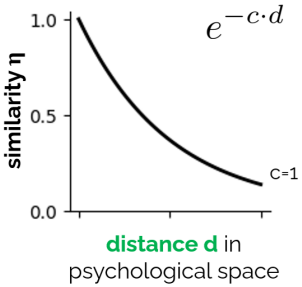
probability of responding A to stimulus S

B_A response bias

Categorization: Exemplar Model

psychological space





similarities

$$\eta_{i,j} = \exp[-c\sqrt{(x_{i,1} - x_{j,1})^2 + (x_{i,2} - x_{j,2})^2)}]$$

$d_{i,j}$

similarity of $\mathbf{x}_i$ to training sample j

response model

$$P(R_A|S) = \frac{B_A \sum_{j \in A} \eta_{i,j}}{B_A \sum_{j \in A} \eta_{i,j} + (1 - B_A) \sum_{j \in B} \eta_{i,j}}$$

probability of responding A to stimulus S

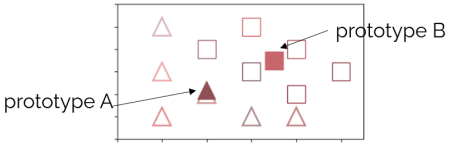
B_A response bias

Categorization: Prototypes vs. Exemplar Model

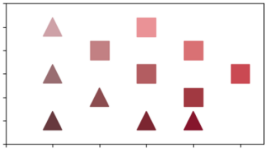
Prototype model

Exemplar model

Learned representation



One prototype (centroid of training examples) per category in memory.



All training examples are stored in memory.

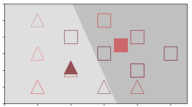
Categorization

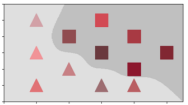
$$\eta_{i,P_A} = \exp[-c\sqrt{(x_{i,1} - P_{A,1})^2 + (x_{i,2} - P_{A,2})^2)}]$$

$$P(R_A|S) = \frac{B_A \eta_{i,P_A}}{B_A \eta_{i,P_A} + (1 - B_A)\eta_{i,P_B}}$$

$$\eta_{i,j} = \exp[-c\sqrt{(x_{i,1} - x_{j,1})^2 + (x_{i,2} - x_{j,2})^2)}]$$

$$P(R_A|S) = \frac{B_A \sum_{j \in A} \eta_{i,j}}{B_A \sum_{j \in A} \eta_{i,j} + (1 - B_A) \sum_{j \in B} \eta_{i,j}}$$





Categorization: Interim Summary

Compared to the prototype model, the exemplar model can express much more complex decision boundaries.

However, what matters is not only how we make categorization decisions based on similarity, but also the representations we use to calculate similarity.

To evaluate this, we will look at more complex, real world stimuli.

Time for a short quiz on WooClap!

Go to wooclap.com

Enter the event code in the top banner

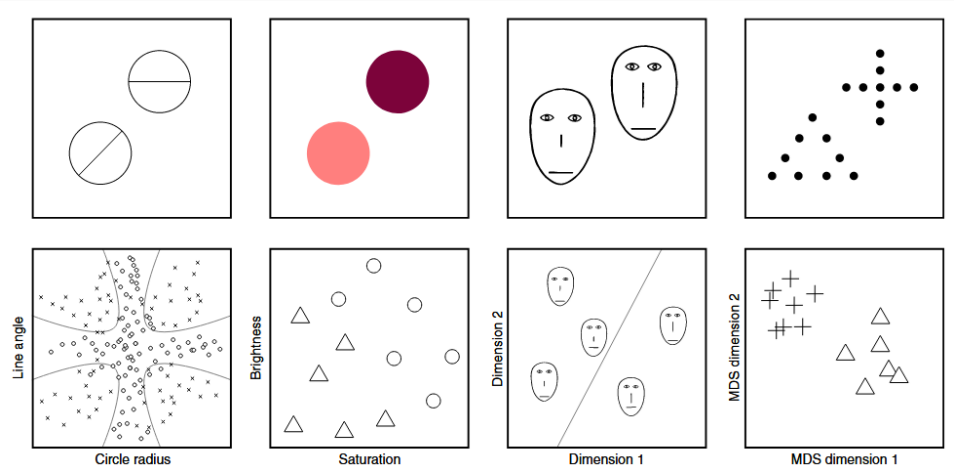
Event code

KOMZAYN



Categorization of Real-World Stimuli

Categorization of Real-World Stimuli



Studies have used highly controlled stimuli, which entails the use of very simple representations. These studies mostly show support for exemplar models.

Categorization of Real-World Stimuli

But do exemplar models account mode how we categorize real-world stimuli?

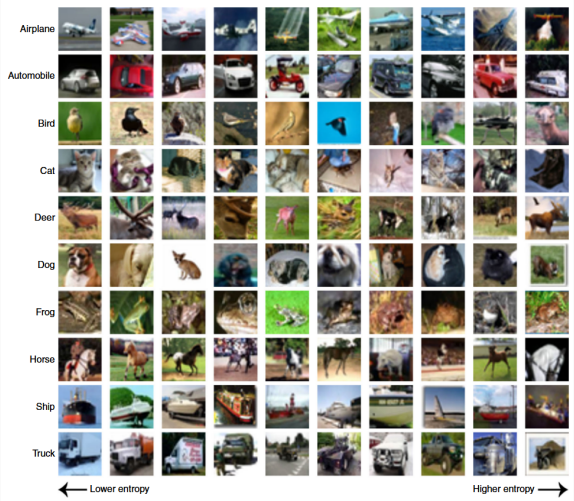
- Maybe categorization experiments on simplified stimuli do not capture how we categorize in our everyday environment (ecological validity).
- Real-world stimuli require more high-dimensional representations than controlled stimuli. Dimensionality matters for computing similarity.

Are the exemplar models still better for real-world stimuli?

Battleday et al. (2020) tested this in a study that used photographic images for categorization.

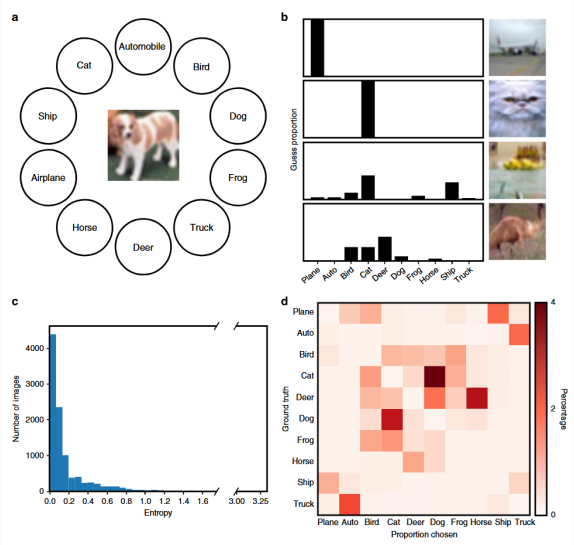
Human Behavior

Categorization of Real-World Stimuli



Battleday et al. (2020) test categorization on images of a machine learning dataset (CIFAR10), which contains 10 categories of objects.

Human Behavior

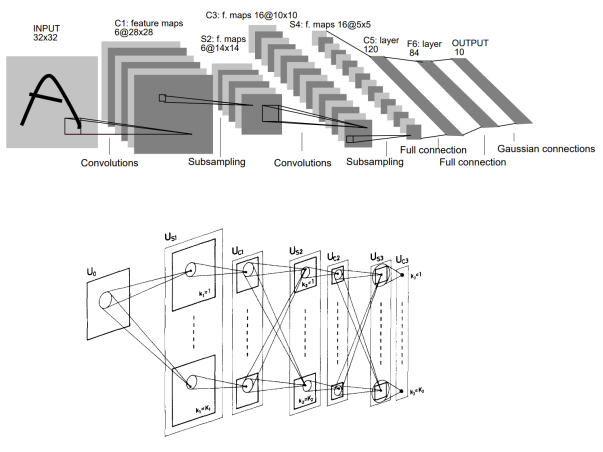


10,000 images, 50 repetitions each (across 2570 participants).

Stimulus Space

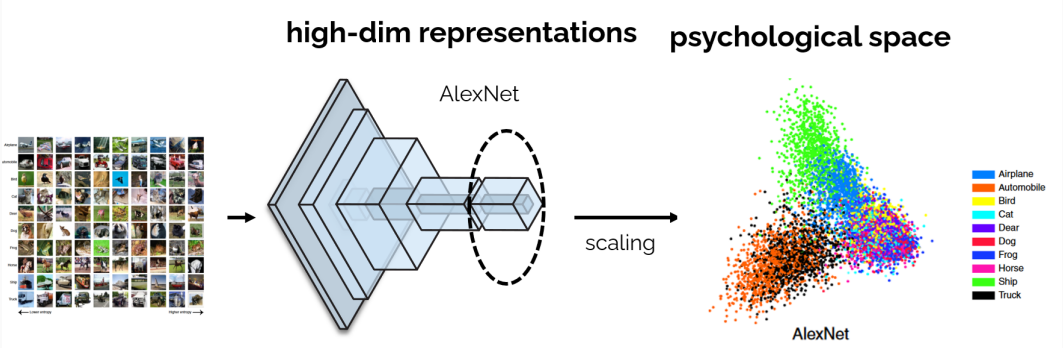
Stimulus Space

The authors chose the latent dimensions of convolutional neural networks (CCNs), pre-trained on image classification.



Top: LeNet (LeCun et al. 1989). Bottom: Neocognitron (Fukushima 1980).

Psychological Space



2D projection of the high-dimensional psychological space (for visualization)

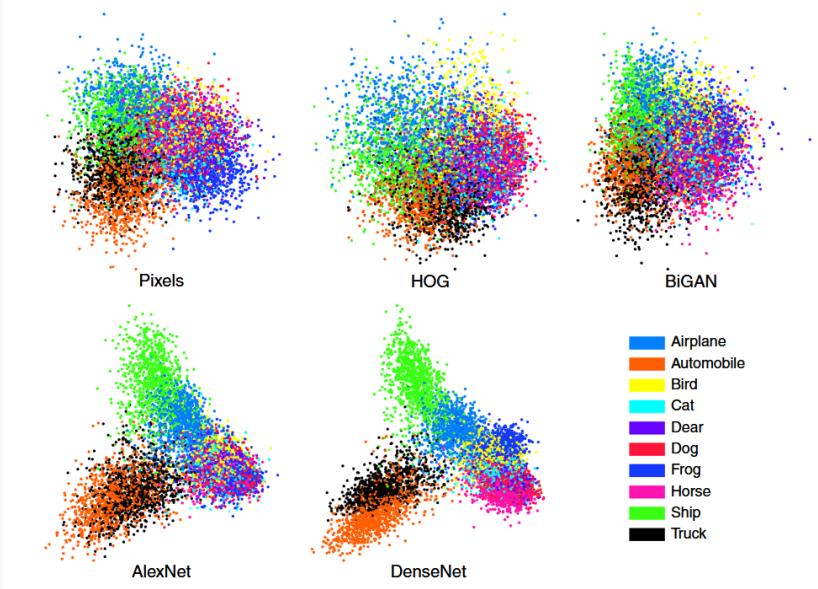
A K-dimensional vector (AlexNet: $K = 4096$) of the last layer of a CNN trained on ImageNet serves as the stimulus representation. After optional scaling (see next slide), this is our psychological space and we can compute psychological distance and transform them into similarities (as before).

Scaling

Each image is represented by a K-dimensional vector (AlexNet: $K = 4096$). Some dimensions have more variability across stimuli than others. The authors consider transformations of the stimulus space before computing the Euclidean distance as psychological distance.

- classic: no transformation
- linear: stretch and rotate such that the variations across images are equal in all directions (whitening)
- quadratic: as linear, but transform for each category.

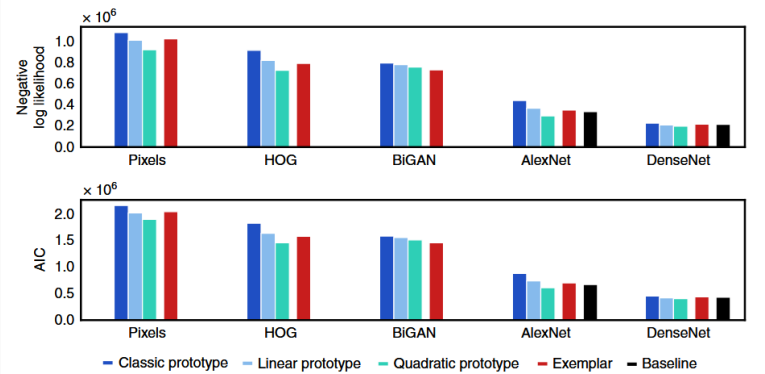
Other Stimulus Spaces



Results

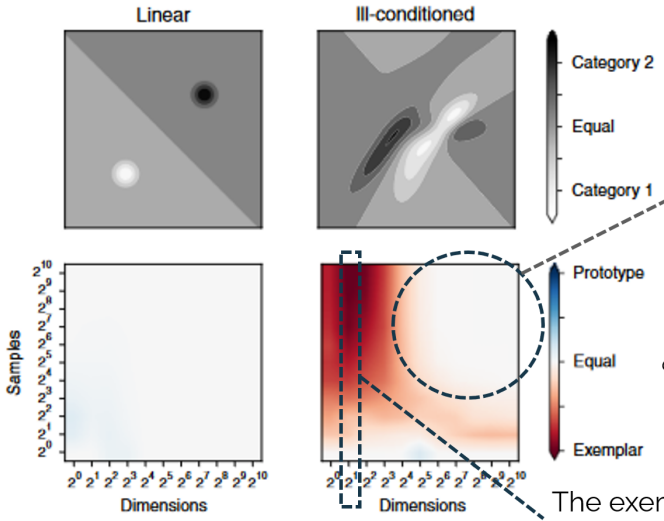
Results

Now compare the predicted categorization from classic prototype and exemplar model for different stimulus representations with human behavior via log-likelihood and AIC.



- No substantial benefit for exemplar over prototype models.
- Large effect of the stimulus representations.

Exemplar and Prototype Models in High Dimensions



The advantage of the exemplar model for complex problems disappears in high dimensions.

color: which model is better?

The exemplar model outperforms the prototype model in 2D.

Summary

- We revisited exemplar and prototype models and introduced the intuition of decision boundaries in psychological space.
- Models have so far been tested in abstracted, low-dimensional stimulus spaces. Here, exemplar models often outperform prototype models.
- In high-dimensional, real-world stimulus spaces, both models perform comparably.

Studying real-world complexity and behavior:

- A fundamental challenge of the brain is to deal with the complexity of the world. This challenge shapes the mechanisms of the mind.
- Principles gleaned from abstracted tasks may not scale to real-world settings.

References

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