THE UNIVERSITY of EDINBURGH
informatics

Advanced Robotics

Motion Planning I

Steve Tonneau
School of Informatics
University of Edinburgh

Reading for this week

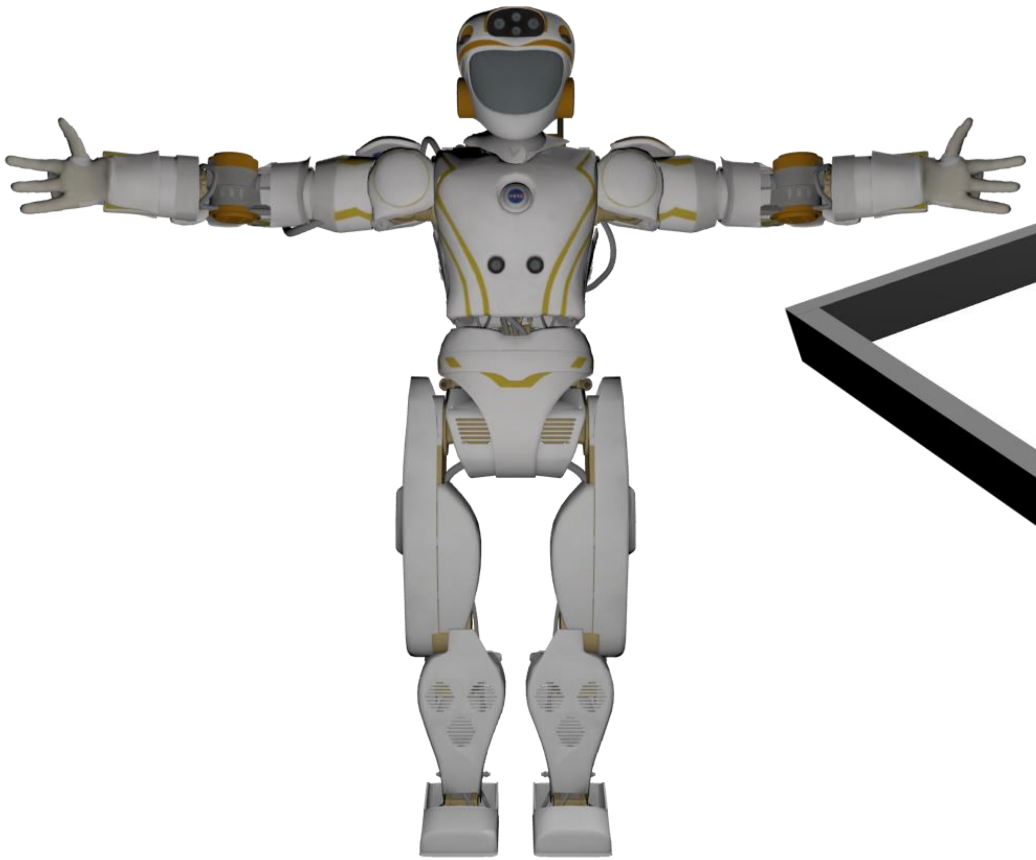
- ❑ Siciliano, B., et al., Robotics: Modelling, Planning and Control.
  Chapter 12.5 – 12.6

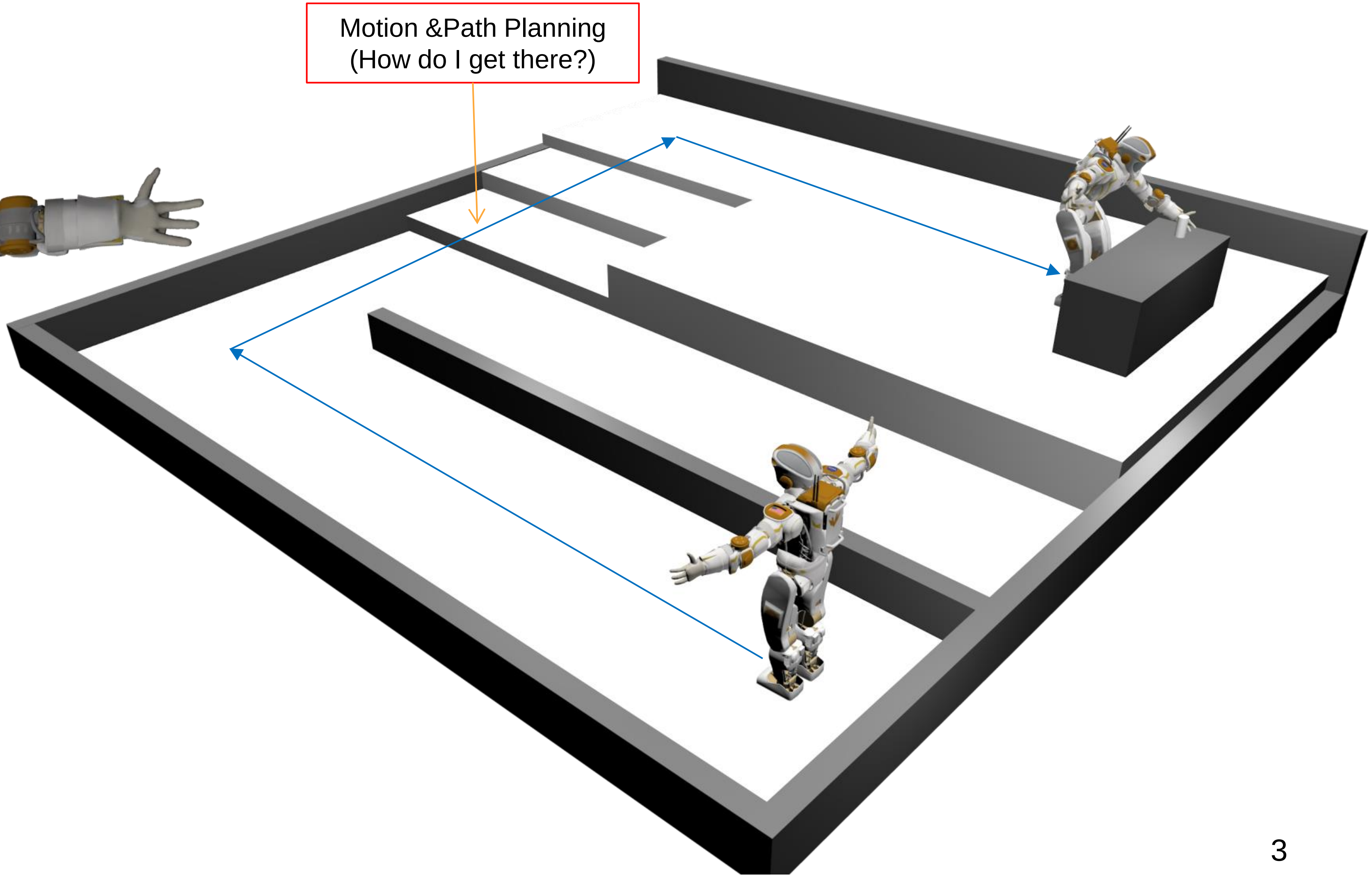
- ❑ ‘Probabilistic roadmaps for path planning in high-dimensional configuration spaces’
  Kavraki et al. IEEE Transactions on Robotics and Automation

- ❑ Rapidly-exploring random trees: A new tool for path planning
  Lavalle, Research Report 9811

- ❑ RRT-connect: An efficient approach to single-query path planning
  Kuffner et Lavalle, ICRA 2002

Recap: What Tools Do We Need?





Outline

- ❑ **Motion planning: goals and challenges**

- ❑ **An intuitive approach: Potential Fields**

- ❑ Sampling based motion planning
  - ❑ Rapidly exploring Random Tree (RRT): principle and code demo (in tutorial)

    Variants of RRT algorithms
  - ❑ Multi-Query Planner: Probabilistic Roadmap (PRM)

Motion Planning

Fundamental task: plan collision-free motions for complex systems from a start to a goal position among a set of obstacles. For mobile robots, it is also referred as a path planning problem.

Today we’ll talk about the geometric aspect of the problem (again)

We’ll start on dynamics next week

The concepts presented here will extend to the dynamic context

What is a Path Planning Problem?

- ❑Elements of the problem:
  - ❑Description of environment, e.g., a map
  - ❑Positions of obstacles, terrain properties, etc.
  - ❑Description of the robot and its capabilities, e.g., geometry of body, ability to move, etc.

- ❑Problem: Given the above elements and start & goal points (sets), compute a path from start to end
  - ❑How to define feasible paths?*
  - ❑Can we evaluate the quality of a path?

* For now, feasibility will be defined in terms of self-collisions / collisions with the environment

Background

Two families of approaches:

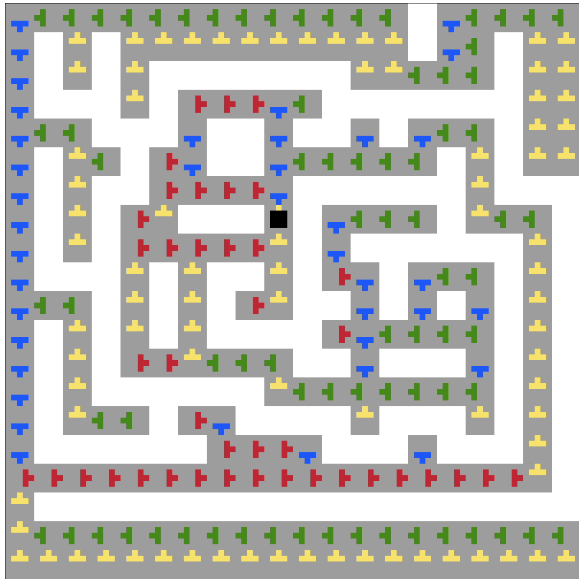
- ❑ Local methods that reactively adapt the behaviour of the robot

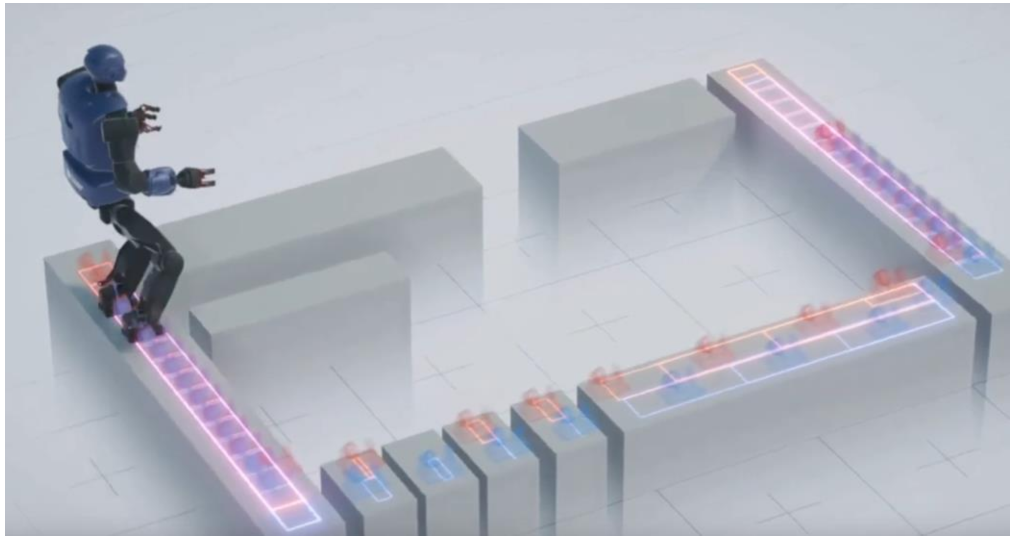
- ❑ Global methods, slower to execute, specially designed to escape local minimas (remember the IG lecture)

- ❑ In the state of the art, most robot motion generation techniques employ both local and global approaches at different frequencies

Intro to the Bellman principle

- *"An optimal policy has the property that, whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."*



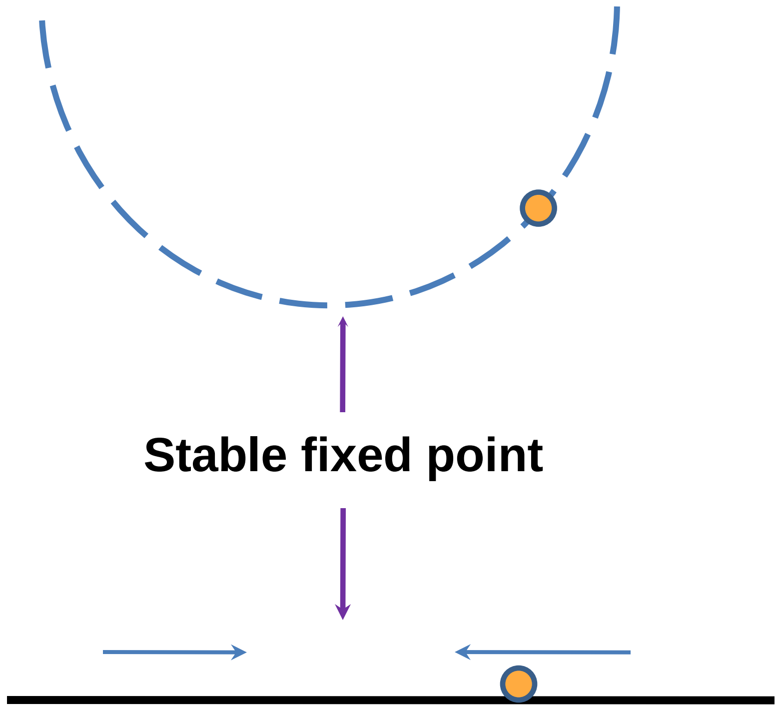


https://www.puzzlescript.net/play.html?p=7441823

Artificial Potential Fields

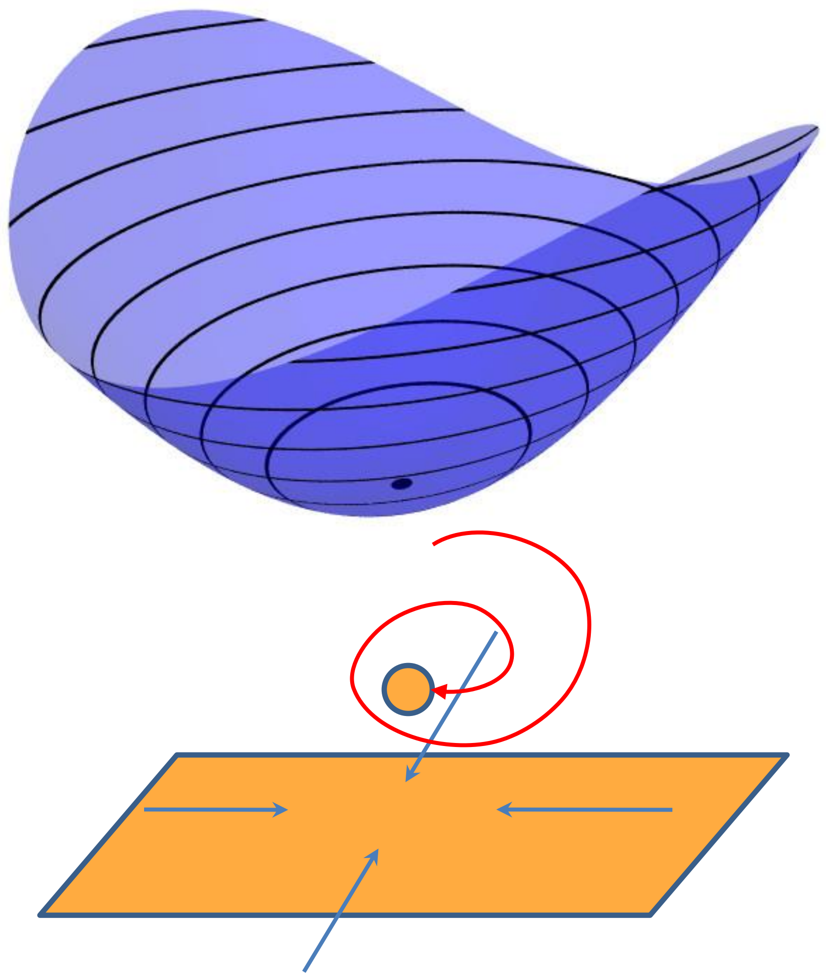
Potential Fields

- ❑Imagine a 1-dim ball rolling within a flat bowl
- ❑Where will it eventually end up after long time interval?
- ❑What happens if you push the ball around with your finger?
- ❑If you "create" such a field on your 1-dim flat world, where will the ball go?



Higher-dimensional bowl

- You could play the same game in higher dimensions
- With contours that shrink down to a point, the ball will move in the direction that decreases a measure of height
- The effect on a 2-dim workspace is that the ball will converge to a fixed point



Why is this useful?

- ❑Imagine a mobile robot in situations where low-level control isn't perfect
  - ❑Wheels can slip
  - ❑Support surfaces may change
  - ❑Rover could get pushed around in high winds

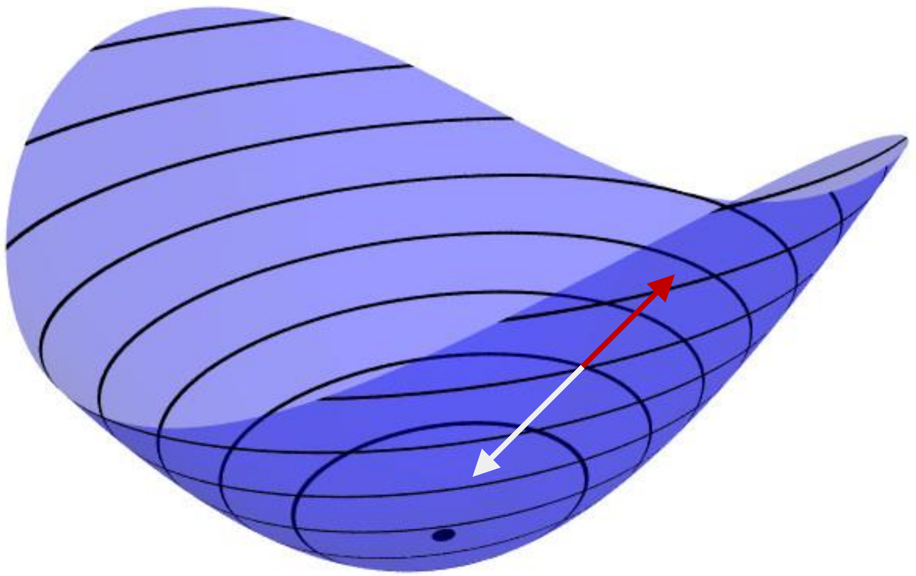
- ❑Nice to have the notion of convergence to goal right at the planning level

Potential Fields

- Artificial potential field approach is originally proposed for collision avoidance. It constructs a differentiable real-valued function

  $\boldsymbol{U} : \boldsymbol{R}^m \rightarrow \boldsymbol{R}$

  called a potential function, which guides the motion of the moving object.



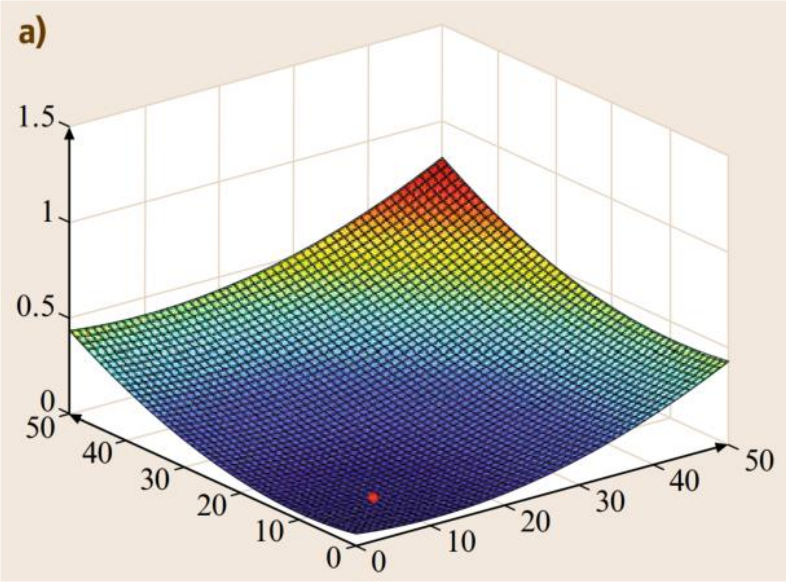
- Treat the value as 'energy'
- Then, gradient is the vector,

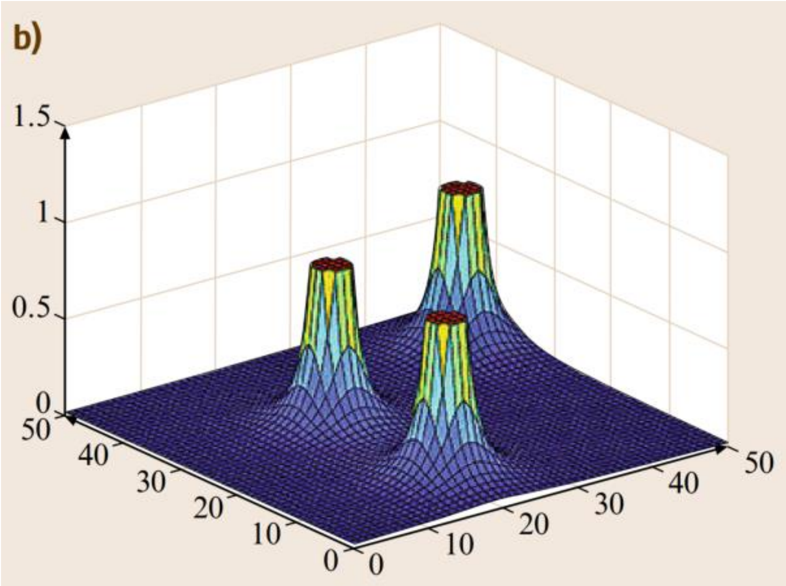
$$\nabla U(q) = DU(q)' = [\frac{\partial U}{\partial q_1}(q), \dots, \frac{\partial U}{\partial q_m}(q)]'$$

Attractive and Repulsive Components

Potential field consists of:

1. an attractive component $\boldsymbol{U_a}$, which pulls the robot towards the goal;
2. and a repulsive component $\boldsymbol{U_r}$, which pushes the robot away from the obstacles.





(Picture source: Springer Handbook of Robotics ISBN 978-3-319-32552-1)

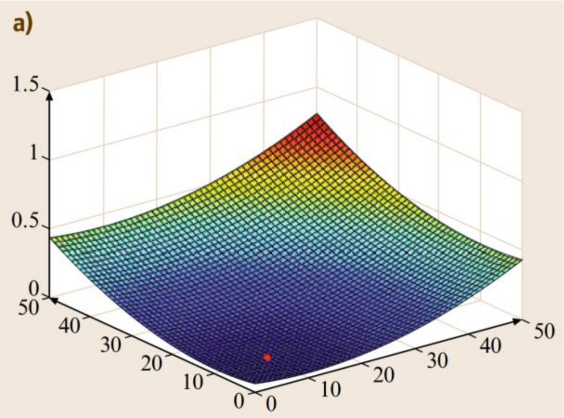
Attractive / Repulsive Components (per dimension)

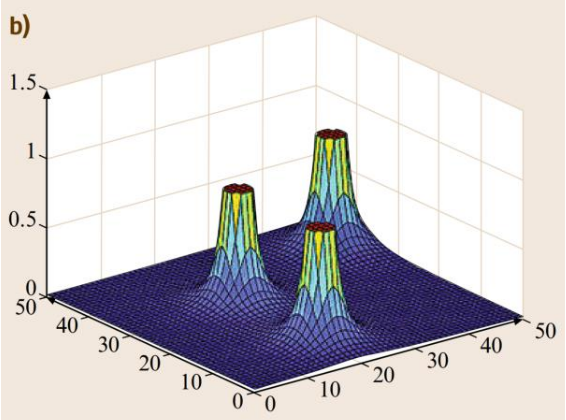
Attractive Component:

$$U_{\boldsymbol{a}}(x) = \frac{1}{2}k_p(x - x_d)^2$$

$$\nabla U_a(x) = k_p(x - x_d)$$

Following negative gradient direction will get us to the goal





Attractive / Repulsive Components (per dimension)

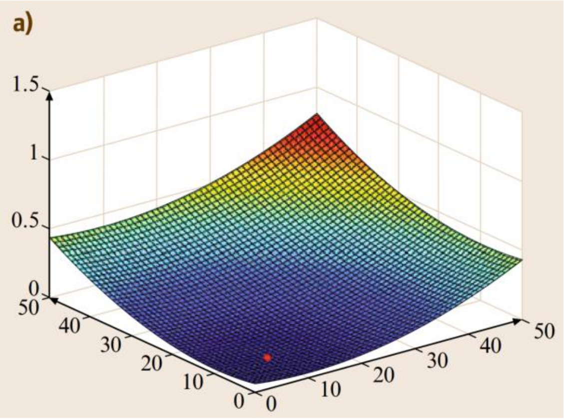
Attractive Component:

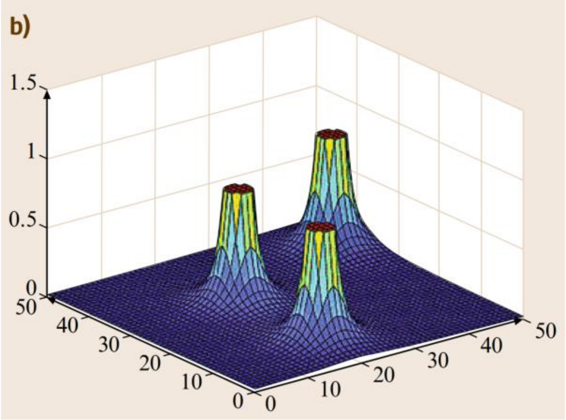
$$\boldsymbol{U_a}(x) = \frac{1}{2}k_p(x - x_d)^2$$

$$\nabla U_a(x) = k_p(x - x_d)$$

Does that sound familiar?

Following negative gradient direction will get us to the goal





Attractive and Repulsive Components

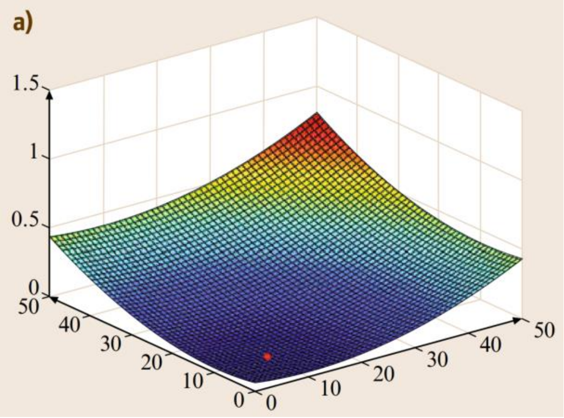
Repulsive Component:

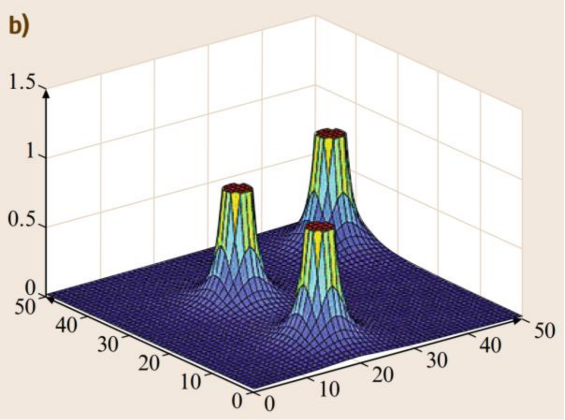
$$U_r(x) = \frac{1}{2}\eta(1/\rho(x) - 1/\rho_0)^2 \text{ if } \quad \rho \leq \rho_0$$

$\boldsymbol{U_r}(x) = 0$, otherwise. Where:

$\rho(x)$: shortest distance to the obstacle

ρ_0: limit distance of the potential field influence





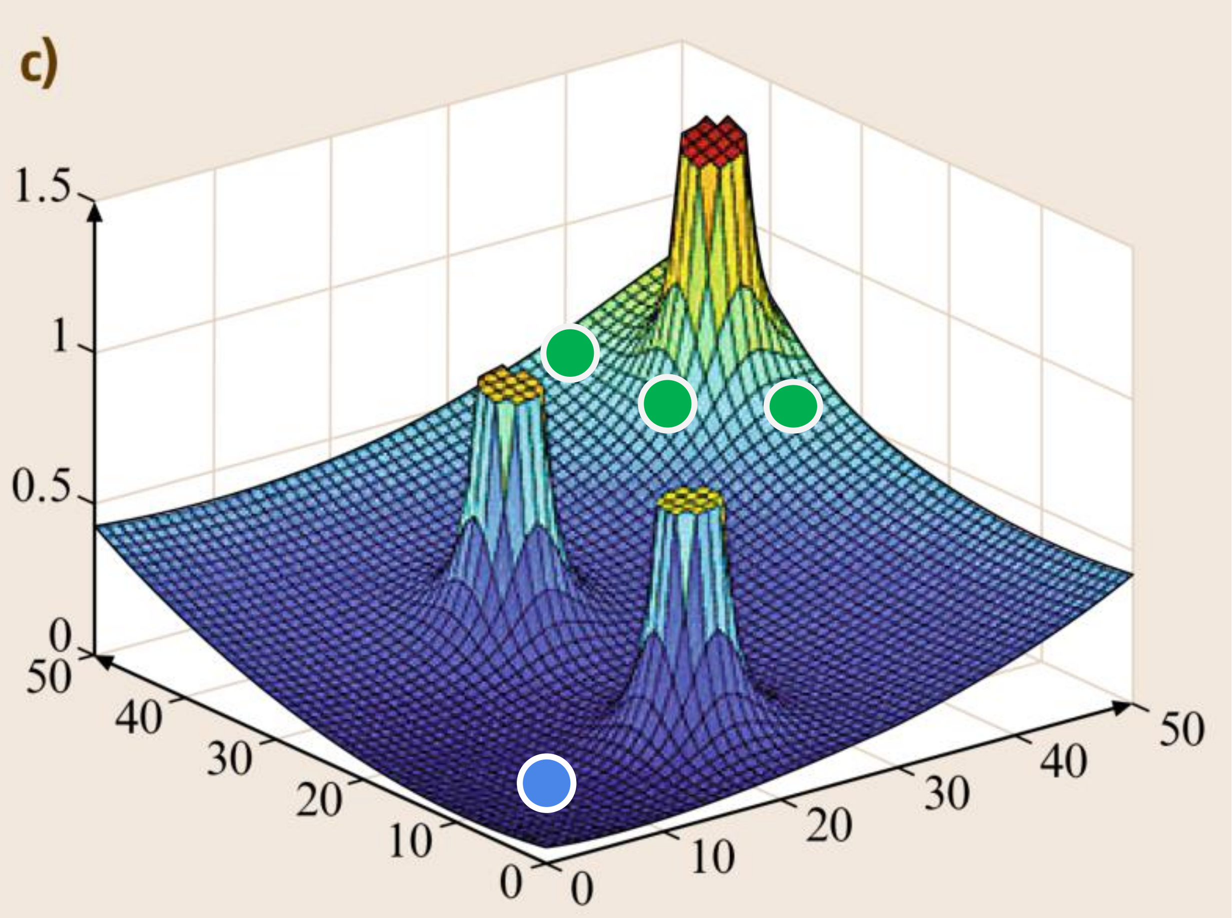
Verify as and exercise that the negative of the gradient will point away from the obstacles

Potential Field: attractive + repulsive components

Online motion planning with PF

Input: Function, $\nabla U(q)$
Output: Sequence $[q(0), q(1), ...q(i)]$

1. $q(0) = q_{start}$
2. i = 0
3. **while** $\nabla U(q(i)) \neq 0$
4. $\quad q(i+1) = q(i) - \alpha(i)\nabla U(q(i))$
5. $\quad i = i + 1$
6. **end while**

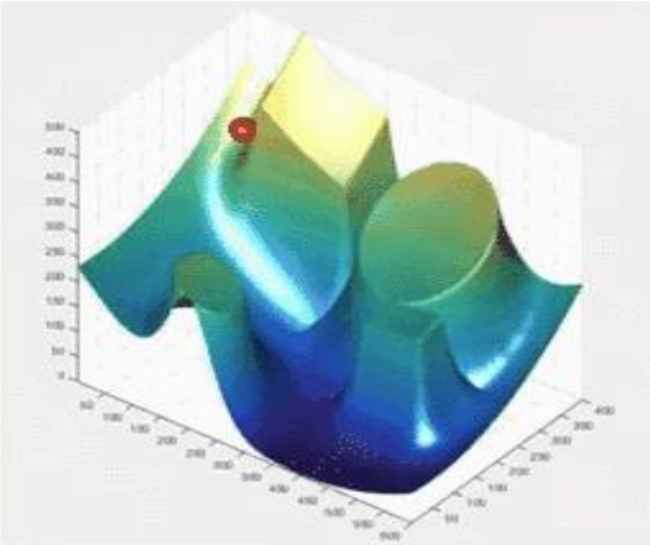


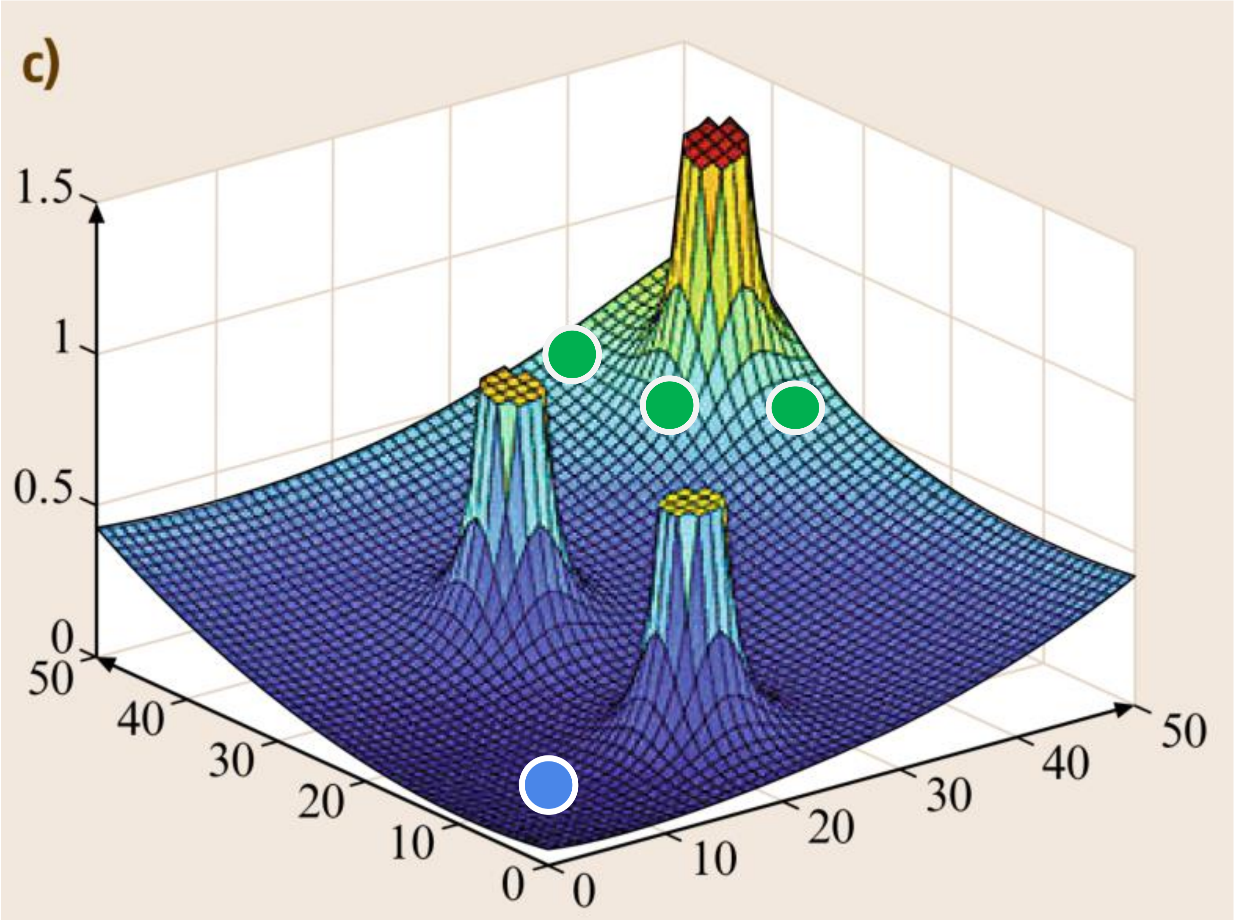
(Picture source: Springer Handbook of Robotics ISBN 978-3-319-32552-1)

Potential Field: attractive + repulsive components

What are the possible paths given different initial configurations?

Can you get any insight of possible drawbacks of this approach?

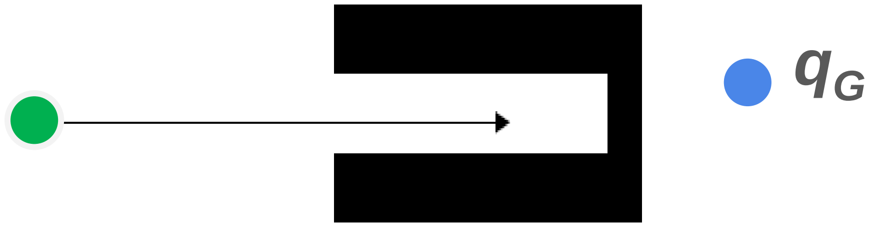


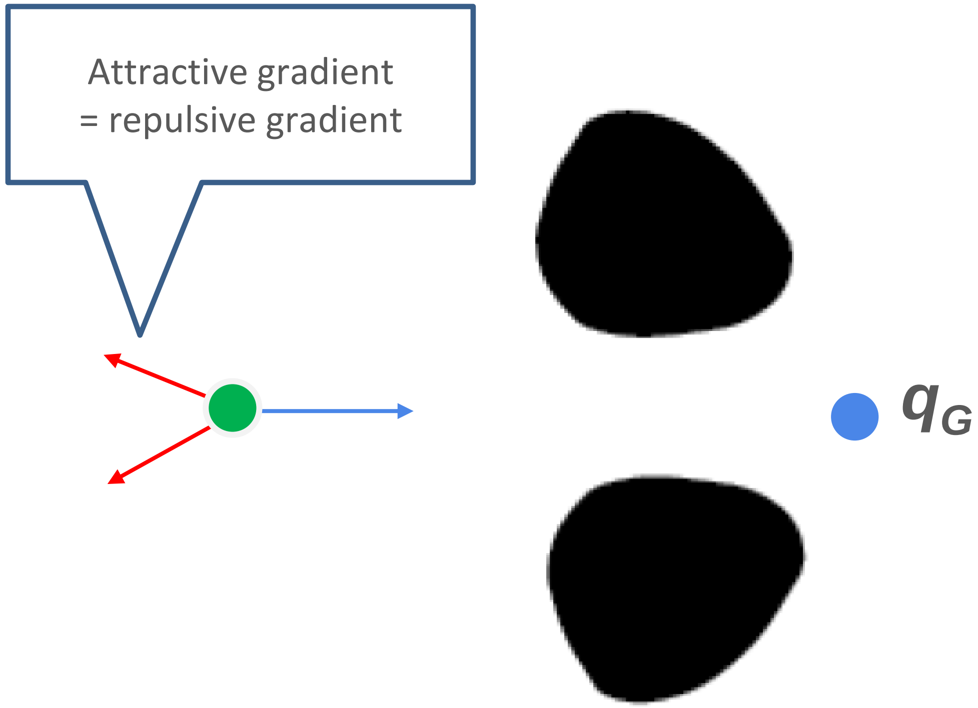


(Picture source: Springer Handbook of Robotics ISBN 978-3-319-32552-1)

Limitations: local minima

- An issue with all gradient descent procedures: trapped in **local minima**.

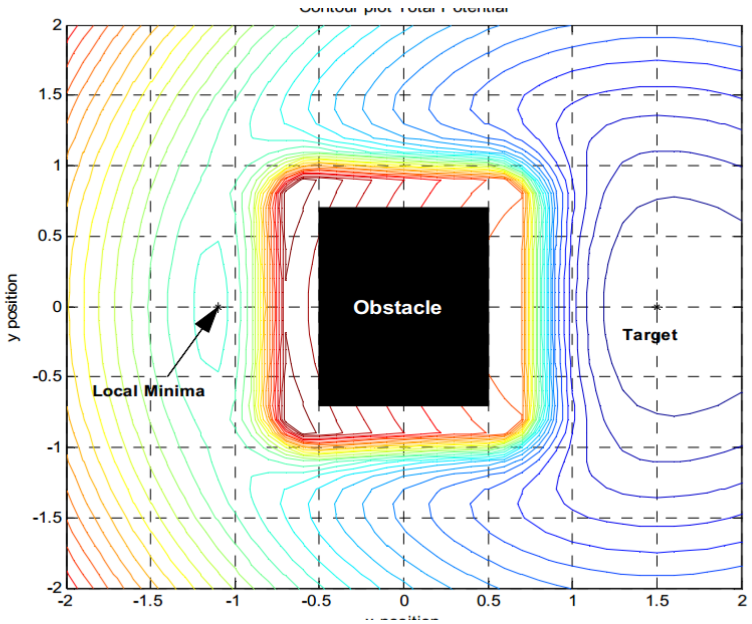


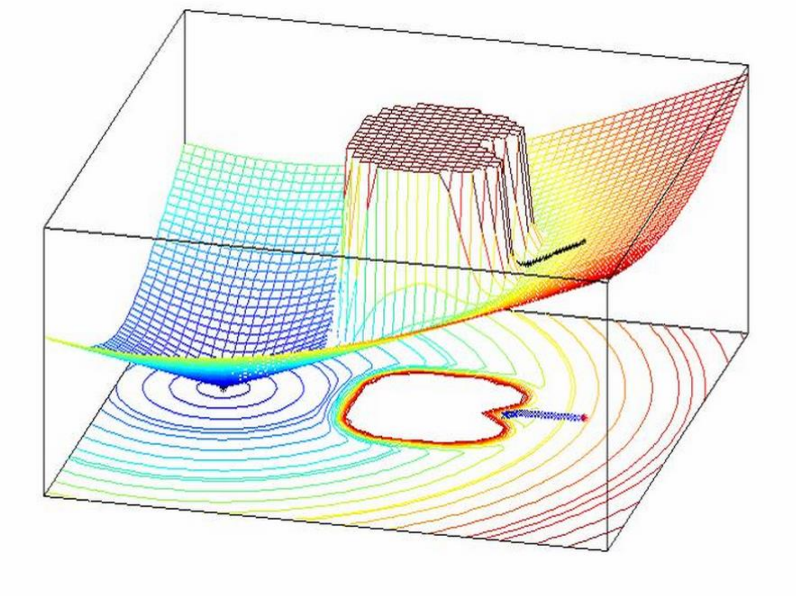


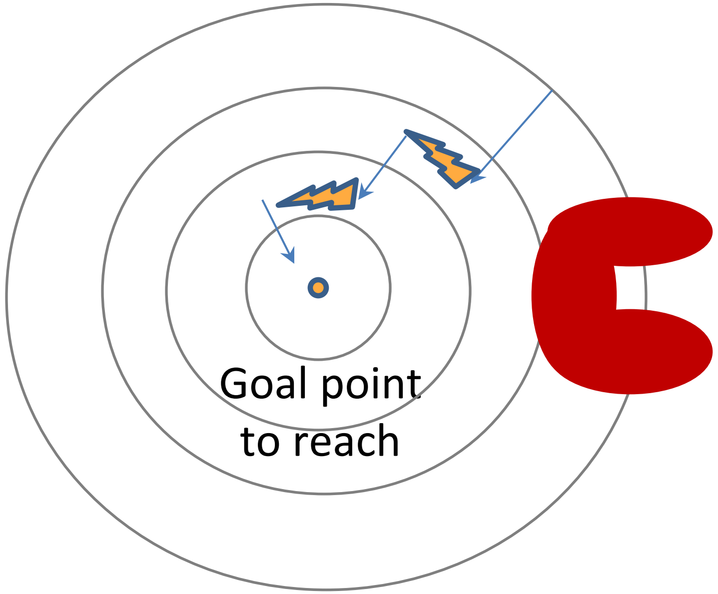
- This issue is not limited to concave obstacles acting as traps, e.g., see the dual obstacles.

- There are solutions such as adding random walks to get out of local minima, but this problem is generally better resolved by sampling based methods.

Limitations: local minima



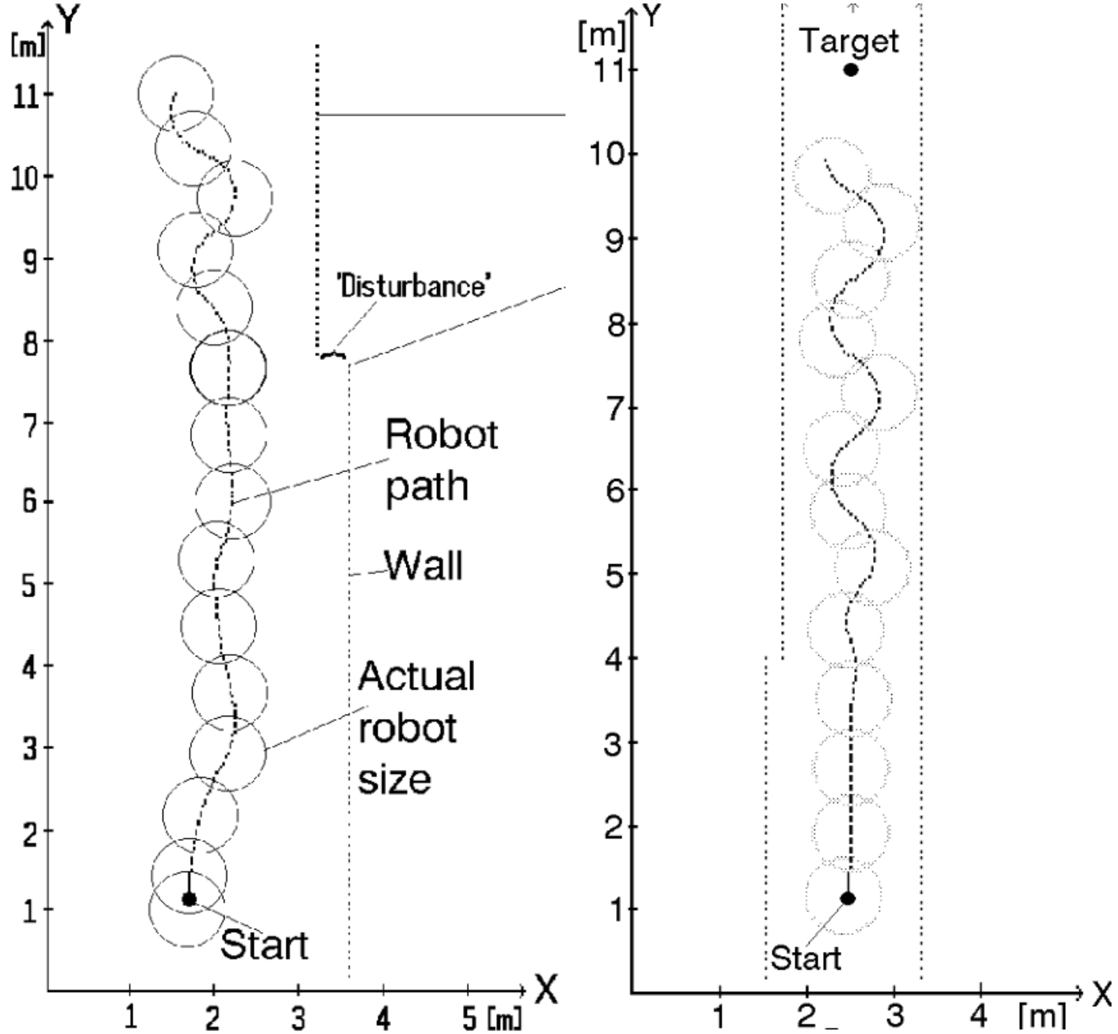




Limitations: unstable oscillations

Oscillations caused by disturbances or the discontinuity of the obstacles.

Oscillations in narrow passages: oscillation motion occurs when the robot is traveling in a narrow passage with high speed, because the robot receives repulsive forces from both side of the wall.



An example of using potential field

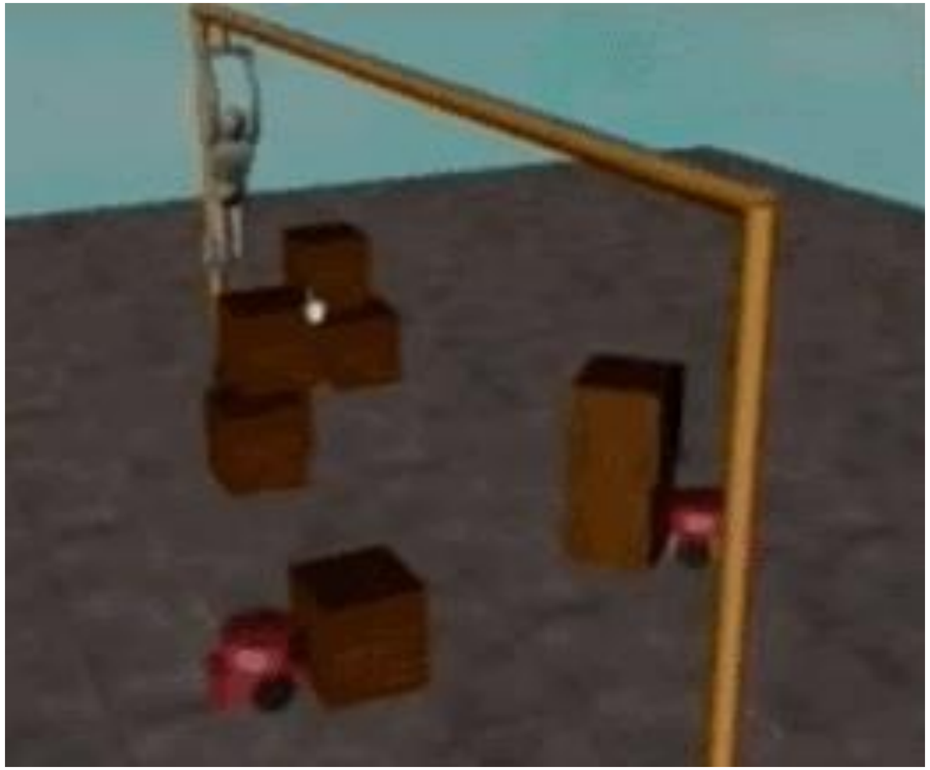
Pros:

- ➢ Computationally fast
- ➢ If it works, usually it produces quite natural motions

Cons:

- ➢ Solution is not guaranteed, needs manual tuning
- ➢ Solution is neither complete nor optimal

* *Completeness*: the solution must be found if it exists.



Picture source: course slides from Prof. Osama Khatib.

Quiz: How to implement in an articulated robot?