



# Advanced Database Systems

Spring 2026

## Lecture #10: Hash-Based Indexing

R&G: Chapter 11

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## RECAP: IN-MEMORY HASH TABLE

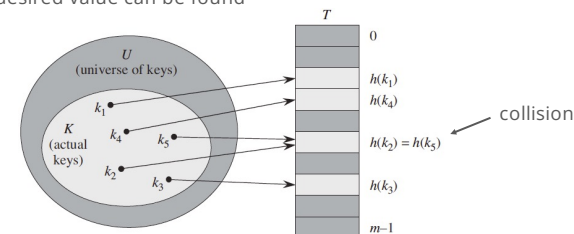
(FROM ALGORITHMS & DATA STRUCTURES COURSE)

A hash table implements an associative array (dictionary)

Data is stored as a collection of **key-value** pairs

It uses a **hash function** to compute an offset into an array of buckets (slots)

From which the desired value can be found



Source: Introduction to Algorithms, 3rd edition

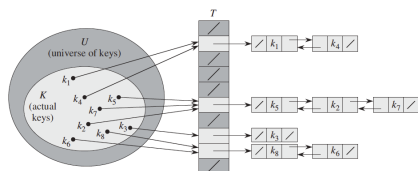
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## COLLISION RESOLUTION

### By chaining

Link together entries hashed to the same value

Long chains can degrade search performance



### Open Addressing

Single giant table of slots

Hash to slot, then probe until a free slot is found

Variants: Linear Probing, Cuckoo, Robin Hood, ...

keys	buckets
John Smith	000
Lisa Smith	001 Lisa Smith 521-8976
Sam Doe	002
Sandra Dee	151
Ted Baker	152 John Smith 521-1234
	153 Sandra Dee 521-9655
	154 Ted Baker 418-4165
	155
	253
	254 Sam Doe 521-5030
	255

Source: Introduction to Algorithms, 3rd edition

Source: [https://en.wikipedia.org/wiki/Hash\\_table](https://en.wikipedia.org/wiki/Hash_table)

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## HASHING IN DATABASES

We want to be able to group together tuples with the same key value

Partition the data with hash function(s) applied on the key

All tuples with a certain key will be in the same partition

Useful for:

Removing duplicates (all duplicates will be grouped together)

Grouping data (for GROUP BY)

Looking up data using hash indexes

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## HASH-BASED INDEXING

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Suitable for **equality-based predicates**

```
SELECT * FROM Customer WHERE A = constant
```

**Cannot** support range queries

Other query operations internally generate a flood of equality tests

E.g.: nested loop join, where hash index can make a real difference

Support in commercial DBMSs

Tree-structured indexes preferred since they cover the more general range predicates

But hash-based indexes are used for (index) nested loop joins

## OVERVIEW

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Static and dynamic hashing techniques exist

Trade-offs similar to ISAM vs. B+ trees

**Static hashing schemes**

Chained hashing

**Dynamic hashing schemes**

Extendible hashing

Linear hashing (not covered)

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## STATIC CHAINED HASHING

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Hash index is a collection of **buckets**

Build static hash index on column A

Allocate a fixed area of N (successive) pages, the so-called **primary buckets**

In each bucket, install a pointer to a chain of **overflow** pages (initially set to null)

Define a **hash function**  $h$  with range  $[0, \dots, N-1]$

The domain of  $h$  is the type of A

e.g.,  $h : \text{INTEGER} \rightarrow [0, \dots, N-1]$ , if A is of type INTEGER

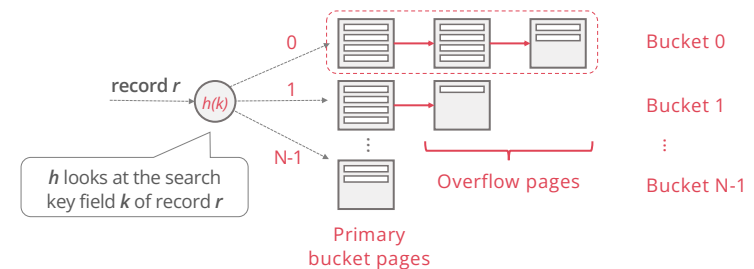
The hash function determines the bucket where the desired value can be found

## STATIC CHAINED HASH TABLE

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Bucket = **primary page** plus zero or more **overflow pages**

Buckets contain index entries  $k^*$  implemented using any of the variants **A**, **B**, or **C**



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## STATIC CHAINED HASH TABLE MANAGEMENT

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Operations: **search**, **insert**, **delete**

Compute  $h(k)$  on the search key field  $k$  of record  $r$

Access the primary bucket page with number  $h(k)$

Search for/insert/delete record on this page or, if needed, access the overflow pages

If overflow chain access is avoidable

**search** requires a single I/O operation

**insert** and **delete** require two I/O operations

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## HASH COLLISIONS AND OVERFLOW CHAINS

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**Hash collisions** are unavoidable

For search keys  $k$  and  $k'$ , can happen  $h(k) = h(k')$

Search keys may not be unique (e.g., student age)

Even if unique, the search key space is much larger than # of buckets

Having as many primary bucket pages as different search keys in database  $\Rightarrow$  waste of space

**Long overflow chains** can degrade performance

Operation costs become non-uniform and unpredictable for a query optimiser

To reduce this problem,  $h$  needs to scatter search keys evenly across  $[0, \dots, N-1]$

Large # of entries can still cause long chains (dynamic hashing to fix this)

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## HASH FUNCTIONS

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How to map a large key space into a smaller domain

Real distributions of search key values are often non-uniform (skewed)

Trade-off between being fast vs. collision rate

We want a lightweight (non-cryptographic) hash function with a low collision rate

Simple hash function:  $h(k) = k \bmod N$

Guarantees the range of  $h(k)$  to be  $[0, N-1]$

Choosing  $N = 2^d$  for some  $d$  effectively considers the least  $d$  bits of  $k$  only

Prime numbers work best for  $N$

Better hash functions used in practice

[xxHash](#) (+ benchmark), [MurmurHash](#), [Google CityHash](#), [Google FarmHash](#), [CLHash](#).

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## STATIC HASHING AND DYNAMIC FILES

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If the data file **grows**,

the development of overflow chains spoils the index I/O behaviour (1-2 I/O operations)

If the data file **shrinks**,

a significant fraction of primary buckets may be (almost) empty – a waste of space

We may **periodically rehash** the data file to restore the ideal situation (20% free space, no overflow chains)

Expensive – the index not usable while rehashing is in progress

As for ISAM, static hashing has advantages with concurrent access

Only need to lock one bucket page to store a new entry or extend the overflow chain

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## EXTENDIBLE HASHING

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Situation: Bucket (primary page) is full and we want to insert. Why not reorganize the index by doubling # of buckets?

Reading and writing all pages is expensive!

Idea: Use **directory of pointers to buckets**, double # of buckets by **doubling the directory**, splitting just the bucket that overflowed

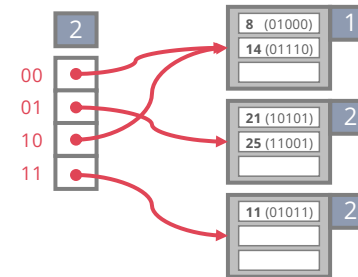
Directory much smaller than file, so doubling it is much cheaper

Only one page of data entries is split

*No overflow pages!*

## EXTENDIBLE HASHING

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Note: we depict as index entries  $h(k)$  instead of  $k^*$

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## GLOBAL AND LOCAL DEPTH

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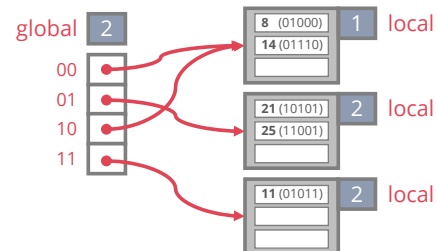
### Global depth ( $n$ at directory)

Use the least  $n$  bits of  $h(k)$  to find a bucket pointer in the directory

The directory size is  $2^n$

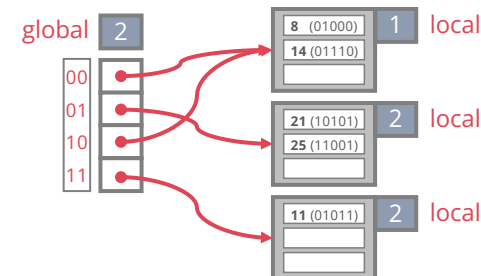
### Local depth ( $d$ at individual buckets)

The hash values  $h(k)$  of all entries in this bucket agree on their least  $d$  bits



## EXTENDIBLE HASHING

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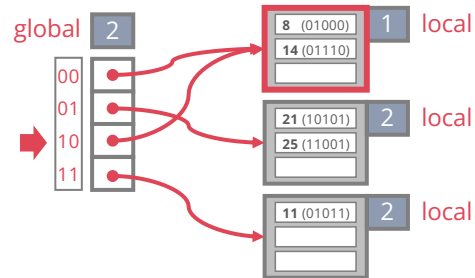
Find A  
 $\text{hash}(A) = 14 = 01110_2$

To find a bucket for A, take the least 2 bits of  $\text{hash}(A)$

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## EXTENDIBLE HASHING

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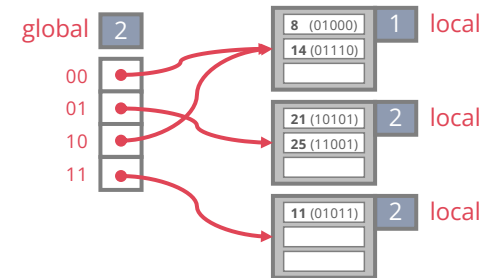
Find A  
hash(A) = **14** = 01110<sub>2</sub>

Check if the bucket contains key A. Need to compare keys due to collisions!

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## EXTENDIBLE HASHING

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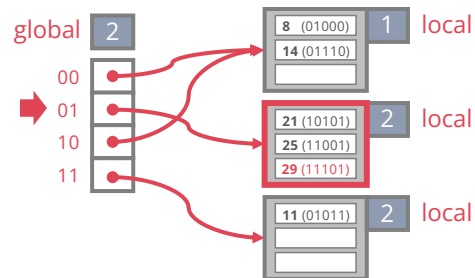
Find A  
hash(A) = **14** = 01110<sub>2</sub>

Insert B  
hash(B) = **29** = 11101<sub>2</sub>

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## EXTENDIBLE HASHING

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Find A  
hash(A) = **14** = 01110<sub>2</sub>

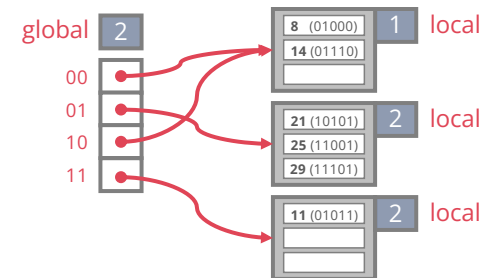
Insert B  
hash(B) = **29** = 11101<sub>2</sub>

If the bucket still has capacity, store the index entry in it

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## EXTENDIBLE HASHING

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Find A  
hash(A) = **14** = 01110<sub>2</sub>

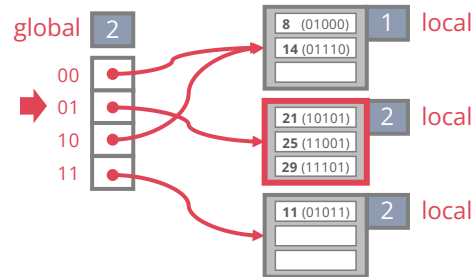
Insert B  
hash(B) = **29** = 11101<sub>2</sub>

Insert C  
hash(C) = **5** = 00101<sub>2</sub>

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## EXTENDIBLE HASHING

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Find A  
hash(A) = **14** = 01110<sub>2</sub>

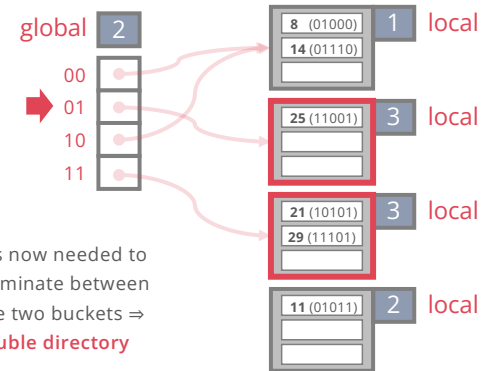
Insert B  
hash(B) = **29** = 11101<sub>2</sub>

Insert C  
hash(C) = **5** = 00101<sub>2</sub>

**Split** bucket if full (allocate new bucket, increase local, redistribute)

## EXTENDIBLE HASHING

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Find A  
hash(A) = **14** = 01110<sub>2</sub>

Insert B  
hash(B) = **29** = 11101<sub>2</sub>

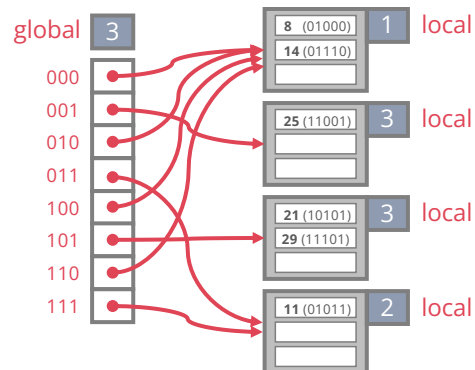
Insert C  
hash(C) = **5** = 00101<sub>2</sub>

3 bits now needed to  
discriminate between  
these two buckets ⇒  
**double directory**

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## EXTENDIBLE HASHING

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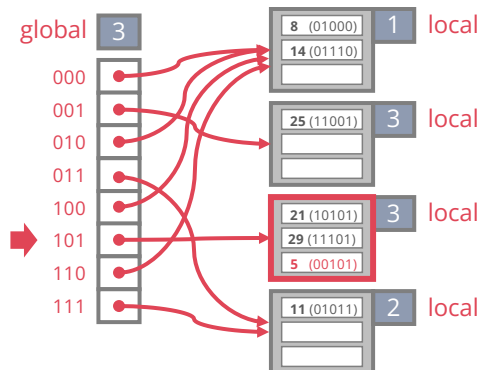
Find A  
hash(A) = **14** = 01110<sub>2</sub>

Insert B  
hash(B) = **29** = 11101<sub>2</sub>

Insert C  
hash(C) = **5** = 00101<sub>2</sub>

## EXTENDIBLE HASHING

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Find A  
hash(A) = **14** = 01110<sub>2</sub>

Insert B  
hash(B) = **29** = 11101<sub>2</sub>

Insert C  
hash(C) = **5** = 00101<sub>2</sub>

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## DIRECTORY DOUBLING

Double directory by **copying** its original pointers and "fixing" pointer to split bucket

Use of least significant bits enables efficient doubling via copying!

Splitting a bucket does not always require doubling the directory

Buckets with local depth < global depth have multiple pointers to them

Splitting such buckets does not require doubling

Modifying one or more bucket pointers in directory is sufficient

Directory can also shrink when buckets become empty

## SUMMARY

Hash-based indexes

Best for equality searches, cannot support range searches

Static hashing

Can lead to long overflow chains

Extendible hashing

Avoids overflow chains by splitting a full bucket when a new entry is to be added to it