

# AR Coursework QA

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# Information

- Demonstrator/TA: Filip Smola `f.smola@ed.ac.uk`
- Lab sessions: Mondays from 9am to 11am in 4.12, Appleton Tower
- Submission deadline: 12 noon on 18th November

# Structured Higher-Order Statements

## Lemmas

Higher-order logic (meta-level) statement:

**lemma** finite\_UnI:

```
" $\wedge F\ G. \llbracket \text{finite } F; \text{ finite } G \rrbracket \implies \text{finite } (F \cup G)$ "
```

**proof** -

```
fix F G :: "'a set"
```

```
assume "finite F"
```

```
assume "finite G"
```

```
show "?thesis F G" sorry
```

**qed**

# Structured Higher-Order Statements

## Lemmas

Letting Isabelle take care of universally quantifying free variables:

**lemma** finite\_UnI:

```
"[[finite F; finite G]]  $\implies$  finite (F  $\cup$  G)"
```

**proof** -

```
  assume "finite F"
```

```
  assume "finite G"
```

```
  show ?thesis sorry
```

**qed**

# Structured Higher-Order Statements

## Lemmas

Structured statement using outer syntax:

```
lemma finite_UnI:
  assumes "finite F"
  and finite_G: "finite G"
  shows "finite (F  $\cup$  G)"
proof -
  show ?thesis
  using 'finite F' finite_G sorry
qed
```

# Structured Higher-Order Statements

## Lemmas

Explicitly stating type of free variables:

```
lemma finite_UnI:
  fixes F G :: "'a set"
  assumes "finite F"
  and finite_G: "finite G"
  shows "finite (F  $\cup$  G)"
proof -
  show ?thesis
  using 'finite F' finite_G sorry
qed
```

# Structured Higher-Order Statements

## Lemmas

No need to use structured statements for simple cases:

**lemma** finite\_lt:

```
"finite {x :: nat. x < n}"
```

**by** simp

# Structured Higher-Order Statements

## Intermediate Facts

Single higher-order statement:

```
have " $\bigwedge F\ G. [\text{finite } F; \text{finite } G] \implies \text{finite } (F \cup G)$ "
```

```
proof -
```

```
  fix F G :: "'a set"
```

```
  assume "finite F"
```

```
  assume "finite G"
```

```
  show "?thesis F G" sorry
```

```
qed
```

```
then have ...
```

# Structured Higher-Order Statements

## Intermediate Facts

Raw proof block:

```
{  
  fix F G :: "'a set"  
  assume "finite F"  
  assume "finite G"  
  show "finite (F  $\cup$  G)" sorry  
}  
then have ...
```

# Structured Higher-Order Statements

## Intermediate Facts

Structured statement:

```
have "finite (F ∪ G)"  
  if "finite F" and "finite G" for F G :: "'a set"  
proof -  
  show ?thesis  
    using that sorry  
qed  
then have ...
```

# Gathering Facts in Isar

Manually:

```
have a: "x < 0  $\implies$  P"
```

```
  sorry
```

```
have b: "x = 0  $\implies$  P"
```

```
  sorry
```

```
have c: "x > 0  $\implies$  P"
```

```
  sorry
```

```
have P
```

```
  using a b c by blast
```

# Gathering Facts in Isar

Automatically:

have "x < 0  $\implies$  P"

sorry

moreover have "x = 0  $\implies$  P"

sorry

moreover have "x > 0  $\implies$  P"

sorry

ultimately have P

by blast

# Overview

- Submission deadline: 12 noon on 18th November
- Automated tactics (`auto`, `simp`, ...) can be used from Part 2 onwards
- Use “Query” panel or [find\\_theorems](#) to search for useful theorems
- Break things into sublemmas, especially in Part 2
- Ask questions (labs, Piazza, email)