



Advanced Database Systems

Spring 2026

Lecture #10: Hash-Based Indexing

R&G: Chapter 11

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Source: Introduction to Algorithms, 3rd edition

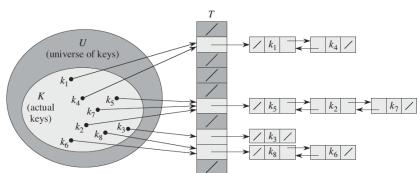
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COLLISION RESOLUTION

By chaining

Link together entries hashed to the same value

Long chains can degrade search performance



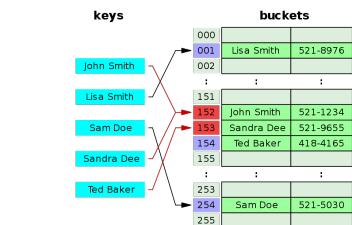
Source: Introduction to Algorithms, 3rd edition

Open Addressing

Single giant table of slots

Hash to slot, then probe until a free slot is found

Variants: Linear Probing, Cuckoo, Robin Hood, ...



Source: https://en.wikipedia.org/wiki/Hash_table

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RECAP: IN-MEMORY HASH TABLE

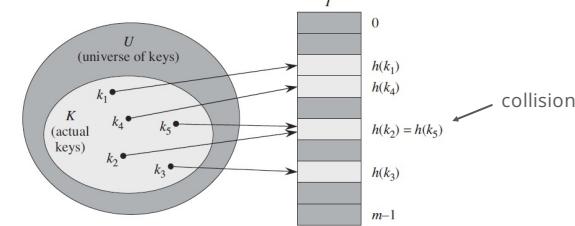
(FROM ALGORITHMS & DATA STRUCTURES COURSE)

A hash table implements an associative array (dictionary)

Data is stored as a collection of **key-value** pairs

It uses a **hash function** to compute an offset into an array of buckets (slots)

From which the desired value can be found



Source: Introduction to Algorithms, 3rd edition

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HASHING IN DATABASES

We want to be able to group together tuples with the same key value

Partition the data with hash function(s) applied on the key

All tuples with a certain key will be in the same partition

Useful for:

Removing duplicates (all duplicates will be grouped together)

Grouping data (for GROUP BY)

Looking up data using hash indexes

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HASH-BASED INDEXING

Suitable for **equality-based predicates**

```
SELECT * FROM Customer WHERE A = constant
```

Cannot support range queries

Other query operations internally generate a flood of equality tests

E.g.: nested loop join, where hash index can make a real difference

Support in commercial DBMSs

Tree-structured indexes preferred since they cover the more general range predicates

But hash-based indexes are used for (index) nested loop joins

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OVERVIEW

Static and dynamic hashing techniques exist

Trade-offs similar to ISAM vs. B+ trees

Static hashing schemes

Chained hashing

Dynamic hashing schemes

Extendible hashing

Linear hashing (not covered)

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STATIC CHAINED HASHING

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Hash index is a collection of **buckets**

Build static hash index on column A

Allocate a fixed area of N (successive) pages, the so-called **primary buckets**

In each bucket, install a pointer to a chain of **overflow** pages (initially set to null)

Define a **hash function h** with range $[0, \dots, N-1]$

The domain of h is the type of A

e.g., $h : \text{INTEGER} \rightarrow [0, \dots, N-1]$, if A is of type INTEGER

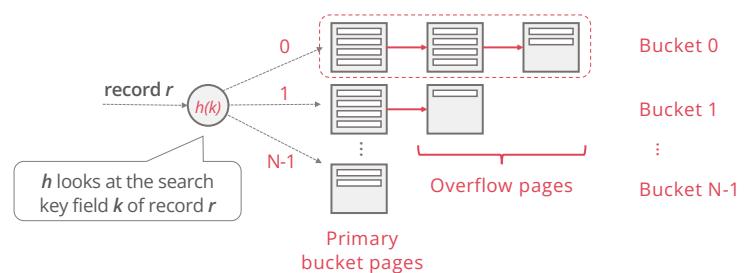
The hash function determines the bucket where the desired value can be found

STATIC CHAINED HASH TABLE

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Bucket = **primary page** plus zero or more **overflow pages**

Buckets contain index entries **k^*** implemented using any of the variants **A, B, or C**



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STATIC CHAINED HASH TABLE MANAGEMENT

Operations: **search, insert, delete**

Compute $h(k)$ on the search key field k of record r

Access the primary bucket page with number $h(k)$

Search for/insert/delete record on this page or, if needed, access the overflow pages

If overflow chain access is avoidable

search requires a single I/O operation

insert and **delete** require two I/O operations

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HASH COLLISIONS AND OVERFLOW CHAINS

Hash collisions are unavoidable

For search keys k and k' , can happen $h(k) = h(k')$

Search keys may not be unique (e.g., student age)

Even if unique, the search key space is much larger than # of buckets

Having as many primary bucket pages as different search keys in database \Rightarrow waste of space

Long overflow chains can degrade performance

Operation costs become non-uniform and unpredictable for a query optimiser

To reduce this problem, h needs to scatter search keys evenly across $[0, \dots, N-1]$

Large # of entries can still cause long chains (dynamic hashing to fix this)

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HASH FUNCTIONS

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How to map a large key space into a smaller domain

Real distributions of search key values are often non-uniform (skewed)

Trade-off between being fast vs. collision rate

We want a lightweight (non-cryptographic) hash function with a low collision rate

Simple hash function: $h(k) = k \bmod N$

Guarantees the range of $h(k)$ to be $[0, N-1]$

Choosing $N = 2^d$ for some d effectively considers the least d bits of k only

Prime numbers work best for N

Better hash functions used in practice

[xxHash](#) (+ benchmark), [MurmurHash](#), [Google CityHash](#), [Google FarmHash](#), [CLHash](#)

STATIC HASHING AND DYNAMIC FILES

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If the data file **grows**,

the development of overflow chains spoils the index I/O behaviour (1-2 I/O operations)

If the data file **shrinks**,

a significant fraction of primary buckets may be (almost) empty – a waste of space

We may **periodically rehash** the data file to restore the ideal situation
(20% free space, no overflow chains)

Expensive – the index not usable while rehashing is in progress

As for ISAM, static hashing has advantages with concurrent access

Only need to lock one bucket page to store a new entry or extend the overflow chain

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EXTENDIBLE HASHING

Situation: Bucket (primary page) is full and we want to insert. Why not reorganize the index by doubling # of buckets?

Reading and writing all pages is expensive!

Idea: Use **directory of pointers to buckets**, double # of buckets by **doubling the directory**, splitting just the bucket that overflowed

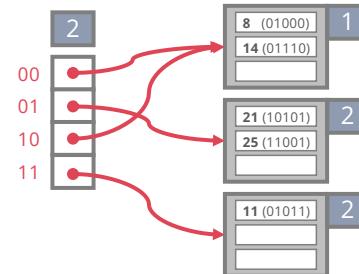
Directory much smaller than file, so doubling it is much cheaper

Only one page of data entries is split

No overflow pages!

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EXTENDIBLE HASHING



Note: we depict as index entries $h(k)$ instead of k^*

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GLOBAL AND LOCAL DEPTH

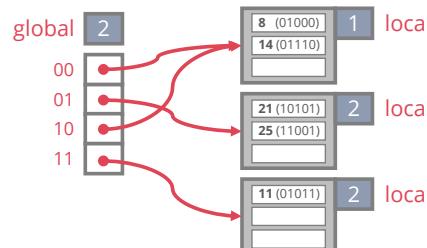
Global depth (n at directory)

Use the least n bits of $h(k)$ to find a bucket pointer in the directory
The directory size is 2^n

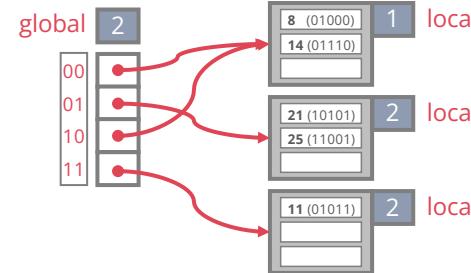
Local depth (d at individual buckets)

The hash values $h(k)$ of all entries in this bucket agree on their least d bits

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EXTENDIBLE HASHING

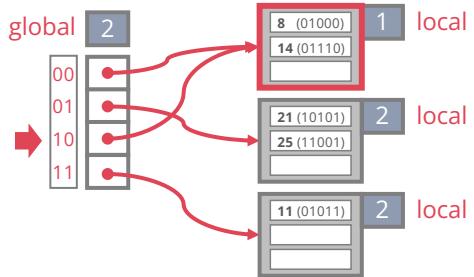


To find a bucket for A, take the least 2 bits of hash(A)

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EXTENDIBLE HASHING

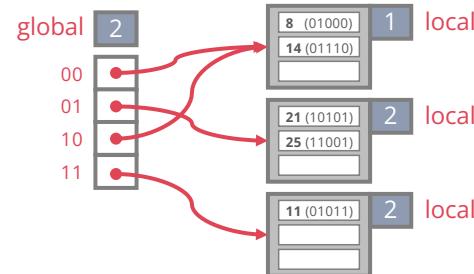


Find A
hash(A) = 14 = 01110₂

Check if the bucket contains key A. Need to compare keys due to collisions!

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EXTENDIBLE HASHING

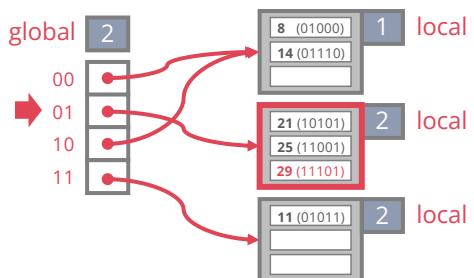


Find A
hash(A) = 14 = 01110₂
Insert B
hash(B) = 29 = 11101₂

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EXTENDIBLE HASHING

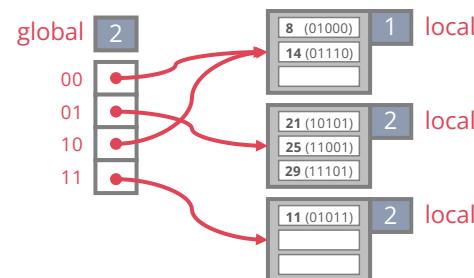


Find A
hash(A) = 14 = 01110₂
Insert B
hash(B) = 29 = 11101₂

If the bucket still has capacity, store the index entry in it

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EXTENDIBLE HASHING



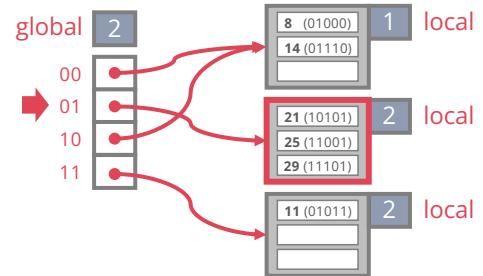
Find A
hash(A) = 14 = 01110₂
Insert B
hash(B) = 29 = 11101₂
Insert C
hash(C) = 5 = 00101₂

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EXTENDIBLE HASHING

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Find A
hash(A) = **14** = 01110₂

Insert B
hash(B) = **29** = 11101₂

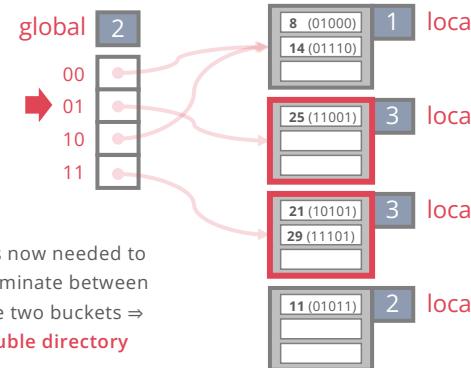
Insert C
hash(C) = **5** = 00101₂

Split bucket if full (allocate new bucket, increase local, redistribute)

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EXTENDIBLE HASHING

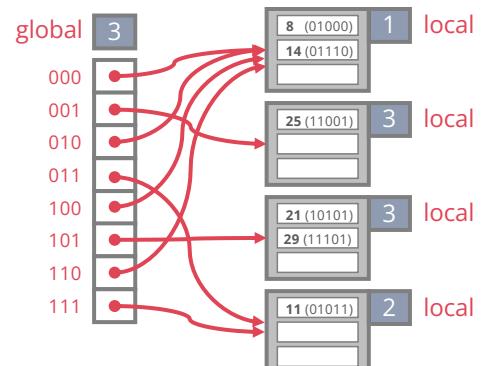
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EXTENDIBLE HASHING

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Find A
hash(A) = **14** = 01110₂

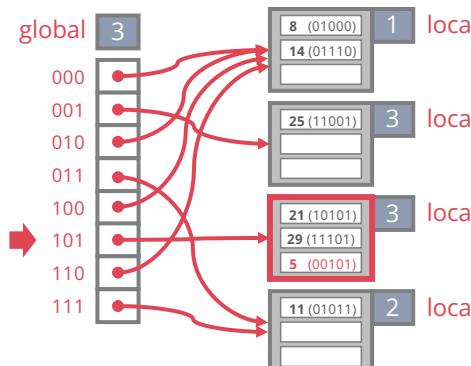
Insert B
hash(B) = **29** = 11101₂

Insert C
hash(C) = **5** = 00101₂

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EXTENDIBLE HASHING

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Find A
hash(A) = **14** = 01110₂

Insert B
hash(B) = **29** = 11101₂

Insert C
hash(C) = **5** = 00101₂

DIRECTORY DOUBLING

Double directory by **copying** its original pointers and "fixing" pointer to split bucket

Use of least significant bits enables efficient doubling via copying!

Splitting a bucket does not always require doubling the directory

Buckets with local depth < global depth have multiple pointers to them

Splitting such buckets does not require doubling

Modifying one or more bucket pointers in directory is sufficient

Directory can also shrink when buckets become empty

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SUMMARY

Hash-based indexes

Best for equality searches, cannot support range searches

Static hashing

Can lead to long overflow chains

Extendible hashing

Avoids overflow chains by splitting a full bucket when a new entry is to be added to it

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