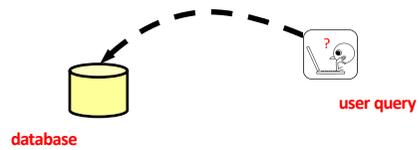


understand how a DBMS efficiently evaluates a query over a database
(the main focus of this course)



A Quick Recap

Flight	origin	destination	airline
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Airport	code	city
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A Quick Recap

List all the airlines

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

A Quick Recap

List all the airlines

Flight	origin	destination	airline
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	LGW	GLA	U2
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Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

↓
{BA, U2, OS}

$\pi_{airline}$ Flight

A Quick Recap

List the codes of the airports in London

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
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A Quick Recap

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Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇓
{LHR, LGW}

$\pi_{code} (\sigma_{city='London'} Airport)$

A Quick Recap

List the airlines that fly directly from London to Glasgow

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

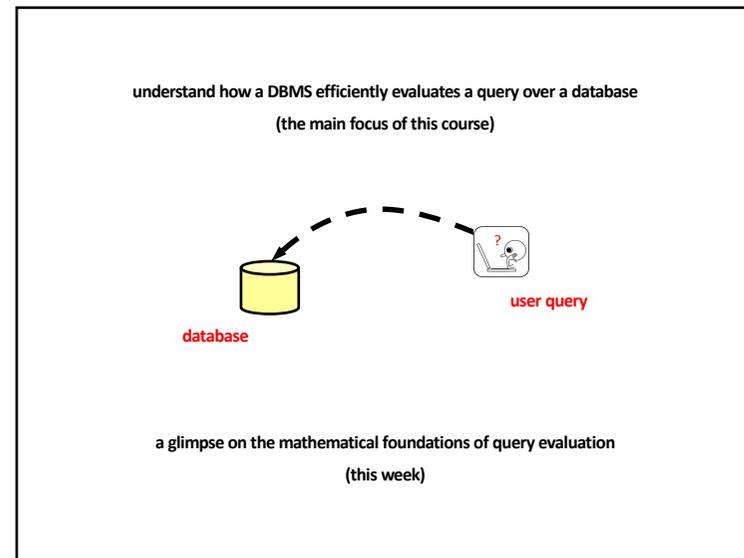
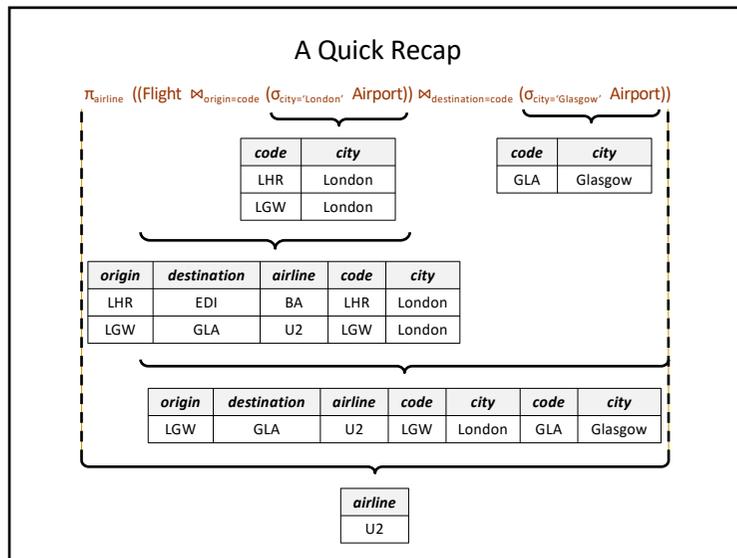
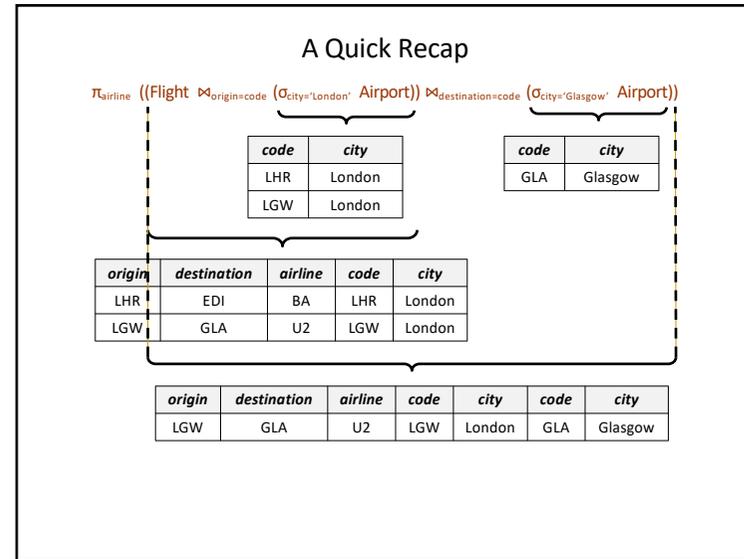
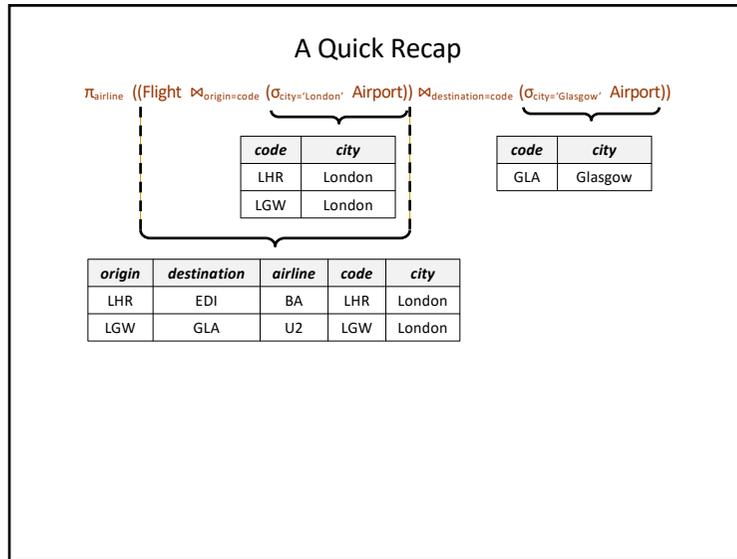
$\pi_{airline} ((Flight \bowtie_{origin=code} (\sigma_{city='London'} Airport)) \bowtie_{destination=code} (\sigma_{city='Glasgow'} Airport))$

A Quick Recap

$\pi_{airline} ((Flight \bowtie_{origin=code} (\sigma_{city='London'} Airport)) \bowtie_{destination=code} (\sigma_{city='Glasgow'} Airport))$

code	city
LHR	London
LGW	London

code	city
GLA	Glasgow



Conjunctive Queries: Syntax and Semantics

(Chapters 12 and 13 of DBT)

[DBT] Database Theory, <https://github.com/pdm-book/community>

Codd's Theorem

Relational Algebra = Relational Calculus



Edgar F. Codd
(1923 - 2003)
Turing Award 1981

- Queries are written using a **declarative language**
- DBMS converts declarative queries into **procedural queries** that are optimized and executed

Relational Calculus

List all the airlines

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇓
{BA, U2, OS}

$\{z \mid \exists x \exists y (\text{Flight}(x,y,z))\}$

Relational Calculus

List the codes of the airports in London

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇓
{LHR, LGW}

$\{x \mid \text{Airport}(x, \text{London})\}$

Relational Calculus

List the airlines that fly directly from London to Glasgow

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇓
{U2}

$\{z \mid \exists x \exists y (\text{Airport}(x, \text{London}) \wedge \text{Airport}(y, \text{Glasgow}) \wedge \text{Flight}(x, y, z))\}$

A Core Relational Query Language

Conjunctive Queries (CQ)

- = $\{\sigma, \pi, \bowtie\}$ -fragment of relational algebra
- = relational calculus without \neg, \forall, \neq
- = simple SELECT-FROM-WHERE SQL queries (only AND and equality in the WHERE clause)

Syntax of Conjunctive Queries

$$Q(x) := \exists y (R_1(v_1) \wedge \dots \wedge R_m(v_m))$$

- R_1, \dots, R_m are relation names
- x, y, v_1, \dots, v_m are tuples of variables
- each variable mentioned in v_i appears either in x or y
- the variables in x are free called *distinguished* or *output variables*

It is very convenient to see conjunctive queries as rule-based queries of the form

$$Q(x) :- \underbrace{R_1(v_1), \dots, R_m(v_m)}$$

this is called the *body* of Q that can be seen as a set of atoms

Conjunctive Queries: Example 1

List all the airlines

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇓
{BA, U2, OS}

$\pi_{\text{airline}} \text{Flight}$
 $\{z \mid \exists x \exists y \text{Flight}(x, y, z)\}$

$Q(z) :- \text{Flight}(x, y, z)$

Conjunctive Queries: Example 2

List the codes of the airports in London

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh



{LHR, LGW}

$\pi_{code} (\sigma_{city='London'} \text{ Airport})$

{x | Airport(x,London)}

Q(x) :- Airport(x,London)

Conjunctive Queries: Example 3

List the airlines that fly directly from London to Glasgow

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh



{U2}

$\pi_{airline} ((\text{Flight} \bowtie_{origin=code} (\sigma_{city='London'} \text{ Airport})) \bowtie_{destination=code} (\sigma_{city='Glasgow'} \text{ Airport}))$

{z | $\exists x \exists y (\text{Airport}(x,London) \wedge \text{Airport}(y,Glasgow) \wedge \text{Flight}(x,y,z))$ }

Conjunctive Queries: Example 3

List the airlines that fly directly from London to Glasgow

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh



{U2}

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

{	Flight(VIE,LHR,BA),	Airport(VIE,Vienna),
	Flight(LHR,EDI,BA),	Airport(LHR,London),
	Flight(LGW,GLA,U2),	Airport(LGW,London),
	Flight(LCA,VIE,OS),	Airport(LCA,Larnaca),
		Airport(GLA,Glasgow),
	Airport(EDI,Edinburgh)	

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

}	Flight(VIE,LHR,BA),	Airport(VIE,Vienna),
	Flight(LHR,EDI,BA),	Airport(LHR,London),
	Flight(LGW,GLA,U2),	Airport(LGW,London),
	Flight(LCA,VIE,OS),	Airport(LCA,Larnaca),
		Airport(GLA,Glasgow),
	Airport(EDI,Edinburgh)	

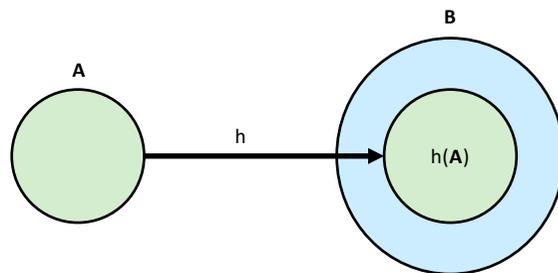
$Q(z) :- \text{Airport}(x,\text{London}), \text{Airport}(y,\text{Glasgow}), \text{Flight}(x,y,z)$

Homomorphism

- Pattern matching - properly formalized via the key notion of **homomorphism**
- A **substitution** from a set of terms S to a set of terms T is a function $h : S \rightarrow T$, i.e., h is a set of **mappings** of the form $s \mapsto t$, where $s \in S$ and $t \in T$
- A **homomorphism** from a set of atoms A to a set of atoms B is a substitution $h : \text{terms}(A) \rightarrow \text{terms}(B)$ such that:
 1. t is a constant value $\Rightarrow h(t) = t$
 2. $R(t_1, \dots, t_k) \in A \Rightarrow h(R(t_1, \dots, t_k)) = R(h(t_1), \dots, h(t_k)) \in B$

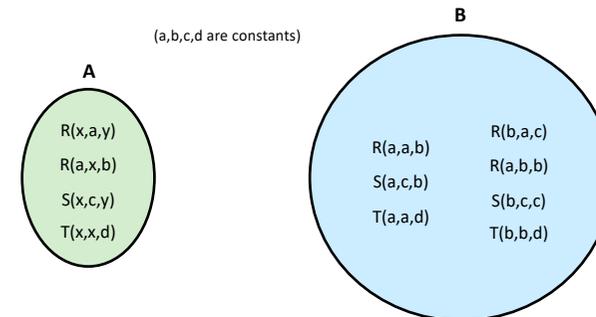
$(\text{terms}(A) = \{t \mid t \text{ is a variable or a constant value that occurs in } A\})$

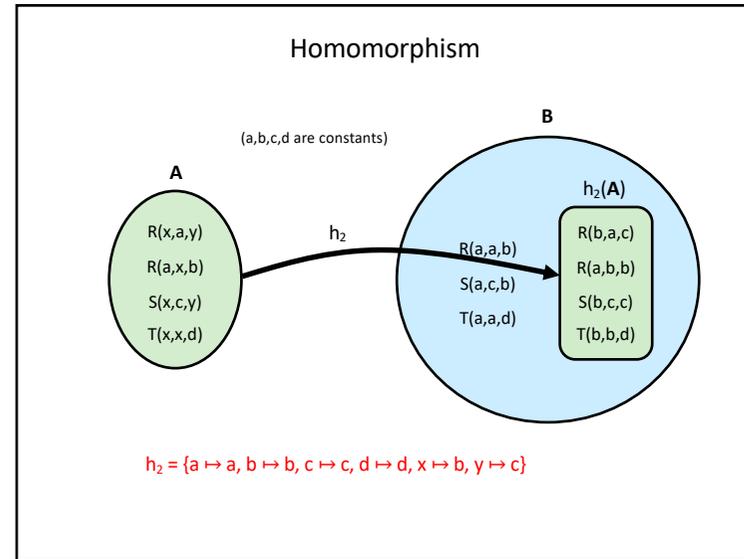
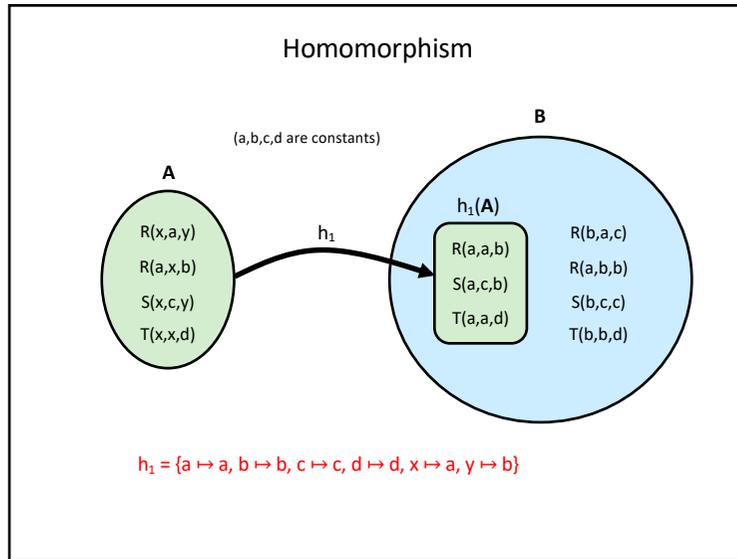
Homomorphism



$h : \text{terms}(A) \rightarrow \text{terms}(B)$ that is the identity on constants

Homomorphism





- ### Semantics of Conjunctive Queries
- A **match** of a conjunctive query $Q(x_1, \dots, x_k) :- \text{body}$ in a database D is a homomorphism h from the set of atoms **body** to the set of atoms D
 - The **answer** to $Q(x_1, \dots, x_k) :- \text{body}$ over D is the set of k -tuples

$$Q(D) := \{(h(x_1), \dots, h(x_k)) \mid h \text{ is a match of } Q \text{ in } D\}$$
 - The answer consists of the witnesses for the **distinguished variables** of Q

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

Flight(VIE,LHR,BA),	Airport(VIE,Vienna),
Flight(LHR,EDI,BA),	Airport(LHR,London),
Flight(LGW,GLA,U2),	Airport(LGW,London),
Flight(LCA,VIE,OS),	Airport(LCA,Larnaca),
	Airport(GLA,Glasgow),
	Airport(EDI,Edinburgh)

$Q(z) :- \text{Airport}(x,\text{London}), \text{Airport}(y,\text{Glasgow}), \text{Flight}(x,y,z)$

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

Flight(VIE,LHR,BA),	Airport(VIE,Vienna),
Flight(LHR,EDI,BA),	Airport(LHR,London),
Flight(LGW,GLA,U2),	Airport(LGW,London),
Flight(LCA,VIE,OS),	Airport(LCA,Larnaca),
	Airport(GLA,Glasgow),
	Airport(EDI,Edinburgh)

$\{x \mapsto \text{LGW}, y \mapsto \text{GLA}, z \mapsto \text{U2},$
 $\text{London} \mapsto \text{London}, \text{Glasgow} \mapsto \text{Glasgow}\}$

$Q(z) :- \text{Airport}(x,\text{London}), \text{Airport}(y,\text{Glasgow}), \text{Flight}(x,y,z)$