University of Edinburgh

INFR11156: Algorithmic Foundations of Data Science (2025)

Homework 2

Problem 1: Compute the right-singular vectors v_i, the left-singular vectors u_i, the singular values σ_i and hence find the *Singular value decomposition* of

1. $A = \begin{pmatrix} 1 & 1 \\ 0 & 3 \\ 3 & 0 \end{pmatrix}$;

2. $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \\ 1 & 3 \\ 3 & 1 \end{pmatrix}$.

Problem 2: Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \\ 1 & -2 \\ -1 & -2 \end{pmatrix}.$$

1. Run the *power method* starting from $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $k = 3$ steps. What does this give as estimates for v_1 and σ_1?

2. What are the actual values of v_i's, σ_i's and u_i's? You might find it helpful to first compute the eigenvalues and eigenvectors of $B = A^\intercal A$.

Problem 3: Let $v \in \mathbb{R}^n$ such that $\|v\| = 1$. Sample uniformly $x \in \{-1, 1\}^n$, and define $S = \langle x, v \rangle$. Prove that

$$\mathbf{E}\left[S^4\right] = 3\sum_{i=1}^{n} v_i^2 - 2\sum_{i=1}^{n} v_i^4 \leq 3.$$

Problem 4: Let $A \in \mathbb{R}^{n \times n}$ be a symmetric and PSD matrix. Show that the power method can be applied to approximately compute the smallest eigenvalue of A.

Problem 5: Let u be a fixed vector. Show that maximising $x^\intercal u u^\intercal (1 - x)$ subject to $x_i \in \{0, 1\}$ is equivalent to partitioning the coordinates of u into two subsets where the sum of the elements in both subsets are as equal as possible.