Conjunctive Queries: Fast Evaluation

(Chapter 18 of DBT)

[DBT] Database Theory, https://github.com/pdm-book/community

Complexity of Query Evaluation

Theorem: CQ-Evaluation is NP-complete and in PTIME in data complexity

Proof:

(NP-membership) Guess-and-check:

- Consider a database D, a CQ $Q(x_1,\ldots,x_k)$:- body, and a tuple $(a_1,\ldots,a_k)$ of values
- Guess a substitution h : terms(body) → terms(D)
- Verify that h is a match of Q in D, i.e., h(body) ⊆ D and $(h(x_1),\ldots,h(x_k)) = (a_1,\ldots,a_k)$

(NP-hardness) Reduction from 3-colorability

(in PTIME) For every substitution h : terms(body) → terms(D), check if h(body) ⊆ D

and $(h(x_1),\ldots,h(x_k)) = (a_1,\ldots,a_k)$

Complexity of Query Evaluation

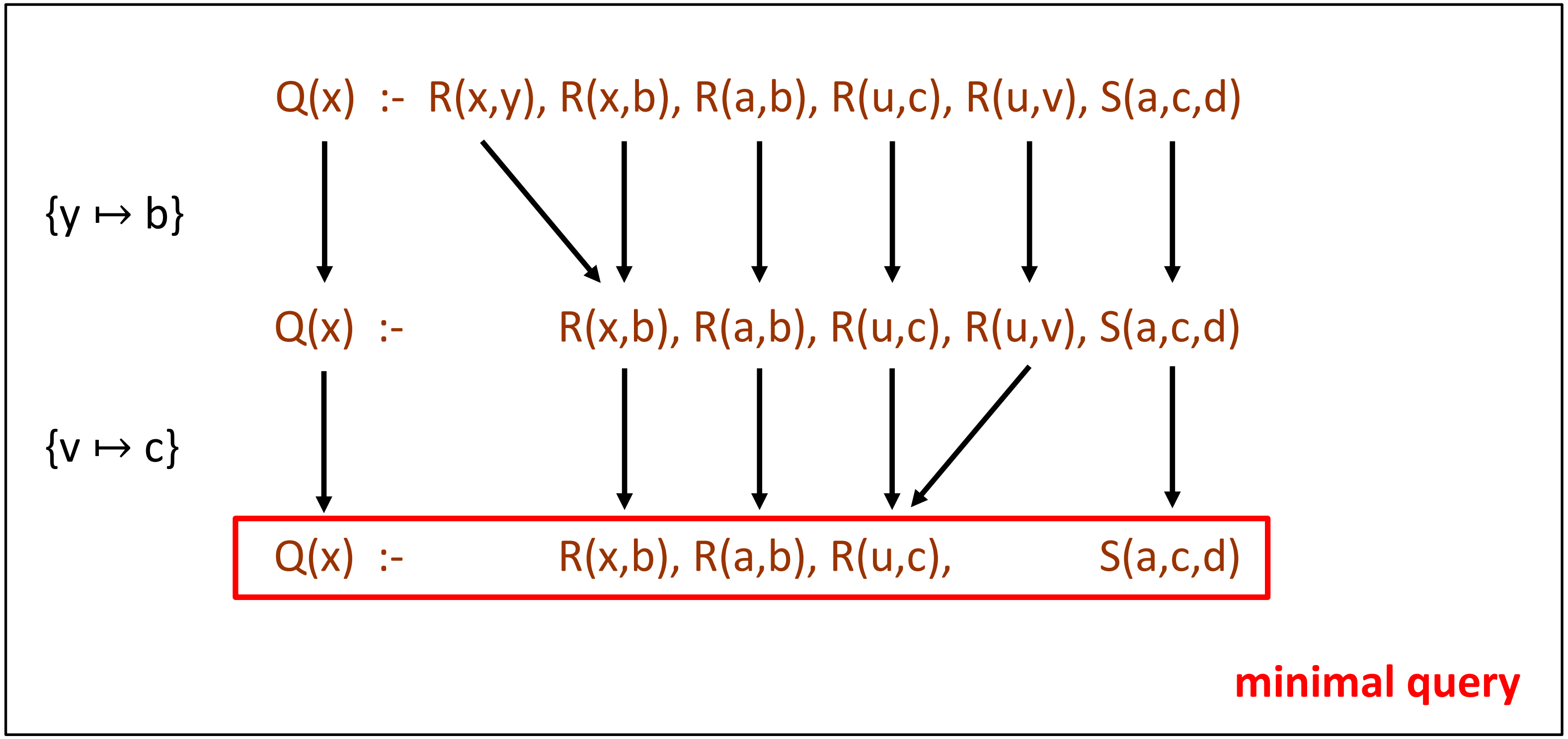
Theorem: CQ-Evaluation is NP-complete and in PTIME in data complexity

Evaluating a CQ Q over a database D takes time $||D||^{O(||Q||)}$

Minimizing Conjunctive Queries

Database theory has developed principled methods for optimizing CQs:

- Find an equivalent CQ with minimal number of atoms (the core)
- Provides a notion of “true” optimality



Minimizing Conjunctive Queries

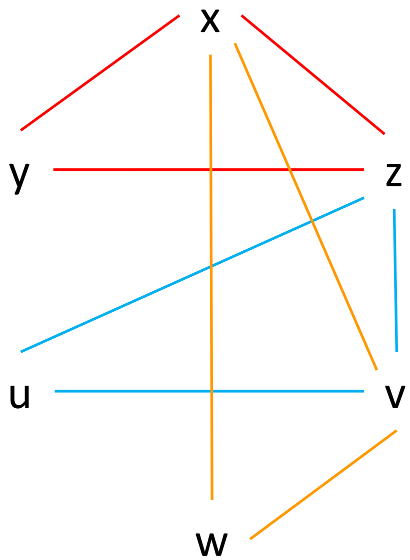
- But, a minimal equivalent CQ might not be easier to evaluate - remains NP-hard

- “Good” classes of CQs for which query evaluation is tractable (*in combined complexity*):
  - Graph-based
  - Hypergraph-based

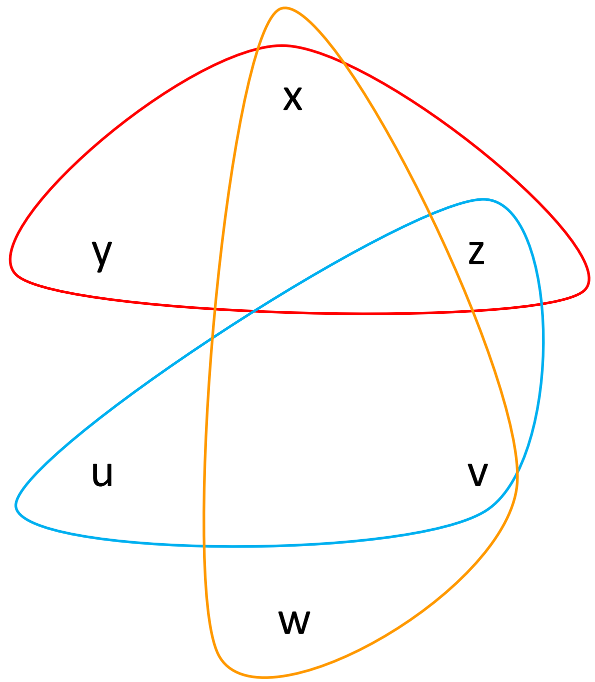
(Hyper)graph of Conjunctive Queries

Q :- R(x,y,z), R(z,u,v), R(v,w,x)

graph of Q - G(Q)



hypergraph of Q - H(Q)



“Good” Classes of Conjunctive Queries

measures how close a graph is to a tree

- Graph-based
  - CQs of bounded treewidth - their graph has bounded treewidth

measures how close a hypergraph is to an acyclic one

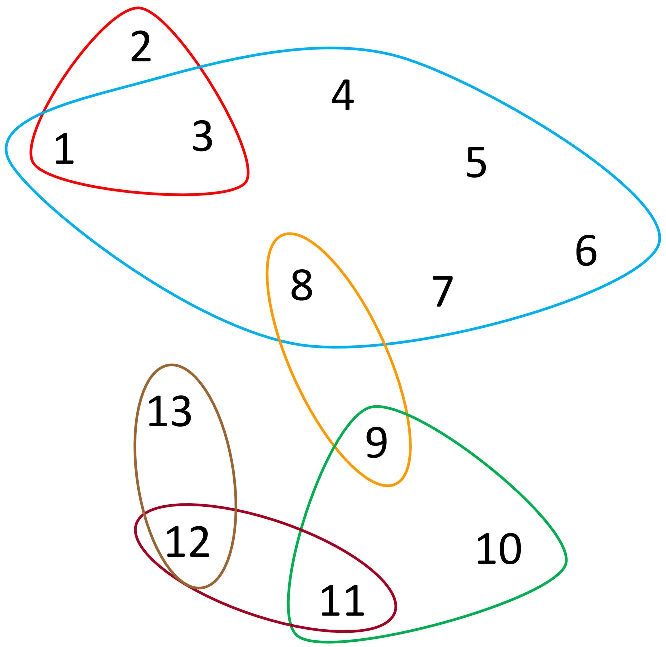
- Hypergraph-based:
  - CQs of bounded hypertree width - their hypergraph has bounded hypertree width
  - **Acyclic CQs - their hypergraph has hypertree width 1**

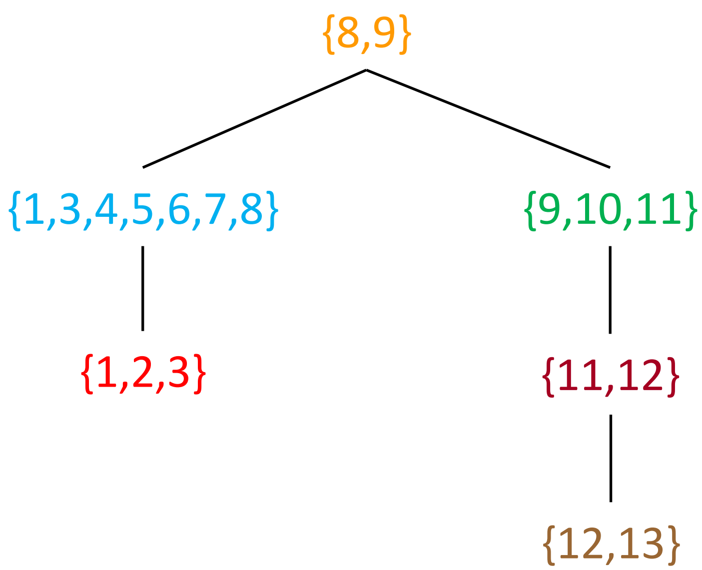
Acyclic Hypergraphs

A join tree of a hypergraph **H** = (V,E) is a labeled tree **T** = (N,F,L), where L : N → E such that:

1. For each hyperedge e ∈ E of **H**, there exists n ∈ N such that e = L(n)

2. For each node u ∈ V of **H**, the set {n ∈ N | u ∈ L(n)} induces a connected subtree of **T**





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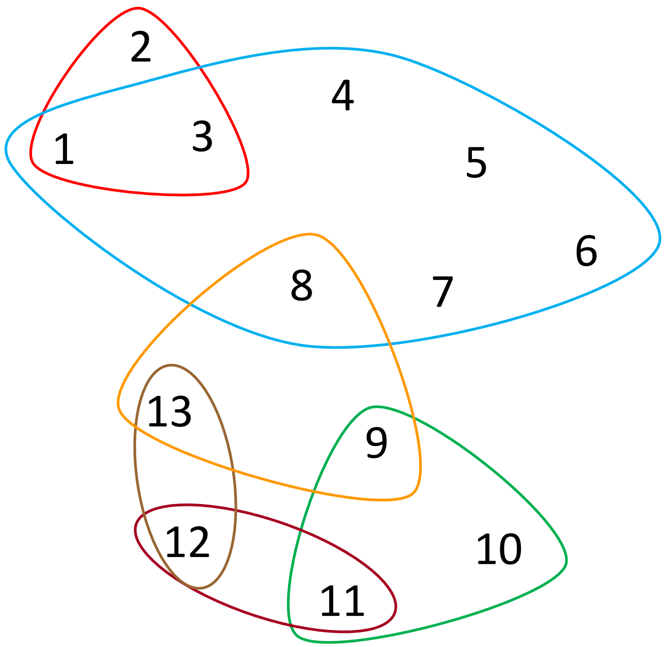

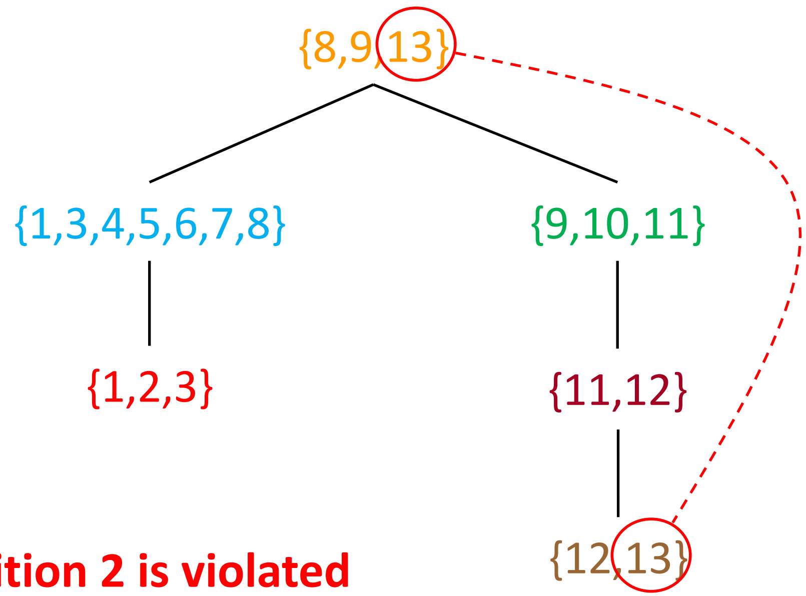

condition 2 is violated

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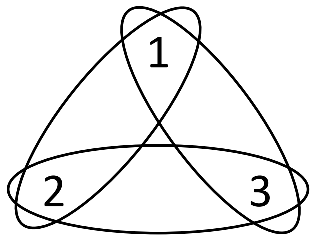
Definition: A hypergraph is acyclic if it has a join tree



prime example of a cyclic hypergraph

Acyclic Hypergraphs

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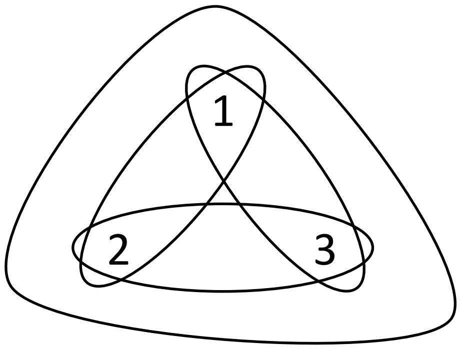


but this is acyclic

Relevant Algorithmic Tasks

ACYCLICITY

Input: a conjunctive query Q

Question: is Q acyclic? or is H(Q) acyclic?

{Q ∈ **CQ** | H(Q) is acyclic}

ACQ-Evaluation

Input: a database D, an acyclic conjunctive query Q, and a tuple $(a_1,\ldots,a_k)$ of values

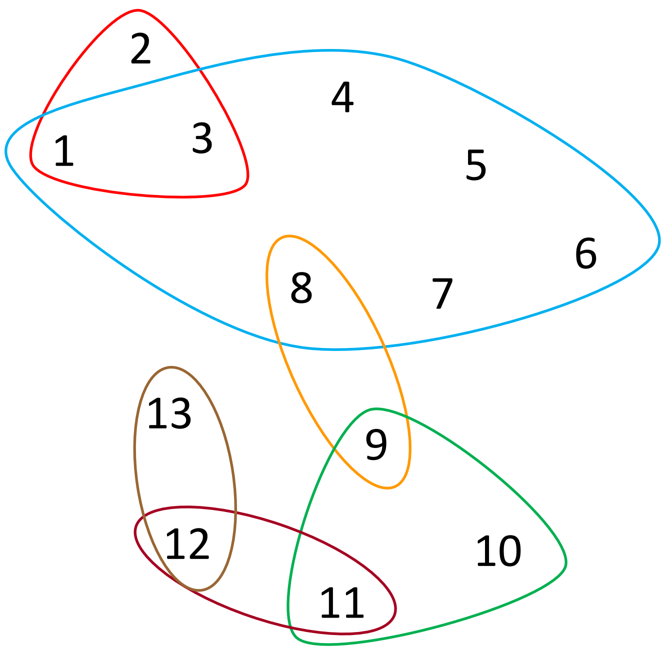
Question: $(a_1,\ldots,a_k) \in Q(D)$?

Checking Acyclicity

Via the GYO-reduction (Graham, Yu and Ozsoyoglu)

1. Eliminate nodes occurring in at most one hyperedge

2. Eliminate hyperedges that are empty or contained in other hyperedges



{8,9}

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{9,10,11}

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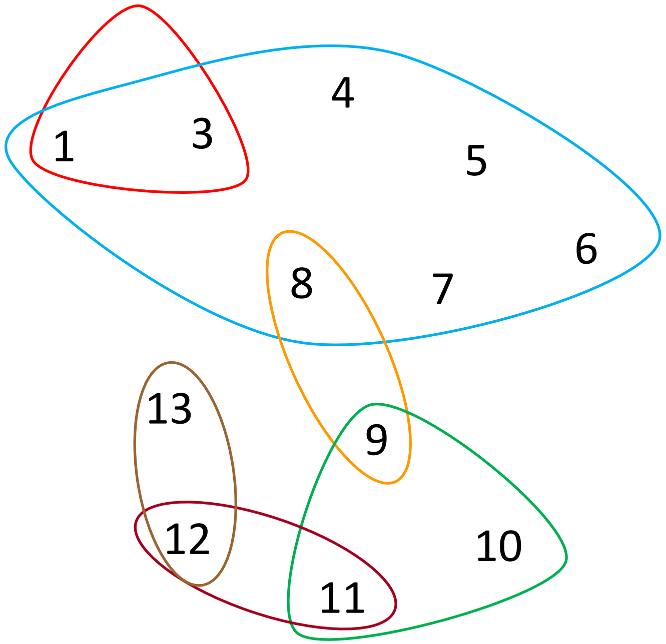

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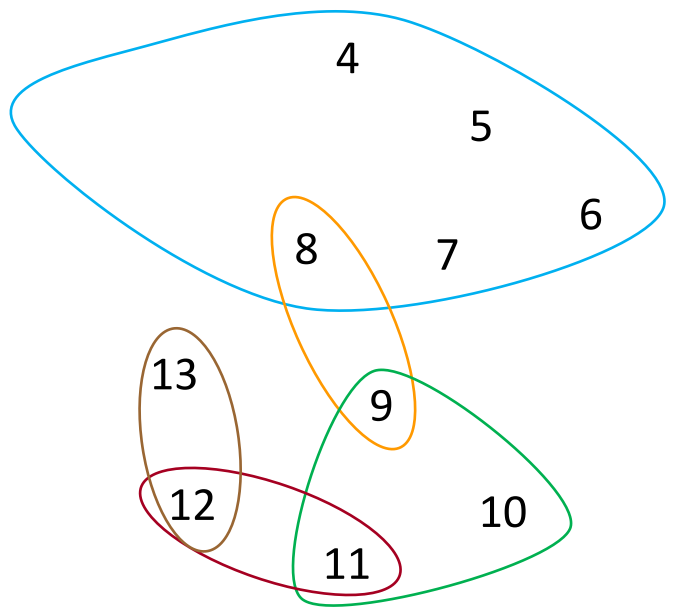

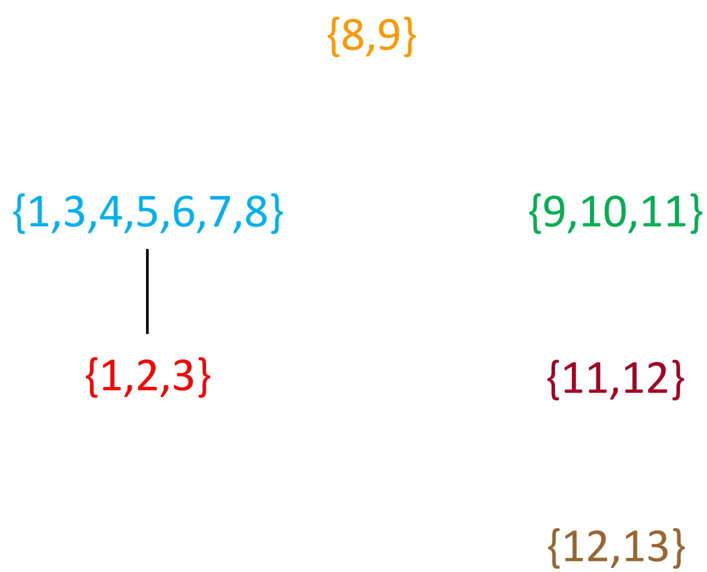

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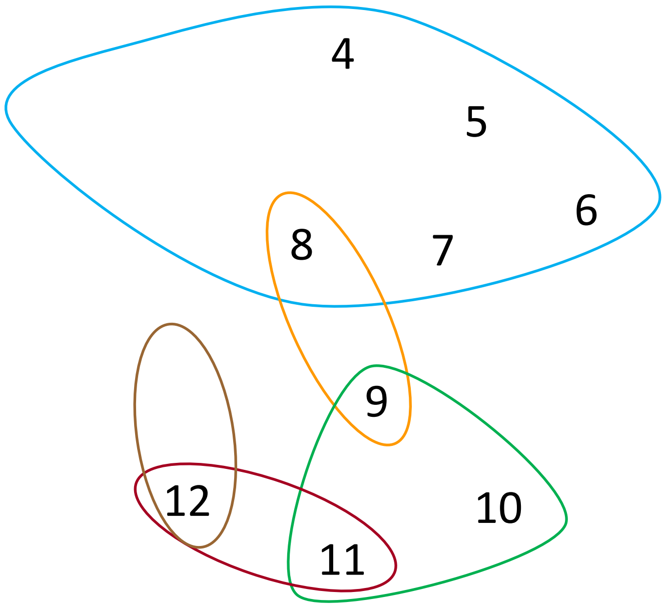

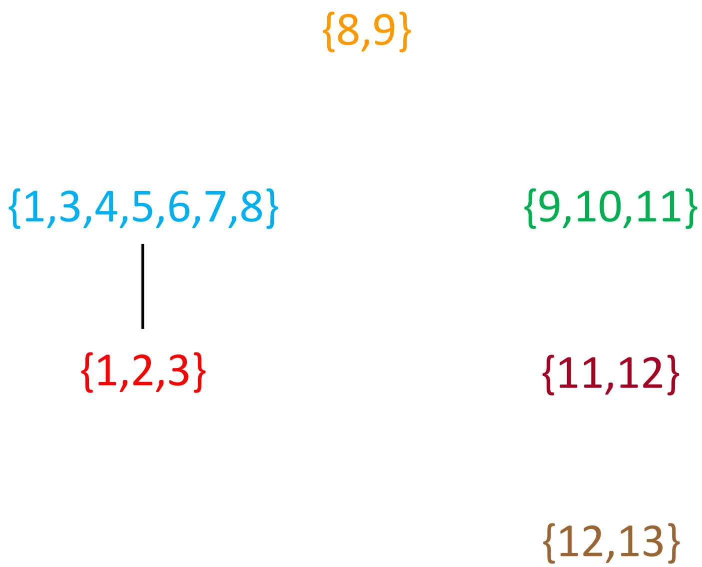

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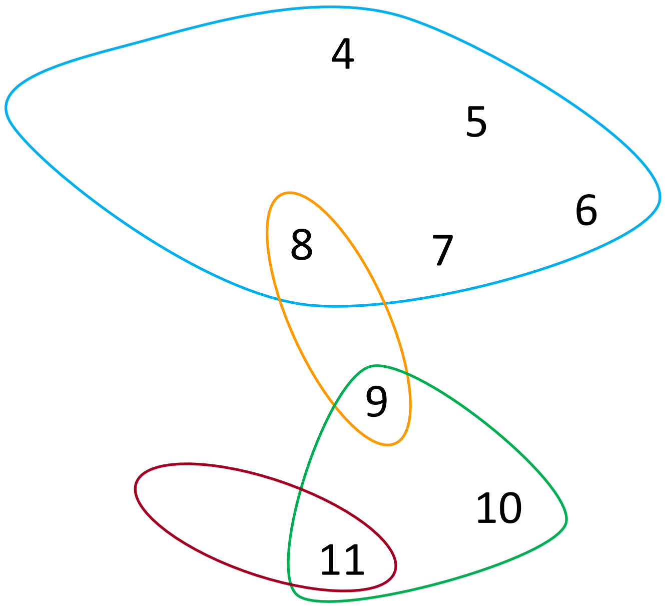

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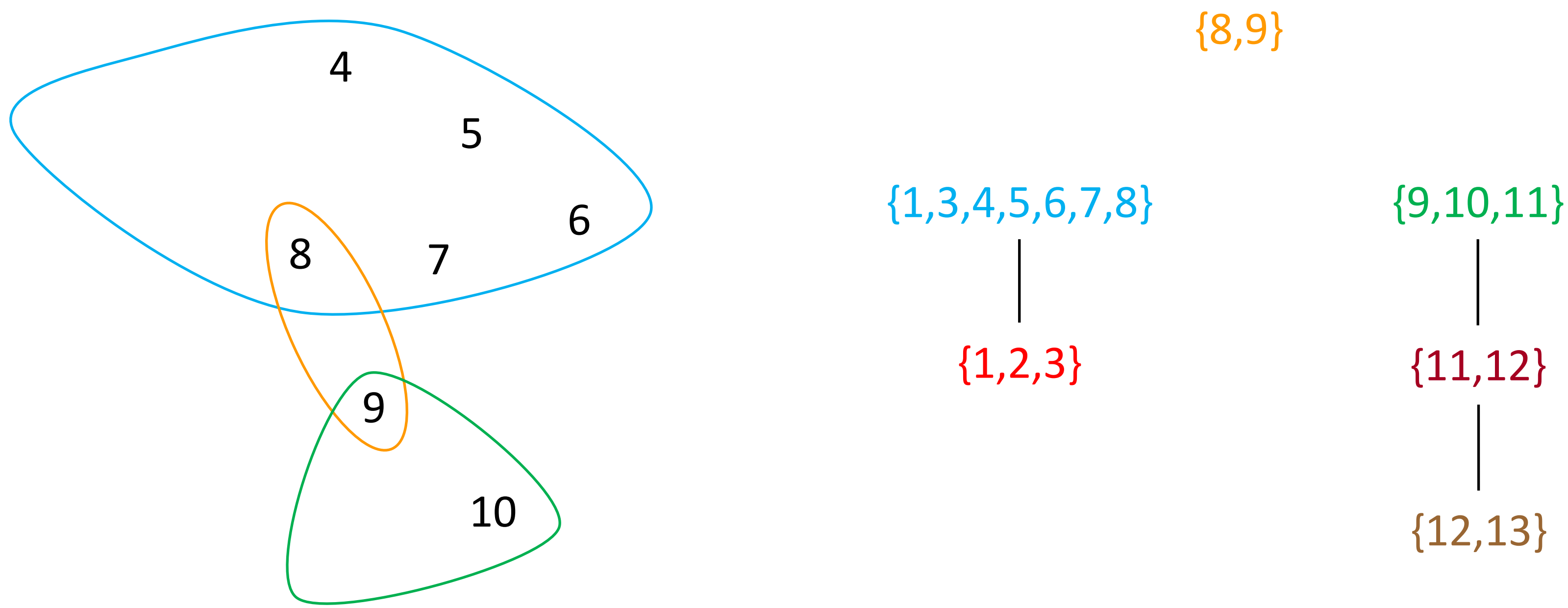

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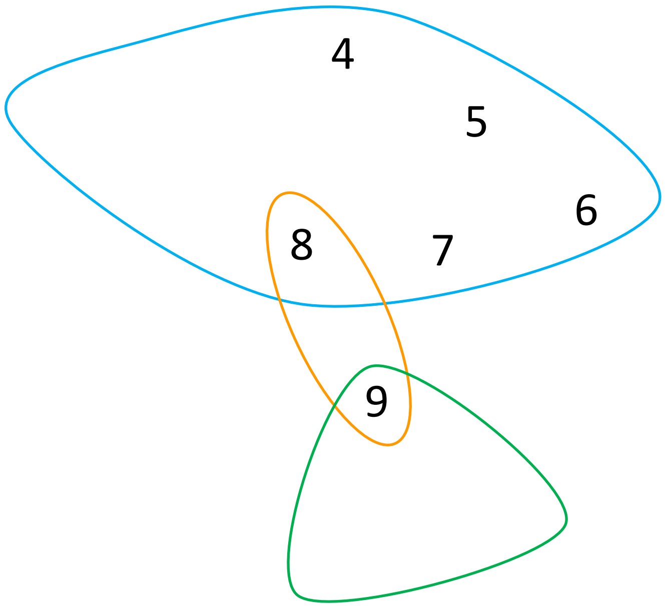

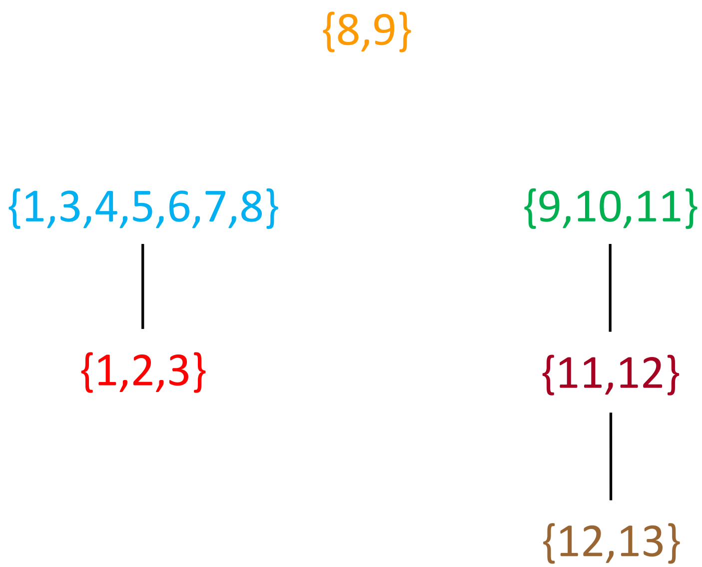

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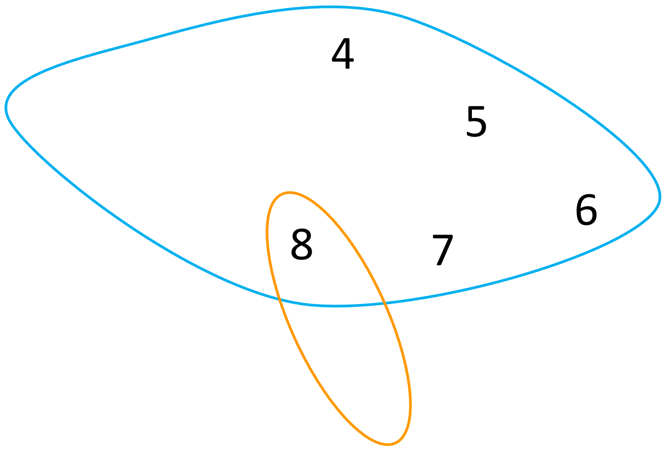

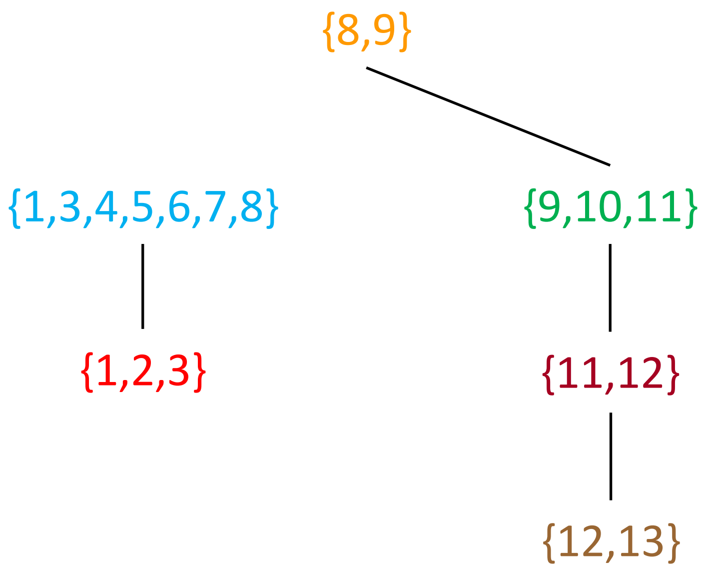

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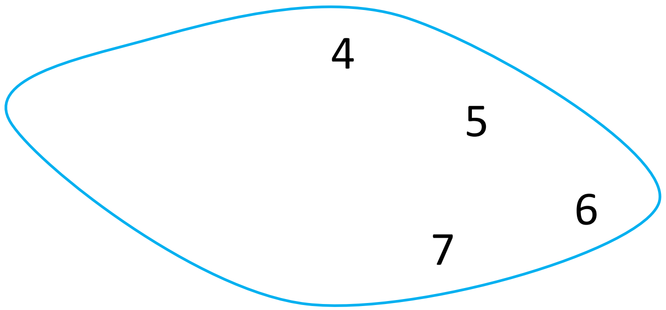

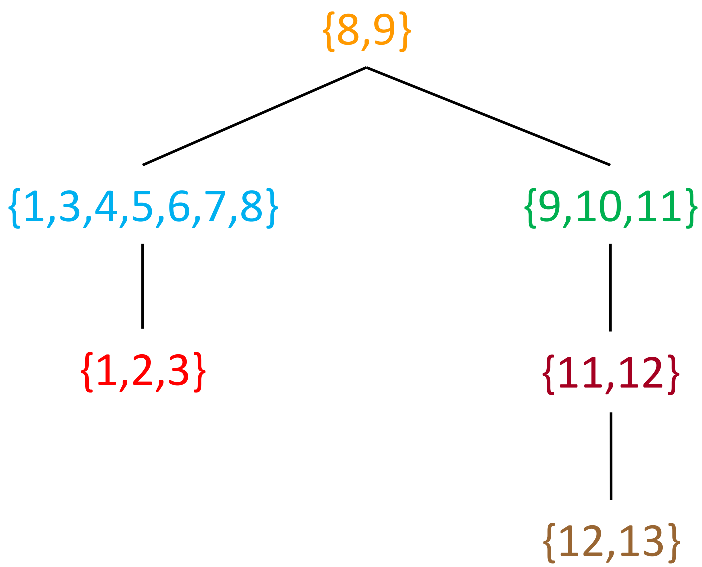

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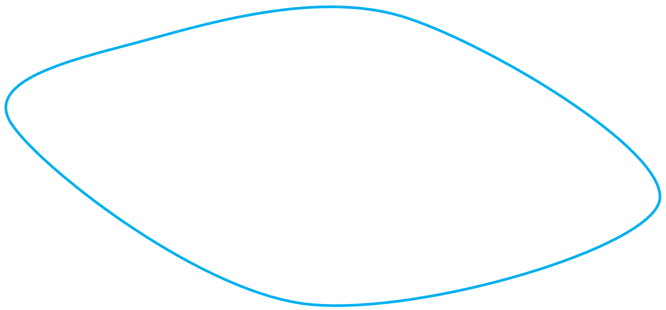

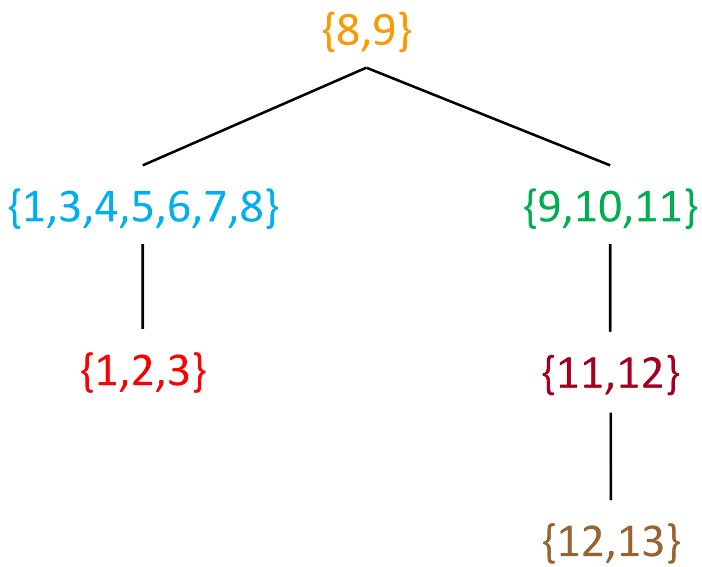

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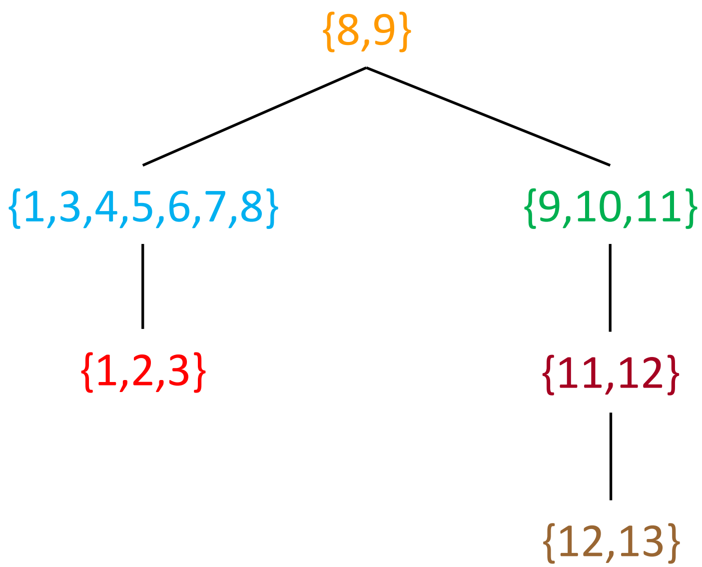
empty hypergraph



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Theorem: A hypergraph **H** is acyclic iff GYO(**H**) = ∅

⇓

checking whether **H** is acyclic is feasible in polynomial time, and if it is the case, a join tree can be found in polynomial time

⇓

Theorem: ACYCLICITY is in PTIME

Checking Acyclicity

Theorem: ACYCLICITY is in PTIME

NOTE: actually, we can check whether a CQ is acyclic in time O(||Q||)

linear time in the size Q

Evaluating Acyclic CQs

Theorem: ACQ-Evaluation is in PTIME

NOTE: actually, if H(Q) is acyclic, then Q can be evaluated in time $O(||D|| \cdot ||Q||)$

linear time in the size of D and Q

Yannakaki's Algorithm

Dynamic programming algorithm over the join tree

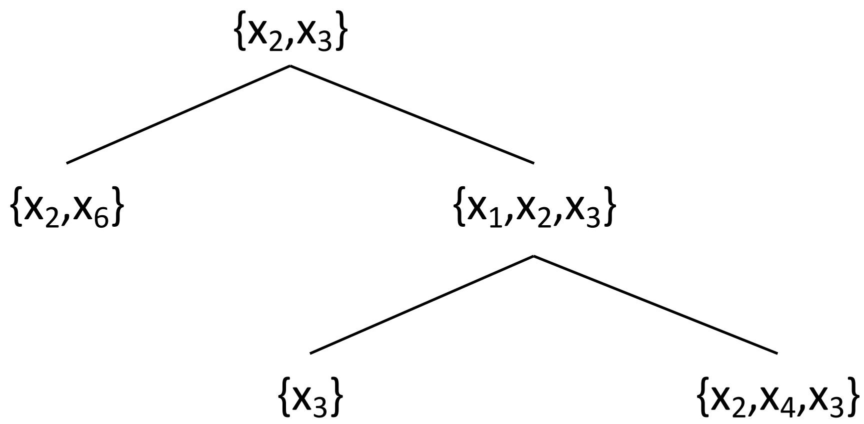
Given a database D and an acyclic Boolean CQ Q

1. Compute the join tree **T** of H(Q)
2. Assign to each node of **T** the corresponding relation of D
3. Compute semi-joins in a bottom up traversal of **T**
4. Return YES if the resulting relation at the root of **T** is non-empty;

   otherwise, return NO

Yannakaki's Algorithm: Step 1

Q :- $R_1(x_1,x_2,x_3)$, $R_2(x_2,x_3)$, $R_3(x_2,x_6)$, $R_4(x_3)$, $R_5(x_2,x_4,x_3)$



Yannakaki’s Algorithm: Step 2

x_2	x_3
c_1	b_2
c_2	b_1
c_4	b_6

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
s_3	c_3	b_1
s_3	c_1	b_4
s_2	c_2	b_3

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki's Algorithm: Step 3

x_2	x_3
c_1	b_2
c_2	b_1
c_4	b_6

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
s_3	c_3	b_1
~~s_3~~	~~c_1~~	~~b_4~~
s_2	c_2	b_3

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki's Algorithm: Step 3

x_2	x_3
c_1	b_2
c_2	b_1
c_4	b_6

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
s_3	c_3	b_1
~~s_3~~	~~c_1~~	~~b_4~~
~~s_2~~	~~c_2~~	~~b_3~~

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki's Algorithm: Step 3

x_2	x_3
c_1	b_2
c_2	b_1
c_4	b_6

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
~~s_3~~	~~c_3~~	~~b_1~~
~~s_3~~	~~c_1~~	~~b_4~~
~~s_2~~	~~c_2~~	~~b_3~~

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki's Algorithm: Step 3

x_2	x_3
c_1	b_2
c_2	b_1
~~c_4~~	~~b_6~~

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
~~s_3~~	~~c_3~~	~~b_1~~
~~s_3~~	~~c_1~~	~~b_4~~
~~s_2~~	~~c_2~~	~~b_3~~

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki’s Algorithm: Step 3

x_2	x_3
c_1	b_2
~~c_2~~	~~b_1~~
~~c_4~~	~~b_6~~

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
~~s_3~~	~~c_3~~	~~b_1~~
~~s_3~~	~~c_1~~	~~b_4~~
~~s_2~~	~~c_2~~	~~b_3~~

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Yannakaki’s Algorithm: Step 4

x_2	x_3
c_1	b_2
~~c_2~~	~~b_1~~
~~c_4~~	~~b_6~~

YES

x_2	x_6
c_1	d_2
c_1	d_1
c_3	d_1

x_1	x_2	x_3
s_1	c_2	b_1
s_1	c_1	b_2
~~s_3~~	~~c_3~~	~~b_1~~
~~s_3~~	~~c_1~~	~~b_4~~
~~s_2~~	~~c_2~~	~~b_3~~

x_3
b_1
b_2

x_2	x_4	x_3
c_2	a_1	b_1
c_1	a_1	b_2
c_1	a_2	b_2

Recap

- “Good” classes of CQs for which query evaluation is tractable - conditions based on the graph or hypergraph of the CQ
- Acyclic CQs - their hypergraph is acyclic, can be checked in linear time
- Evaluating acyclic CQs is feasible in linear time (Yannakaki’s algorithm)