THE UNIVERSITY of EDINBURGH

Advanced Database Systems

Spring 2024

Lecture #22:

Query Optimisation: Costing

R&G: Chapter 15

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What is Needed for Query Optimisation

Given: A closed set of operators

- Relational operators (table in, table out)
- Physical implementations (of those operators and a few more)

Plan space

- Based on relational equivalences, different implementations

Cost estimation

- Cost formula & size estimation for each physical operator

Search algorithm

- To sift through the plan space and find lowest cost option!

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Cost Estimation

For each plan considered, must estimate total cost:

- Must estimate **cost of each operation** in plan tree
  - Depends on input cardinalities
  - Already discussed costs for various operators (sequential scan, index scan, joins, etc.)
- Must estimate **size of result** for each operation in tree!
  - Because it determines downstream input cardinalities!
  - Use information about the input relations
  - For selections and joins, assume independence of predicates

In System R, cost boils down to a single number: **#I/Os + CPU-factor * #tuples**

- Second term estimate the cost of tuple processing

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Statistics and Catalogs

System catalogs store internal statistics about tables, attributes, and indexes

- Typically contain at least:

STATISTIC	MEANING
NTuples	# of tuples in a table (cardinality)
NPages	# of disk pages in a table or index
Low/High	min/max value in a column
NKeys	# of distinct values in a column
Height	the height of an index

Can also keep more detailed statistical information on data values (e.g., histograms)

Catalogs are updated periodically

- Users can also manually refresh them (e.g,. ANALYZE in PostgreSQL)
- Too expensive to do continuously. Lots of approximation anyway, so a little slop is OK

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Size Estimation and Selectivity

Max output cardinality = product of input cardinalities

Selectivity (sel) associated with each term

- Reflects the impact of the term in reducing result size
- Selectivity = |output| / |input|
- Sometimes called "Reduction Factor" (RF)
- Always between 0 and 1

Avoid confusion:

- "highly selective" in common English is opposite of a high selectivity value (|output|/|input| high!)

Selection Estimates

The **selectivity** (sel) of a predicate P is the fraction of tuples that qualify

Equality predicates on unique keys are easy to estimate

What about more complex predicates?
What is their selectivity?

Formula depends on type of predicate

- Equality, range, negation, conjunction, disjunction

```
SELECT * FROM Students
 WHERE sid = 123
```

```
SELECT * FROM Students
 WHERE age = 21
```

```
SELECT * FROM Students
 WHERE age > 22
   AND dept = 'CS'
```


Selections - Complex Predicates

Assume attribute *age* in relation *Students* has five distinct values (20-24)

- NKeys(age) = 5

```
SELECT * FROM Students
 WHERE age = 22
```

```
SELECT * FROM Students
 WHERE age > 22
```

Equality predicate

- sel(A = constant) = 1 / NKeys(A)
- Example: sel(age = 22) = 1/5

Range predicate

- sel(A > a) = (High(A) – a) / (High(A) – Low(A) + 1) *(when A is integer-valued column)*
- sel(A > a) = (High(A) – a) / (High(A) – Low(A)) *(when A is floating-valued column)*
- Example: sel(age > 22) = (24 – 22) / (24 – 20 + 1) = 2/5

Selections - Complex Predicates

Equality predicate

- sel(A = B) = 1 / max{NKeys(A), NKeys(B)} (handy for joins, too)
- Why MAX?
  - Assume that A-values and B-values are **independent**
  - Let there be 2 distinct A-values { v1, v2 } and 10 distinct B-values { v1, ..., v10 }
  - What is the probability of matching values?
  - $P(A = B) = \Sigma_i P(A = v_i, B = v_i) = \Sigma_i P(A = v_i) \cdot P(B = v_i)$
  - $= (1/2 \cdot 1/10) + (1/2 \cdot 1/10) + (0 \cdot 1/10) + \ldots$
  - $= 1/10 = 1 / \max\{ 2, 10 \}$

SELECTIONS - COMPLEX PREDICATES

Negation query

sel(not P) = 1 – sel(P)

Example: sel(age != 22) = 1 – 1/5 = 4/5

Observation: selectivity ≈ probability

```
SELECT * FROM Students
WHERE age != 22
```

Conjunction

sel(P1 ∧ P2) = sel(P1) · sel(P2)

Assumes that the predicates are independent

```
SELECT * FROM Students
WHERE age = 22
  AND name LIKE 'A%'
```

Disjunction

sel(P1 v P2) = sel(P1) + sel(P2) – sel(P1 ∧ P2)

Assumes that the predicates are independent

```
SELECT * FROM Students
WHERE age = 22
   OR name LIKE 'A%'
```


RESULT SIZE ESTIMATION FOR JOINS

How to estimate the size of a join between **R** and **S**?

Key-foreign key join

Example: **S** has a foreign key referencing **R**

The foreign key constraint guarantees $\pi_A(S) \subseteq \pi_A(R)$, thus $|R \bowtie S| = |S|$

Assumes non-null FK values (e.g., if A is part of a primary key in S); otherwise, $|R \bowtie S| \leq |S|$

RESULT SIZE ESTIMATION FOR JOINS

General case: **R** join **S** on A which is not a key for either table

Recall algebraic equivalence: $R \bowtie_p S \equiv \sigma_p(R \times S)$

Hence join selectivity is "just" selectivity sel(p) over a big input |R| · |S|!

Total rows: sel(p) · |R| · |S|

Equi-join on A

Match each **R**-tuple with **S**-tuple: $|R \bowtie S| \approx |R| \cdot |S|$ / NKeys(S.A)

Symmetrically, for **S**: $|R \bowtie S| \approx |S| \cdot |R|$ / NKeys(R.A)

The final estimate is the smaller of the two estimates

Overall: $|R \bowtie S| \approx |R| \cdot |S| \cdot \underbrace{1 / \max\{\text{NKeys(R.A)}, \text{NKeys(S.A)}\}}_{\text{sel(R.A = S.A)}}$

MISSING STATISTICS? USE DEFAULT VALUES

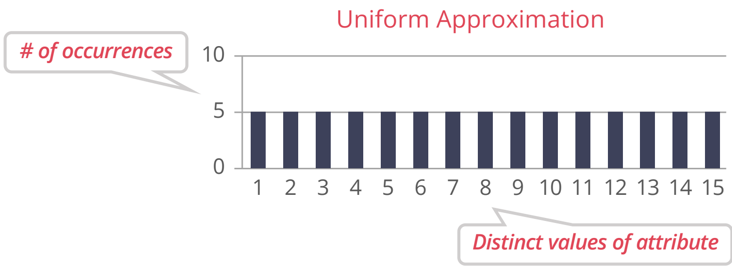
```
/* default selectivity estimate for equalities such as "A = b" */
#define DEFAULT_EQ_SEL  0.005

/* default selectivity estimate for inequalities such as "A < b" */
#define DEFAULT_INEQ_SEL  0.3333333333333333

/* default selectivity estimate for range inequalities "A > b AND A < c" */
#define DEFAULT_RANGE_INEQ_SEL  0.005

/* default selectivity estimate for multirange inequalities "A > b AND A < c" */
#define DEFAULT_MULTIRANGE_INEQ_SEL 0.005

/* default selectivity estimate for pattern-match operators such as LIKE */
#define DEFAULT_MATCH_SEL   0.005

/* default selectivity estimate for other matching operators */
#define DEFAULT_MATCHING_SEL    0.010

/* default number of distinct values in a table */
#define DEFAULT_NUM_DISTINCT  200
```

Postgres 13.0

src/include/utils/selfuncs.h

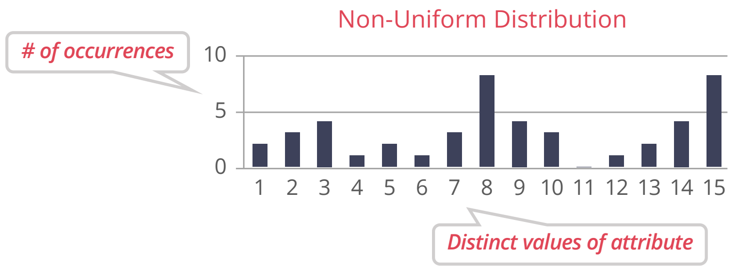
Cost Estimation

Our cost formulas assume that data values are uniformly distributed



Cost Estimation

In practice, attribute values typically have a non-uniform distribution



Histograms

To keep track of this non-uniformity for an attribute *A*, we can maintain a **histogram** to approximate the actual distribution

- Divide the active domain of *A* into adjacent intervals
- Collect statistical parameters for each interval $(b_{i-1}, b_i]$, for example
  - # of tuples r with $b_{i-1} < r.A \leq b_i$
  - # of distinct A values in interval $(b_{i-1}, b_i]$

The histogram intervals are also called **buckets**

Types of Histograms

Equi-width histograms

- All buckets have the **same width w** or number of distinct values
- I.e., boundary $b_{i+1} = b_i + w$ for some fixed width w

Equi-depth histograms

- All buckets contain the **same number of tuples** (their width may vary)
- Able to adapt to data skew (high uniformity)

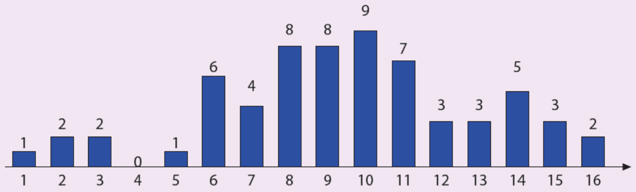
The number of buckets is the tuning knob that defines the trade-off between **histogram resolution** (estimation quality) and **histogram size**

- Catalog space is limited!

Equi-Width Histograms

Example (Actual value distribution)

Column A of SQL type INTEGER (domain $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$).
Actual non-uniform distribution in relation R:



\# of distinct values = 16, # of tuples = 64

Equi-Width Histograms

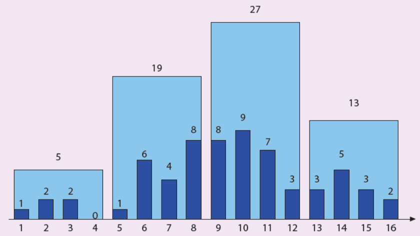
Maintain sum of value frequencies in each bucket
(in addition to bucket boundaries b_i)

Divide active domain of A into B buckets of equal width

Bucket width w:

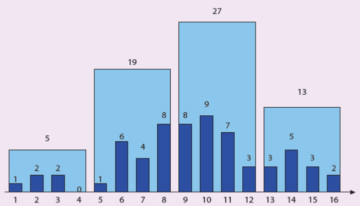
$$w = \frac{High(\mathtt{A}, R) - Low(\mathtt{A}, R) + 1}{B}$$

Example (Equi-width histogram ($B = 4$))



Equality Selection

Example ($Q \equiv \sigma_{\mathtt{A}=5}(R)$)

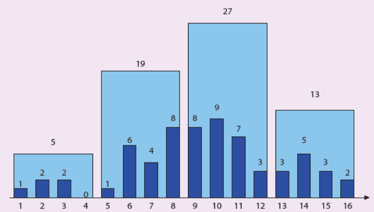


- Value 5 is in bucket $[5, 8]$ (with 19 tuples)
- Assume **uniform distribution within the bucket**:

$$|Q| = 19/w = 19/4 \approx 5 \ .$$

Range Selection

Example ($Q \equiv \sigma_{\mathtt{A}>7 \ \mathtt{AND}\ \mathtt{A}\leqslant 16}(R)$)



- Query interval $(7, 16]$ covers buckets $[9, 12]$ and $[13, 16]$. Query interval touches $[5, 18]$.

$$|Q| = 27 + 13 + {}^{19}\!/_{4} \approx 45 \ .$$

Equi-Depth Histograms

Divide active domain of attribute *A* into B buckets with ***roughly*** the same number of tuples in each bucket

- Depth d of each bucket will be approximately |R|/ B

Maintain depth d and bucket boundaries b_i

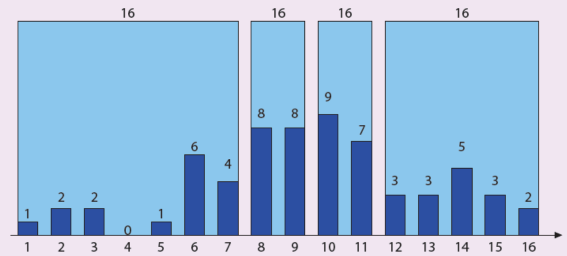
Intuition:

- High-value frequencies are more important than low-value frequencies and put in smaller buckets
- Equi-depth provides better estimates than equi-width for highly frequent values

Resolution of histogram adapts to skewed value distributions

Equi-Depth Histogram

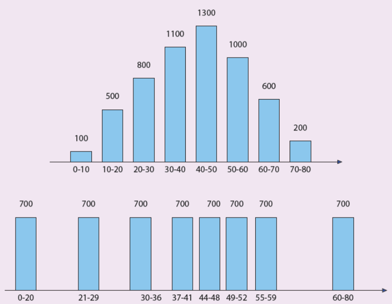
Example (Equi-depth histogram ($B = 4, d = 16$))



Comparison

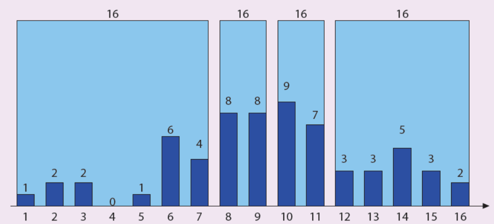
Equi-depth histogram "invests" bytes in the densely populated customer age region between 30 and 59

Example (Histogram on *customer age* attribute ($B = 8, |R| = 5{,}600$))



Equality Selection

Example ($Q \equiv \sigma_{A=5}(R)$)

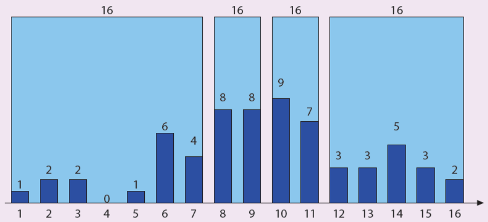


- Value 5 is in first bucket [1, 7] (with $d = 16$ tuples)
- Assume **uniform distribution within the bucket**:

$$|Q| = d/7 = 16/7 \approx 2 \ .$$

Range Selection

Example ($Q \equiv \sigma_{\mathtt{A>5\ AND\ A\leqslant 16}}(R)$)



- Query interval $(5, 16]$ covers buckets $[8, 9]$, $[10, 11]$ and $[12, 16]$ (all with $d = 16$ tuples). Query interval touches $[1, 7]$.

$$|Q| = 16 + 16 + 16 + {}^{2}/{}_{7} \cdot 16 \approx 53 \ .$$

Summary: Selectivity Estimation

We need a way to estimate the size of the intermediate tables

Output size = input size * operator selectivity

Assumption 1: Uniform distribution within histogram bins

- Within a bin, fraction of range = fraction of count

Assumption 2: Independent predicates

- Selectivity of AND = product of selectivities of predicates
- Selectivity of OR = sum of selectivities of predicates – product of selectivities of predicates
- Selectivity of NOT = 1 – selectivity of predicates

General joins

- Simply compute the selectivity of all predicates
- And multiply by the product of the table sizes