



Advanced Database Systems

Spring 2026

Lecture #11:

External Sorting & Aggregation

R&G: Chapters 13 & 14

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QUERY EXECUTION OVERVIEW

SQL Query

```
SELECT S.name
FROM Student S, Enrolled E
WHERE S.sid = E.sid
AND E.cid = 'INF-11199'
```

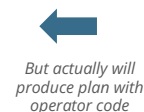
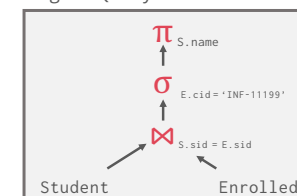


Relational Algebra

```
 $\pi_{S.name}(\sigma_{E.cid = 'INF-11199'}(Student \bowtie_{S.sid = E.sid} Enrolled))$ 
```

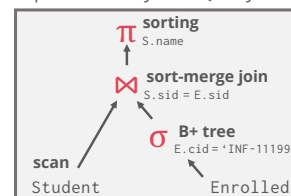


Logical Query Plan



But actually will
produce plan with
operator code

Optimised Physical Query Plan



2

QUERY PLANS AND OPERATORS

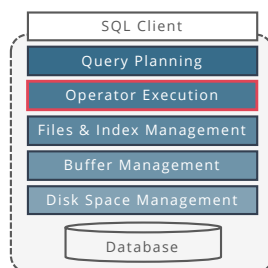
Query plan = Network of operators able to evaluate a query

One operator may have different implementations

- All semantically equivalent
- But with different performance characteristics

Focus of this lecture

- Implementation of **sort**
- Implementation of (grouped by) **aggregation**



WHY DO WE NEED SORTING?

Explicit sorting via the SQL ORDER BY clause

```
SELECT A, B, C FROM R ORDER BY A;
```

Implicit sorting, e.g., for duplicate elimination

```
SELECT DISTINCT A, B, C FROM R;
```

Implicit sorting, e.g., to prepare (sort-merge) equi-join

```
SELECT R.A, S.C FROM R JOIN S ON R.B = S.B;
```

Grouping via **group by**, first step in **bulk loading** tree indexes,
sorted rid scans after access to unclustered indexes, etc.

4

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3

SORTING

5

A file is **sorted** with respect to key k and ordering Θ , if for any two records r_1 and r_2 with r_1 preceding r_2 in the file, their corresponding keys are in Θ -order:

$$r_1 \Theta r_2 \Leftrightarrow r_{1,k} \Theta r_{2,k}$$

A key may be a single attribute or an ordered list of attributes. In the latter case, the order is **lexicographical**

Consider key (A,B) and Θ is <

$$r_1 < r_2 \Leftrightarrow r_{1,A} < r_{2,A} \vee (r_{1,A} = r_{2,A} \wedge r_{1,B} < r_{2,B})$$

SORTING ALGORITHMS

6

If data **fits** in memory, then we can use a standard sorting algorithm like quick-sort

Problem: sort 100GB of data with 1GB of RAM

Why not virtual memory?

If data **does not fit** in memory, then we need to use a technique that is aware of the cost of writing data out to disk

5

6

EXTERNAL SORTING

7

*How can we sort a file of records whose size **exceeds the available main memory space** (let alone the available buffer manager space) by far?*

Idea: **Divide and conquer**

Sort chunks of data that fit in memory, then write back the sorted chunks to disk

Combine sorted chunks into a single larger file

Approach the task in two phases:

1. Sorting a file of arbitrary size is possible using only three buffer pages
2. Refine this algorithm to make effective use of larger buffer sizes

OVERVIEW

8

We will start with a simple example of a 2-way external merge sort

Files are broken up into N pages

The DBMS has a finite number of B fixed-size buffer pages

7

8

2-WAY EXTERNAL MERGE SORT

9

Pass #0

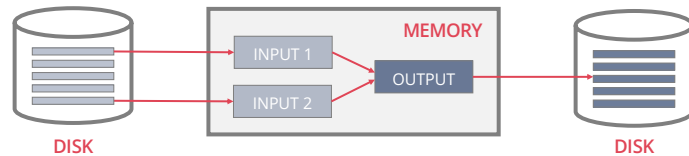
Read a page into memory, sort it, and write it back to disk (uses 1 buffer page)

Each sorted set of pages is called a **run**

Pass #1, #2, #3, ...

Recursively merge pairs of runs into runs twice as long (uses 3 buffer pages)

When input is consumed read next page from disk. When output is full, write to disk



2-WAY EXTERNAL MERGE SORT

10

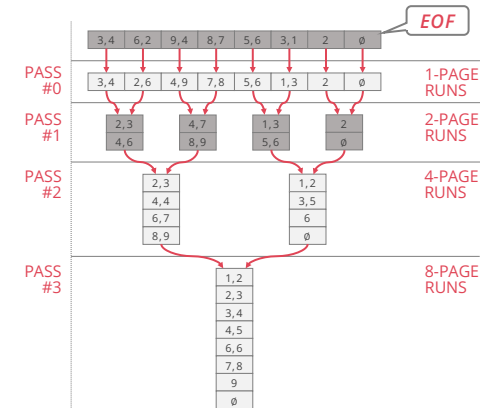
In each pass, we read and write each page in file

Number of passes

$$= 1 + \lceil \log_2 N \rceil$$

Total I/O cost

$$= 2N \cdot (\text{\# of passes})$$



10

EXTERNAL MERGE SORT

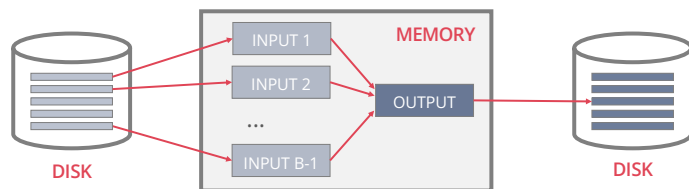
11

Previous algorithm uses only three buffer pages ($B = 3$)

How can we make effective use of a larger buffer pool ($B > 3$)?

Reduce # of initial runs by using the full buffer space during in-memory sort

Reduce # of passes by merging $B - 1$ runs at a time



EXTERNAL MERGE SORT

12

Pass #0

Use B buffer pages

Produce $\lceil N / B \rceil$ sorted runs of size B

Pass #1, #2, #3, ...

Merge $B - 1$ runs (i.e., multi-way merge)

Number of passes = $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$

Total I/O cost = $2N \cdot (\text{\# of passes})$

12

EXAMPLE

13

Sort $N = 108$ page file with $B = 5$ buffer pages

Pass #0: $\lceil 108/5 \rceil = 22$ sorted runs of 5 pages each (last run is only 3 pages)

Pass #1: $\lceil 22/4 \rceil = 6$ sorted runs of 20 pages each (last run is only 8 pages)

Pass #2: $\lceil 6/4 \rceil = 2$ sorted runs of 80 pages and 28 pages

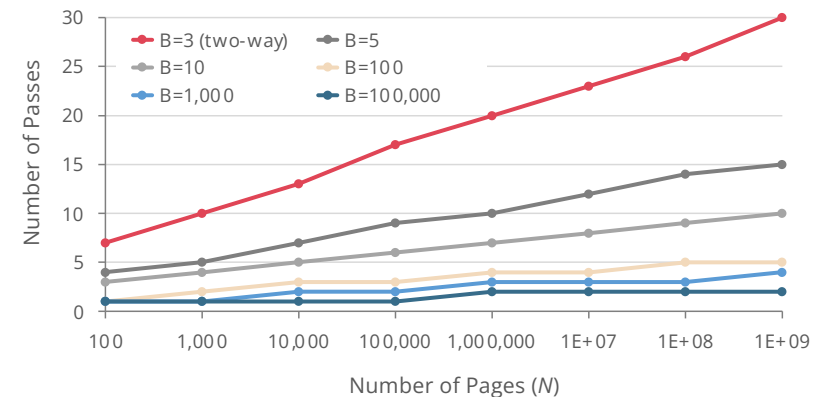
Pass #3: Sorted file of 108 pages

Number of passes = $1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil = 1 + \lceil \log_4 22 \rceil = 1 + \lceil 2.229... \rceil = 4$ passes

Total I/O cost = $2N \cdot (\text{\# of passes}) = 2 \cdot 108 \cdot 4 = 864$

NUMBER OF PASSES OF EXTERNAL SORT

14



14

USING B+ TREES FOR SORTING

17

If the table to be sorted has a B+ tree index on the sort attribute(s), we may be better off by accessing the index and avoid external sorting

Retrieve sorted records by simply traversing the leaf pages of the tree

Cases to consider

- Clustered B+ tree
- Unclustered B+ tree

CASE 1: CLUSTERED B+ TREE

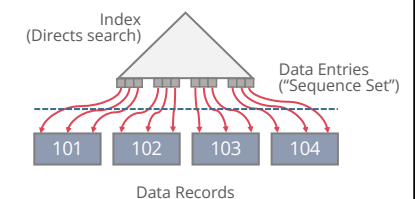
18

Traverse to the left-most leaf page, then retrieve all leaf pages (variant **A**)

If variant **B** is used?

Additional cost of retrieving data records: each page fetched just once

Always better than external sorting!



17

18

CASE 2: UNCLUSTERED B+ TREE

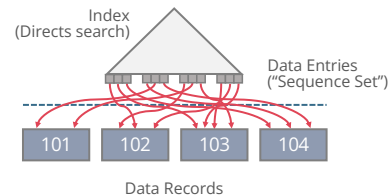
19

Variant **B** for index entries
(each contains *rid* of a data record)

Chase each pointer to the page
that contains the data

This is almost always a bad idea

In general, **one I/O per data record**



DUPLICATE ELIMINATION USING SORTING

20

```
SELECT DISTINCT cid
FROM Enrolled
WHERE grade < 90
```

Enrolled(sid, cid, grade)

sid	cid	grade
123466	INFR-11011	65
123488	INFR-11122	95
123488	INFR-10070	80
123466	INFR-11122	70
123455	INFR-11011	75

Filter

sid	cid	grade
123466	INFR-11011	65
123488	INFR-10070	80
123466	INFR-11122	70
123455	INFR-11011	75

Remove
Columns

cid
INFR-11011
INFR-10070
INFR-11122
INFR-11011

Sort

cid
INFR-10070
INFR-11011
INFR-11011
INFR-11122

Eliminate Duplicates

ALTERNATIVE TO SORTING

21

What if we do not need the data to be ordered?

Forming groups in **GROUP BY** (no ordering)

Removing duplicates in **DISTINCT** (no ordering)

Hashing is a better alternative in this scenario

Only need to remove duplicates, no need for ordering

Can be computationally cheaper than sorting

EXTERNAL HASHING

24

We cannot build an in-memory hash table if there is too much data!

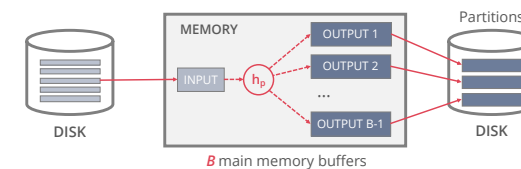
Start by splitting up data into smaller pieces!

Use a hash function h_p to partition the data

Stream partitions to disk

If we have **B** pages of buffer, we can split the data into **B-1** partitions

1 buffer page reserved for streaming data in



19

20

21

24

EXTERNAL HASHING

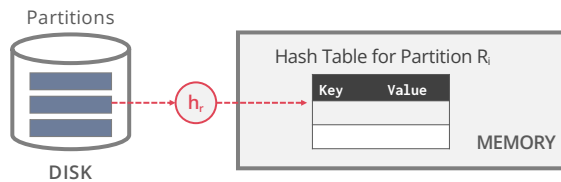
25

If partitions are small enough to fit in memory, we can load them in and make an in-memory hash table for each one, one at a time

Then we can apply duplicate removal, aggregation, etc. in memory

Every tuple in a partition has the same value when h_p is applied!

In-memory hash table must use a different hash function h_r that is independent of h_p



25

AGGREGATIONS

28

Collapse multiple tuples into a single scalar value (SUM, MIN, MAX, ...)

Hashing aggregates:

Populate an ephemeral hash table as the DBMS scans the relation. For each record check whether there is already an entry in the hash table

DISTINCT: Discard duplicate

GROUP BY: Perform aggregate computation

If everything fits in memory, then it's easy

If we have to spill to disk, then we need to be smarter...

```
SELECT A, MAX(B) FROM R
GROUP BY A;
```

28

HASHING AGGREGATE

29

Partition phase

Divide tuples into partitions based on hash key

Rehash phase

Build in-memory hash table for each partition and compute the aggregate

29

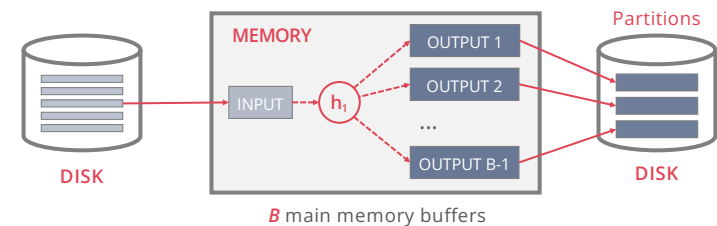
HASHING AGGREGATE PHASE #1: PARTITION

30

Use a hash function h_1 to split tuples into partitions on disk

We know that all matches live in the same partition

Partitions are "spilled" to disk via output buffers



30

HASHING AGGREGATE PHASE #1: PARTITION

31

```
SELECT DISTINCT cid
FROM Enrolled
WHERE grade < 90
```

Enrolled(sid, cid, grade)

sid	cid	grade
123466	INFR-11199	80
123488	INFR-11122	95
123488	INFR-10070	80
123466	INFR-11122	50
123455	INFR-11199	75

Filter

sid	cid	grade
123466	INFR-11199	80
123488	INFR-10070	80
123466	INFR-11122	50
123455	INFR-11199	75

Remove Columns

cid
INFR-11199
INFR-10070
INFR-11122
INFR-11199

h_1

INFR-11011
INFR-10070
INFR-11011
...
INFR-11122

B-1 partitions

HASHING AGGREGATE PHASE #2: REHASH

32

For each partition on disk:

Read it into memory and build an in-memory hash table based on a second hash function $h_2 (\neq h_1)$

Then go through each bucket of this hash table to bring together matching tuples

No need to load the entire partition at once in memory

Can load several pages at a time

But the hash table built for each partition must fit in memory

If not enough memory, repeat Phase #1 on each partition with a different hash function

32

HASHING AGGREGATE PHASE #2: REHASH

33

```
SELECT DISTINCT cid
FROM Enrolled
WHERE grade < 90
```

Enrolled(sid, cid, grade)

sid	cid	grade
123466	INFR-11011	80
123488	INFR-11122	95
123488	INFR-10070	80
123466	INFR-11122	50
123455	INFR-11011	75

Phase #1 Partitions

INFR-11011
INFR-10070
INFR-11011
...
INFR-11122

h_2

Hash Table

Key	Value
INFR-11011	2
INFR-10070	1
...	...
INFR-11122	1

multiplicity

cid
INFR-11011
INFR-10070
INFR-11122

HASHING SUMMARISATION

34

During the Rehash phase, store pairs of the form

GroupKey \rightarrow **RunningValue**

When we want to insert a new tuple into the hash table

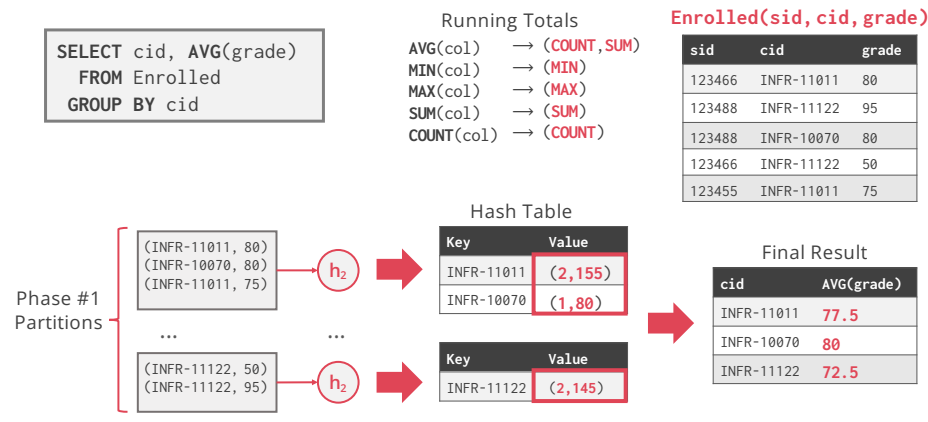
If we find a matching **GroupKey**, just update the **RunningValue** appropriately

Else insert a new **GroupKey** \rightarrow **RunningValue**

34

HASHING SUMMARISATION

35



35

COST ANALYSIS

36

How big of a table can we hash using this approach?

$B-1$ "spill partitions" in Phase #1

Each partition (i.e., its hash table) should be no more than B pages big

Answer: $B \cdot (B-1)$

A table of N pages needs about \sqrt{N} buffer pages

Note: assumes hash distributes records evenly!

Use a "fudge factor" $f > 1$ to capture the (small) increase in size between the partition and a hash table for that partition

Must be $B > f \cdot N / (B-1)$; thus, we need approx. $B > \sqrt{f \cdot N}$ buffer pages

36

CONCLUSION: SORTING VS. HASHING

37

External merge sort often finishes in 1-2 passes

Great if we need output to be sorted anyway

Not sensitive to duplicates or "bad" hash functions

Duplicate elimination

Hashing preferred as it scales with # of distinct values

Delete duplicates in first pass while partitioning

Vs. sort which scales with # of values

Group-by aggregation

Typically computed via hashing

37