THE UNIVERSITY of EDINBURGH

Advanced Database Systems

Spring 2026

Lecture #10:

Hash-Based Indexing

R&G: Chapter 11

Recap: In-Memory Hash Table

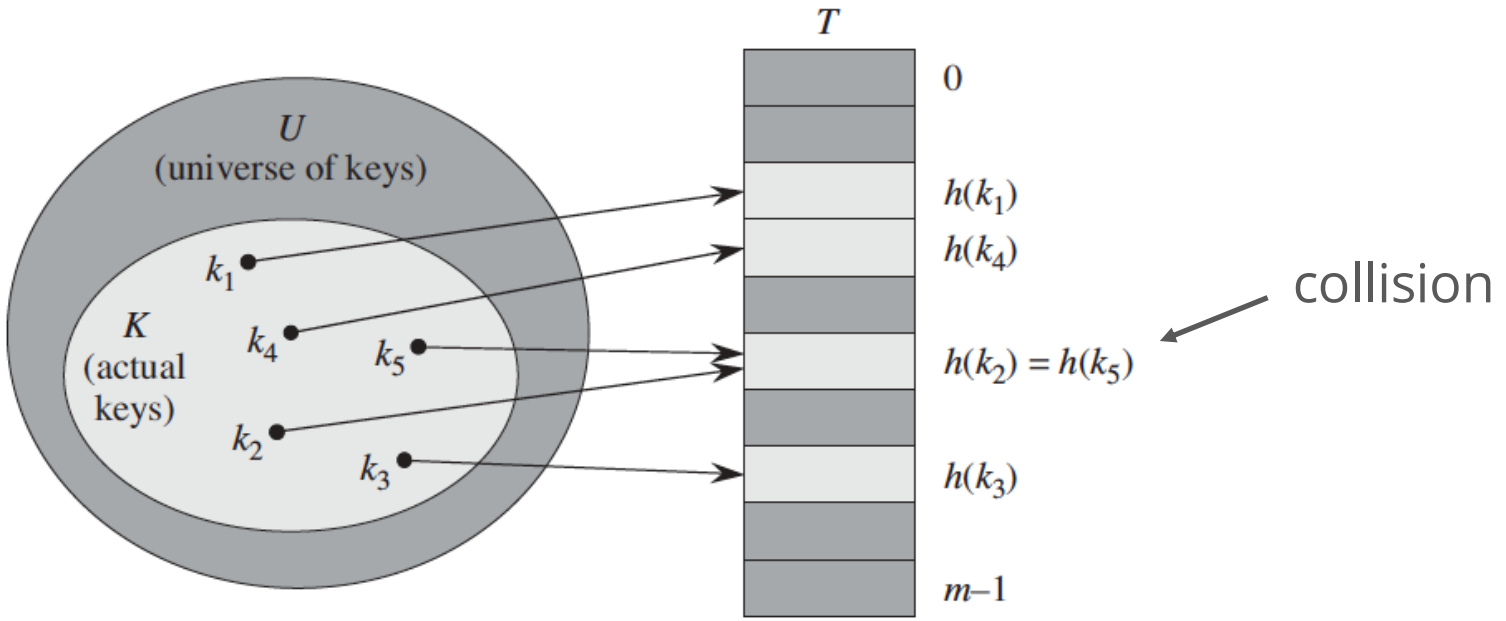
(From Algorithms & Data Structures Course)

A hash table implements an associative array (dictionary)

Data is stored as a collection of **key-value** pairs

It uses a **hash function** to compute an offset into an array of buckets (slots)

From which the desired value can be found



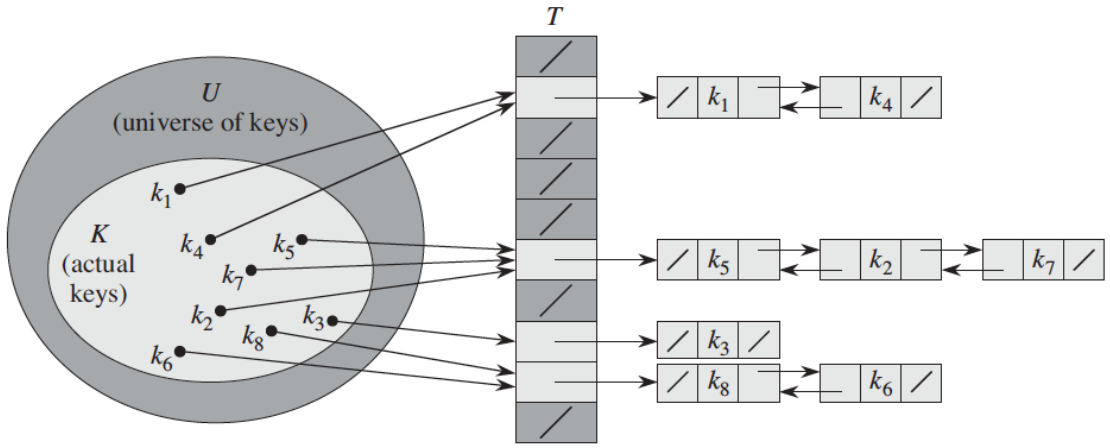
Source: Introduction to Algorithms, 3rd edition

Collision Resolution

By chaining

Link together entries hashed to the same value

Long chains can degrade search performance

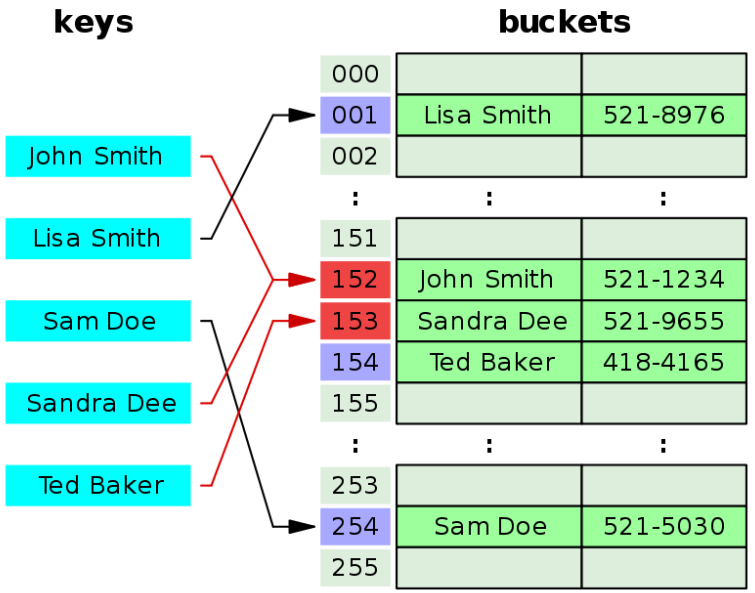


Open Addressing

Single giant table of slots

Hash to slot, then probe until a free slot is found

Variants: Linear Probing, Cuckoo, Robin Hood, ...



Source: Introduction to Algorithms, 3rd edition

Source: https://en.wikipedia.org/wiki/Hash_table

Hashing in Databases

We want to be able to group together tuples with the same key value

Partition the data with hash function(s) applied on the key

- All tuples with a certain key will be in the same partition

Useful for:

- Removing duplicates (all duplicates will be grouped together)
- Grouping data (for GROUP BY)
- Looking up data using hash indexes

Hash-Based Indexing

Suitable for **equality-based predicates**

```
SELECT * FROM Customer WHERE A = constant
```

Cannot support range queries

Other query operations internally generate a flood of equality tests

- E.g.: nested loop join, where hash index can make a real difference

Support in commercial DBMSs

- Tree-structured indexes preferred since they cover the more general range predicates
- But hash-based indexes are used for (index) nested loop joins

Overview

Static and dynamic hashing techniques exist

- Trade-offs similar to ISAM vs. B+ trees

Static hashing schemes

- Chained hashing

Dynamic hashing schemes

- Extendible hashing
- Linear hashing (not covered)

Static Chained Hashing

Hash index is a collection of **buckets**

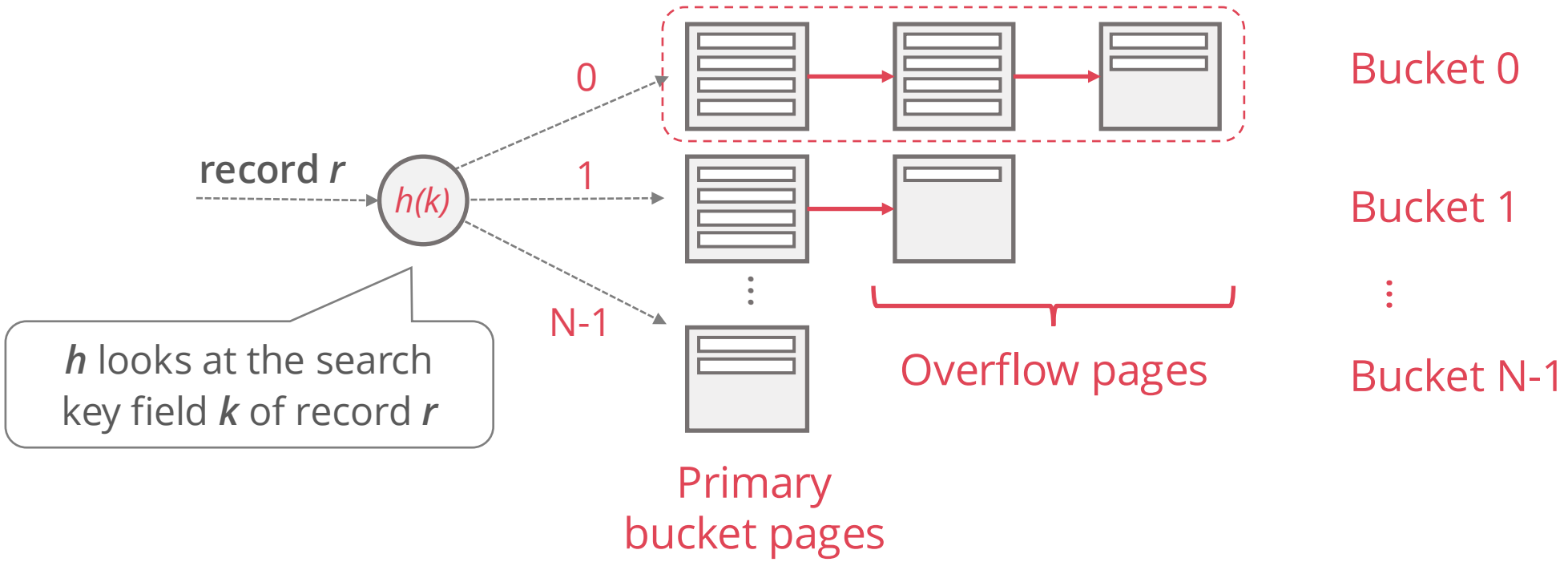
Build static hash index on column A

- Allocate a fixed area of N (successive) pages, the so-called **primary buckets**
- In each bucket, install a pointer to a chain of **overflow** pages (initially set to **null**)
- Define a **hash function *h*** with range [0, ..., N-1]
- The domain of ***h*** is the type of A
  - e.g., *h* : INTEGER ⟶ [0, ..., N-1], if A is of type INTEGER
- The hash function determines the bucket where the desired value can be found

Static Chained Hash Table

Bucket = **primary page** plus zero or more **overflow pages**

Buckets contain index entries **k*** implemented using any of the variants **A**, **B**, or **C**



Static Chained Hash Table Management

Operations: search, insert, delete

- Compute $h(k)$ on the search key field k of record r
- Access the primary bucket page with number $h(k)$
- Search for/insert/delete record on this page or, if needed, access the overflow pages

If overflow chain access is avoidable

- search requires a single I/O operation
- insert and delete require two I/O operations

Hash Collisions and Overflow Chains

Hash collisions are unavoidable

- For search keys *k* and *k'*, can happen *h*(*k*) = *h*(*k'*)
- Search keys may not be unique (e.g., student age)
- Even if unique, the search key space is much larger than # of buckets
  - Having as many primary bucket pages as different search keys in database ⇒ waste of space

Long overflow chains can degrade performance

- Operation costs become non-uniform and unpredictable for a query optimiser
- To reduce this problem, *h* needs to scatter search keys evenly across [0, ..., N-1]
- Large # of entries can still cause long chains (dynamic hashing to fix this)

Hash Functions

How to map a large key space into a smaller domain

- Real distributions of search key values are often non-uniform (skewed)

Trade-off between being fast vs. collision rate

- We want a lightweight (non-cryptographic) hash function with a low collision rate

Simple hash function: $h(k) = k \bmod N$

- Guarantees the range of $h(k)$ to be $[0, N-1]$
- Choosing $N = 2^d$ for some **d** effectively considers the least **d** bits of **k** only
- Prime numbers work best for **N**

Better hash functions used in practice

- xxHash (+ benchmark), MurmurHash, Google CityHash, Google FarmHash, CLHash

Static Hashing and Dynamic Files

If the data file **grows**,

- the development of overflow chains spoils the index I/O behaviour (1–2 I/O operations)

If the data file **shrinks**,

- a significant fraction of primary buckets may be (almost) empty – a waste of space

We may **periodically rehash** the data file to restore the ideal situation (20% free space, no overflow chains)

- Expensive – the index not usable while rehashing is in progress

As for ISAM, static hashing has advantages with concurrent access

- Only need to lock one bucket page to store a new entry or extend the overflow chain

Extendible Hashing

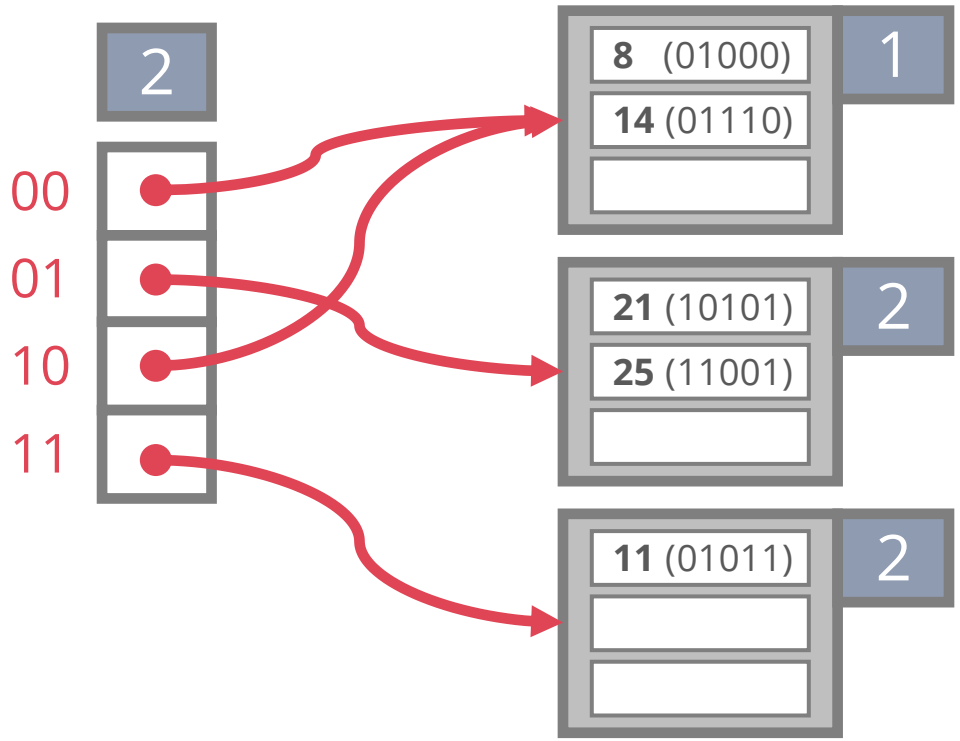
Situation: Bucket (primary page) is full and we want to insert. Why not reorganize the index by doubling # of buckets?

- Reading and writing all pages is expensive!

Idea: Use directory of pointers to buckets, double # of buckets by **doubling the directory**, splitting just the bucket that overflowed

- Directory much smaller than file, so doubling it is much cheaper
- Only one page of data entries is split
- *No overflow pages!*

Extendible Hashing



Note: we depict as index entries $h(k)$ instead of k^*

Global and Local Depth

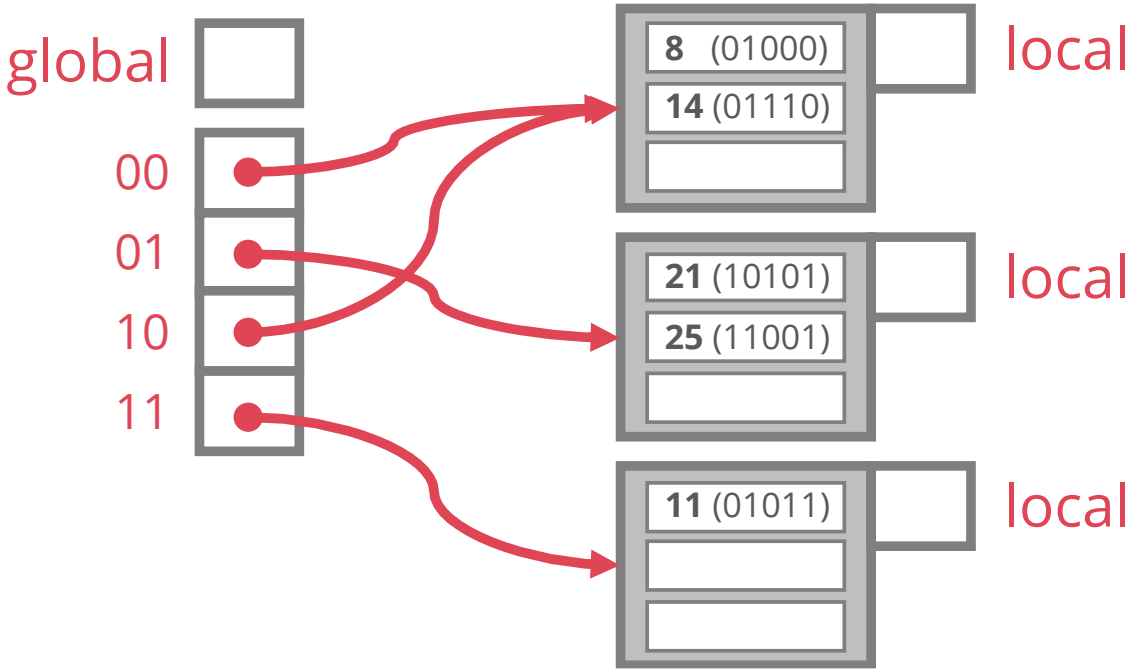
Global depth (*n* at directory)

Use the least *n* bits of *h(k)* to find a bucket pointer in the directory

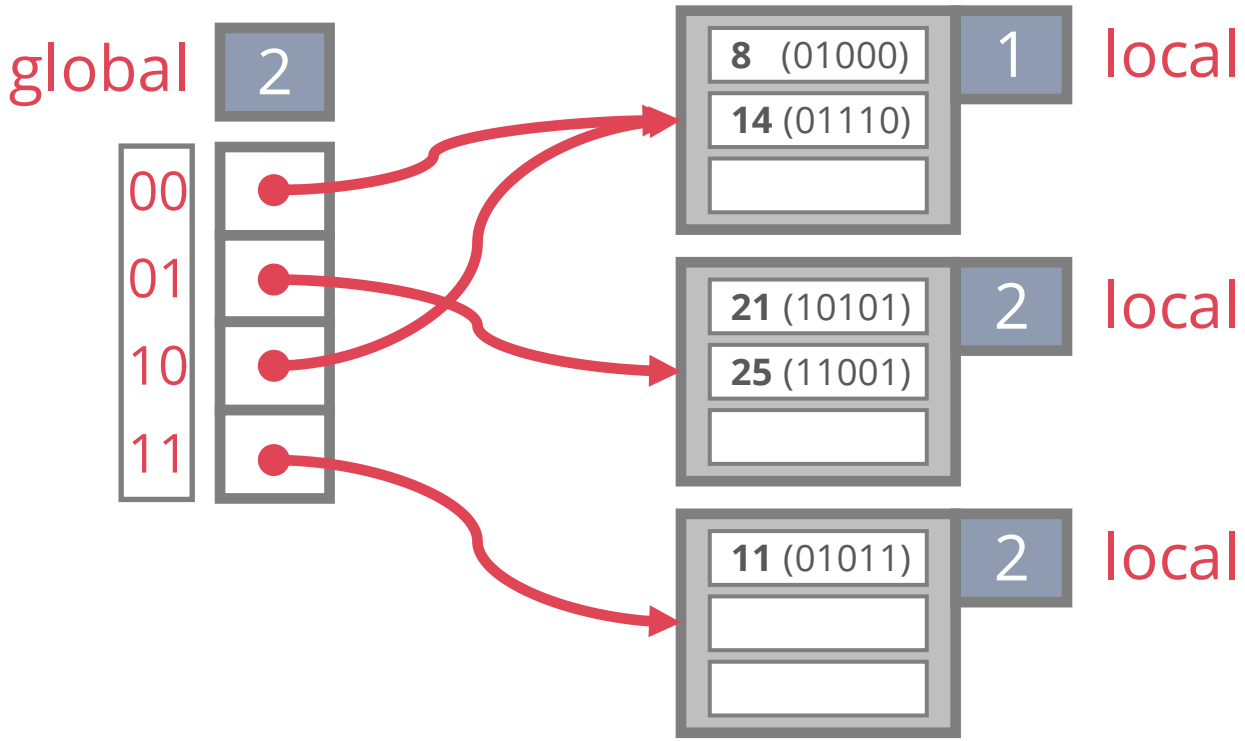
The directory size is 2^n

Local depth (*d* at individual buckets)

The hash values *h(k)* of all entries in this bucket agree on their least *d* bits



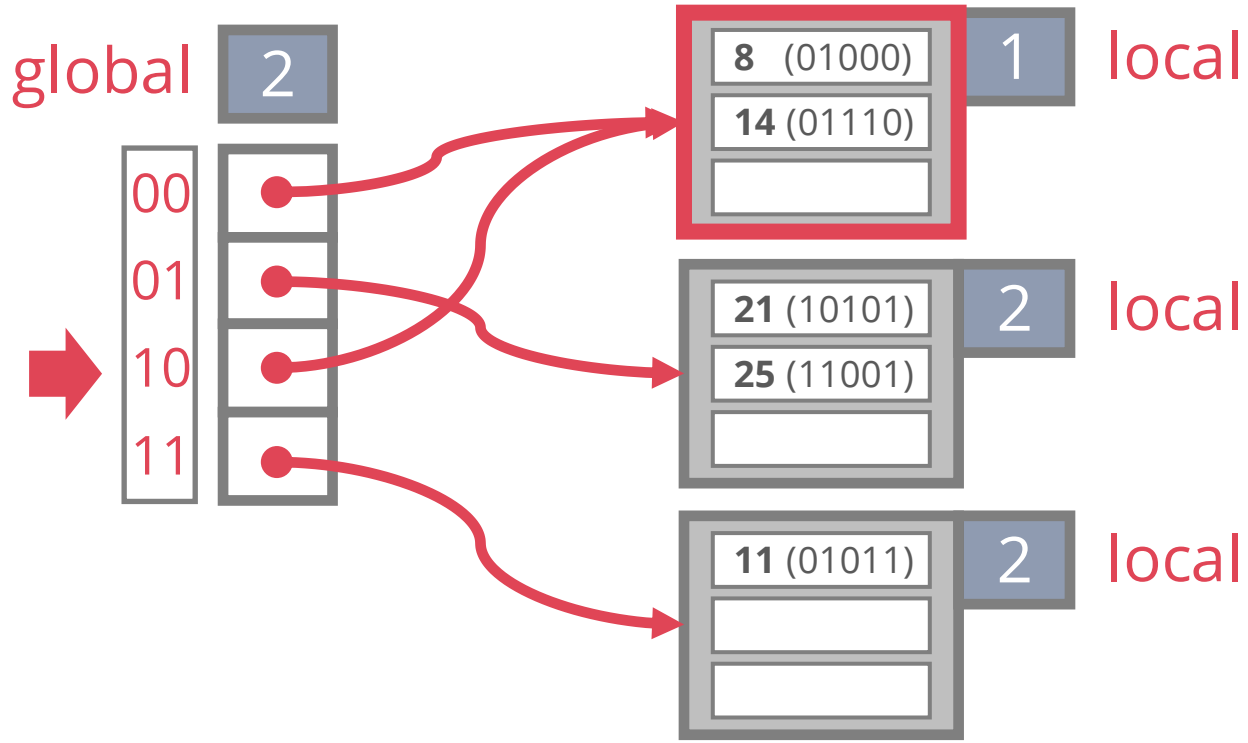
Extendible Hashing



Find A
hash(A) = **14** = 01110_2

To find a bucket for A, take the least 2 bits of hash(A)

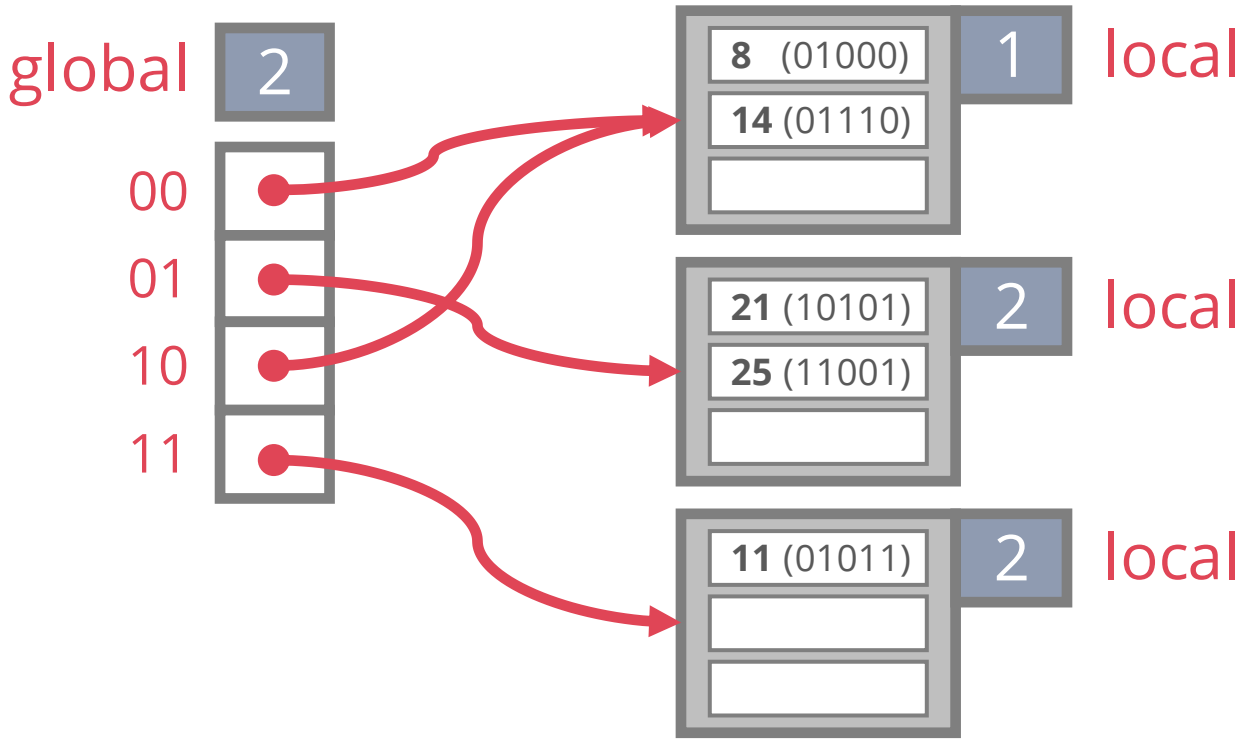
Extendible Hashing



Find A
hash(A) = **14** = 011$10_2$

Check if the bucket contains key A. Need to compare keys due to collisions!

Extendible Hashing



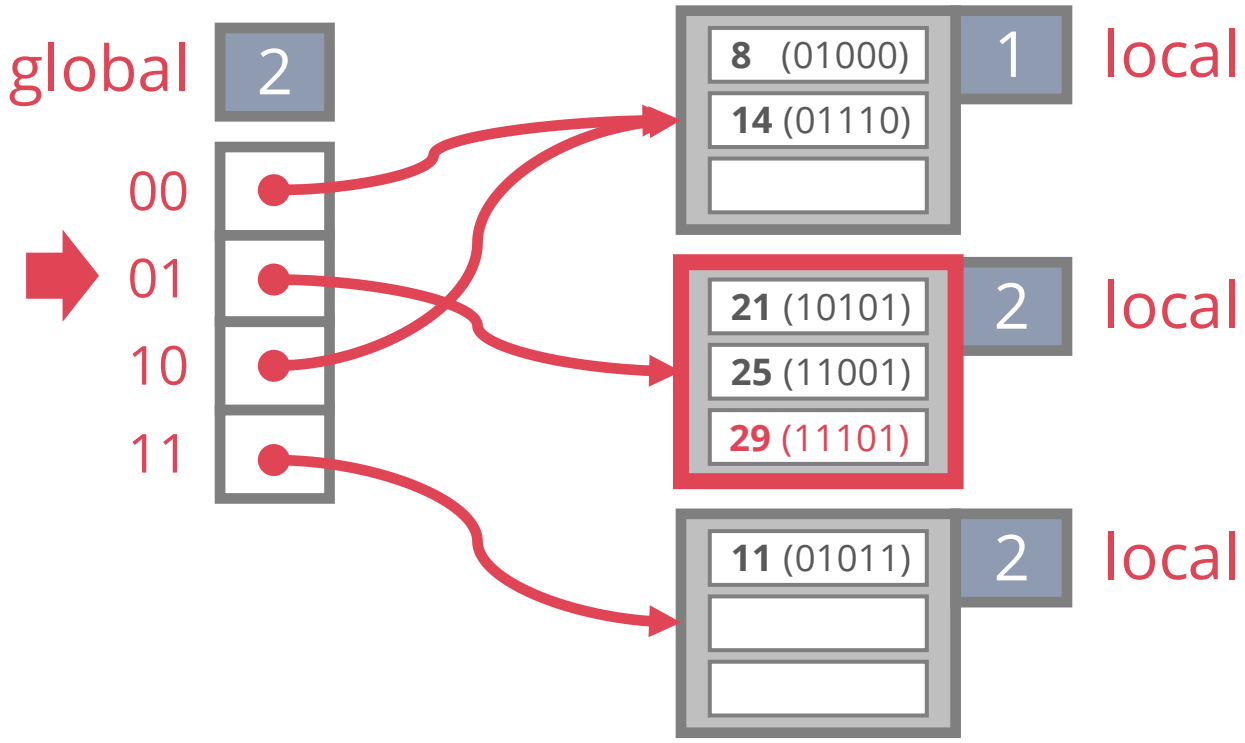
Find A

hash(A) = **14** = 01110_2

Insert B

hash(B) = **29** = 11101_2

Extendible Hashing

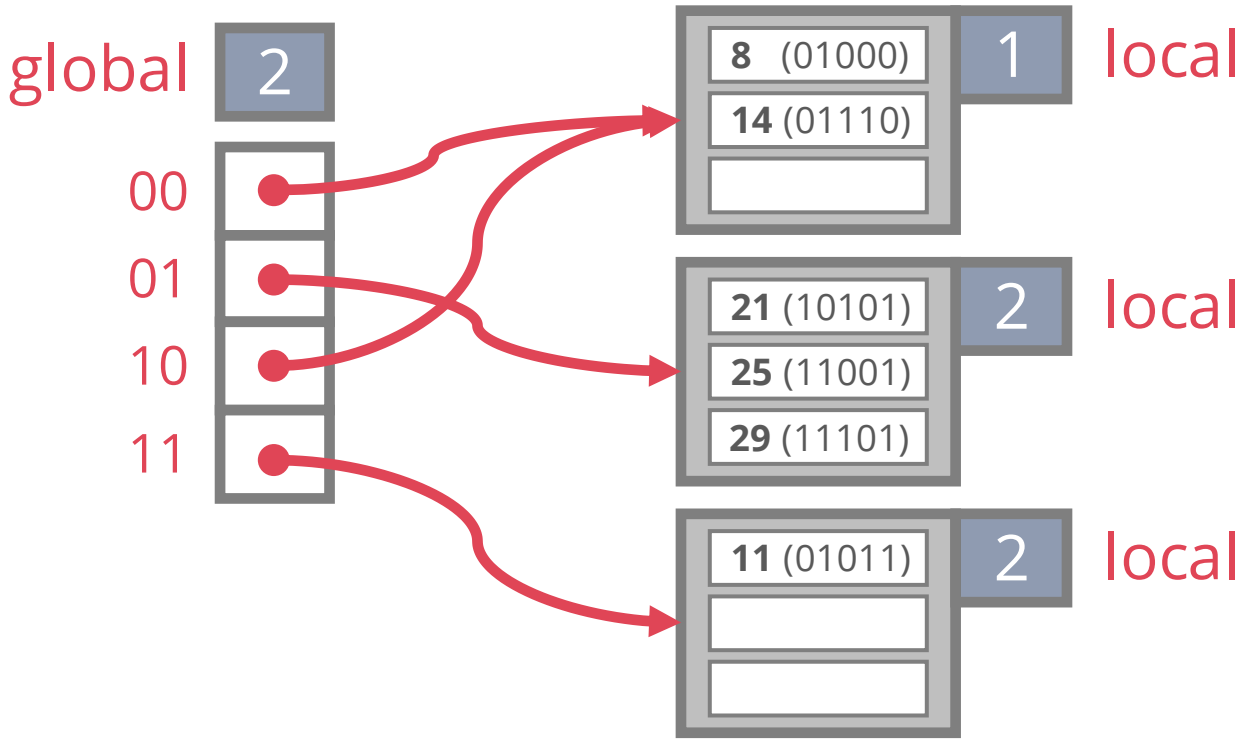


Find A
hash(A) = **14** = 01110_2

Insert B
hash(B) = **29** = $111\boxed{01}_2$

If the bucket still has capacity, store the index entry in it

Extendible Hashing

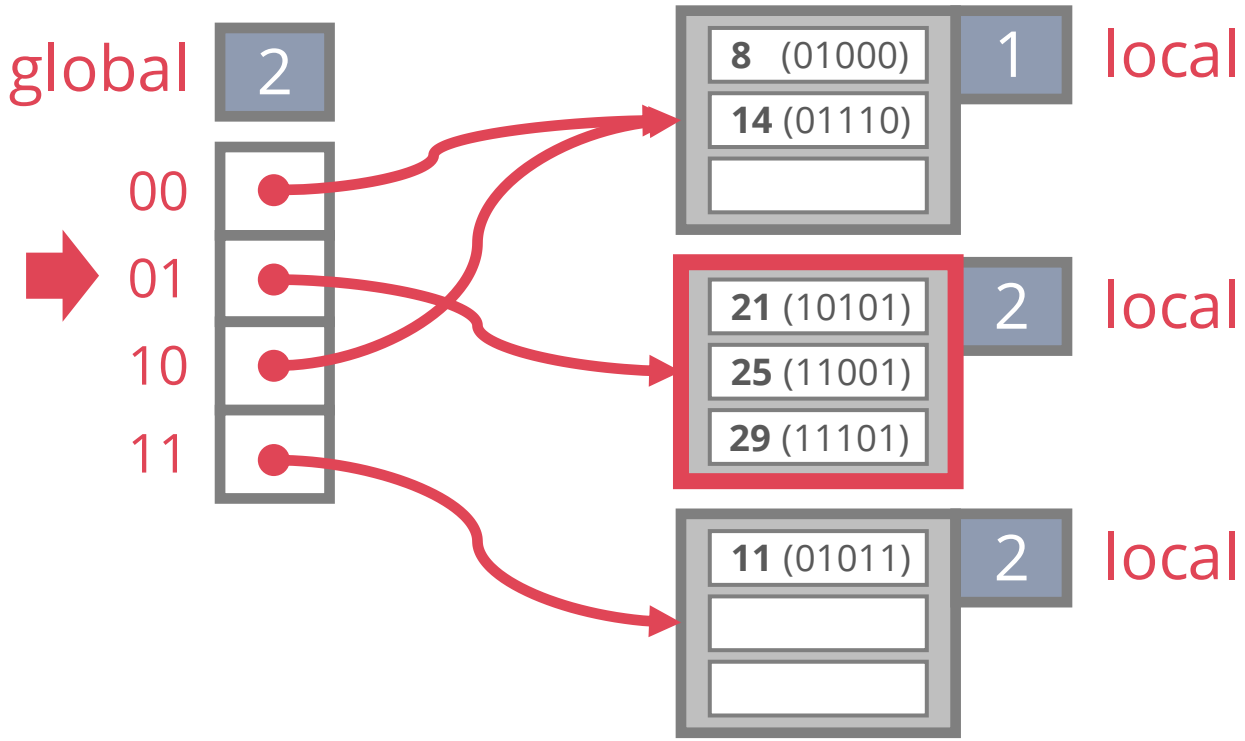


Find A
hash(A) = **14** = 01110_2

Insert B
hash(B) = **29** = 11101_2

Insert C
hash(C) = **5** = 00101_2

Extendible Hashing



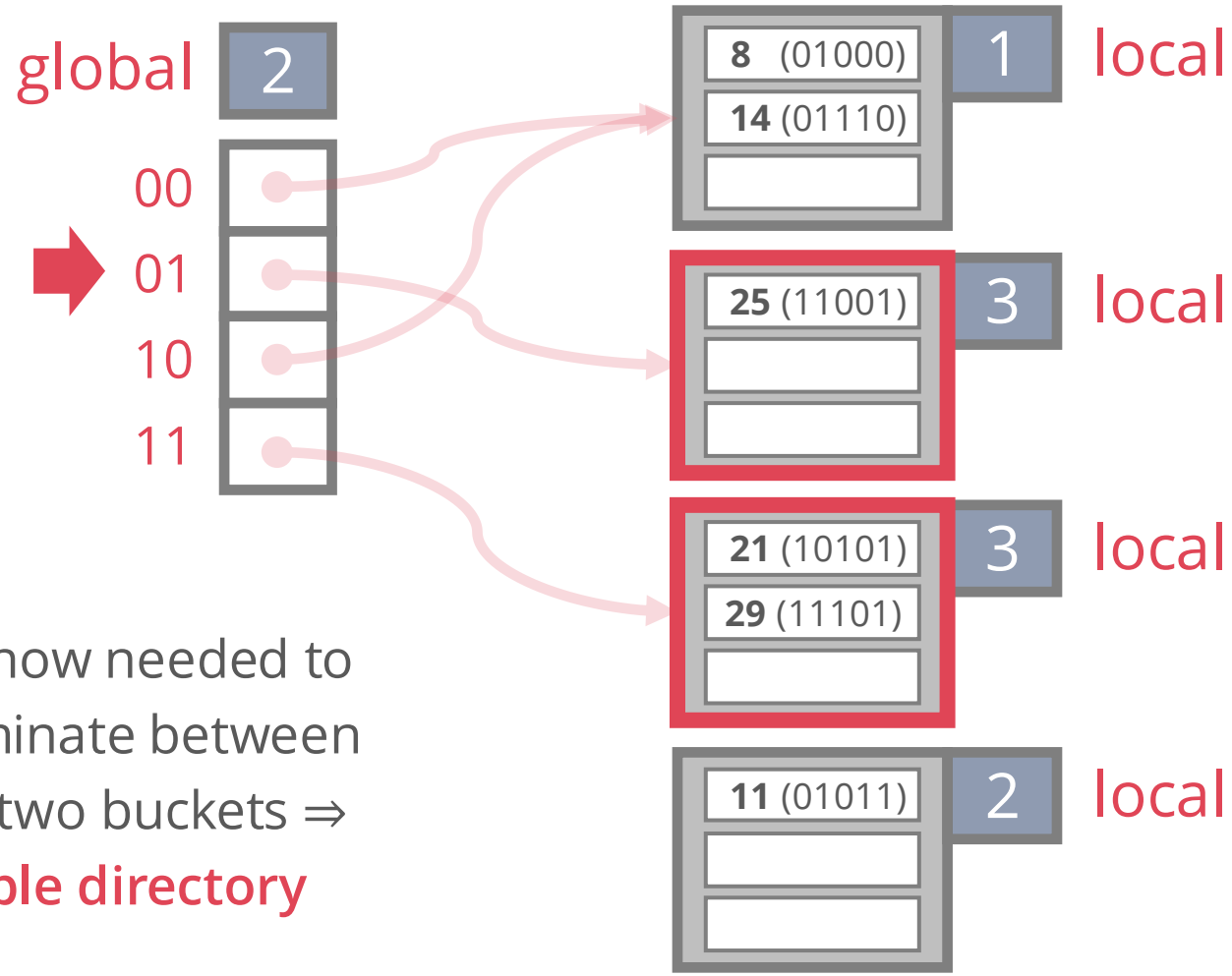
Find A
hash(A) = **14** = 01110_2

Insert B
hash(B) = **29** = 11101_2

Insert C
hash(C) = **5** = $001\boxed{01}_2$

Split bucket if full (allocate new bucket, increase local, redistribute)

Extendible Hashing



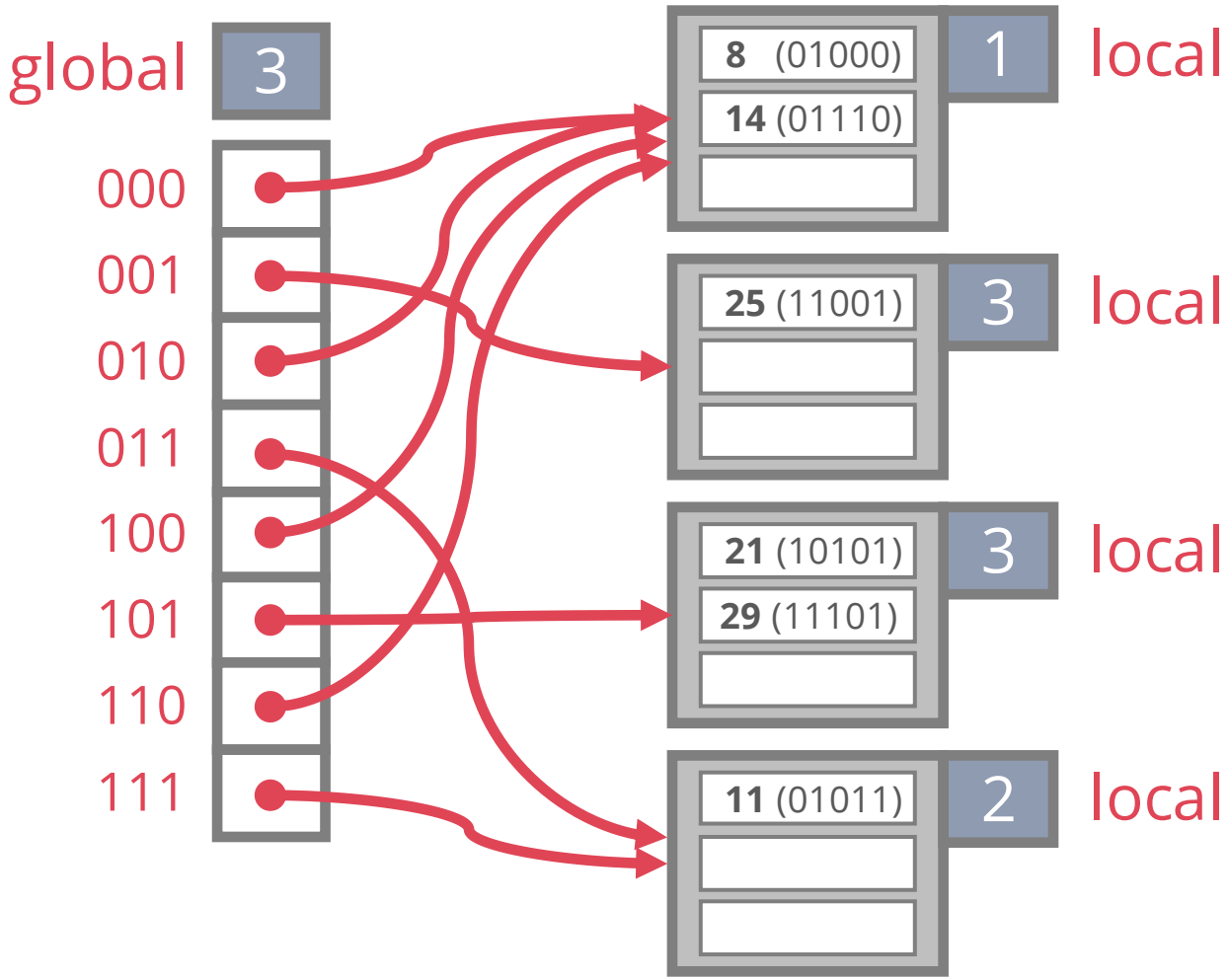
3 bits now needed to discriminate between these two buckets ⇒ **double directory**

Find A
hash(A) = **14** = 01110_2

Insert B
hash(B) = **29** = 11101_2

Insert C
hash(C) = **5** = 00101_2

Extendible Hashing



Find A
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Insert C
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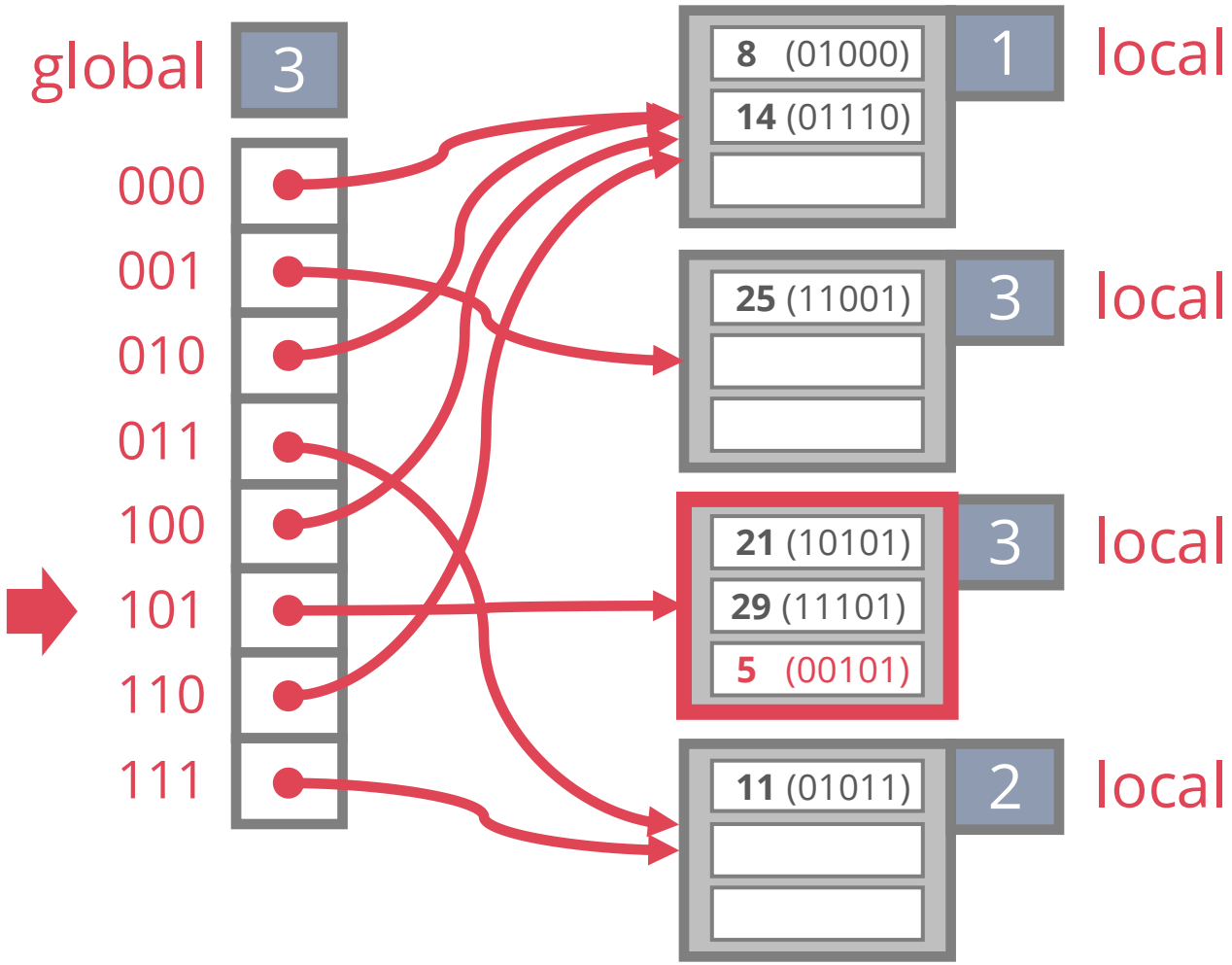
Extendible Hashing



Find A
hash(A) = **14** = 01110_2

Insert B
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Insert C
hash(C) = **5** = 00101_2

Directory Doubling

Double directory by **copying** its original pointers and ”fixing” pointer to split bucket

- Use of least significant bits enables efficient doubling via copying!

Splitting a bucket does not always require doubling the directory

- Buckets with local depth < global depth have multiple pointers to them
- Splitting such buckets does not require doubling
- Modifying one or more bucket pointers in directory is sufficient

Directory can also shrink when buckets become empty

SUMMARY

Hash-based indexes

- Best for equality searches, cannot support range searches

Static hashing

- Can lead to long overflow chains

Extendible hashing

- Avoids overflow chains by splitting a full bucket when a new entry is to be added to it