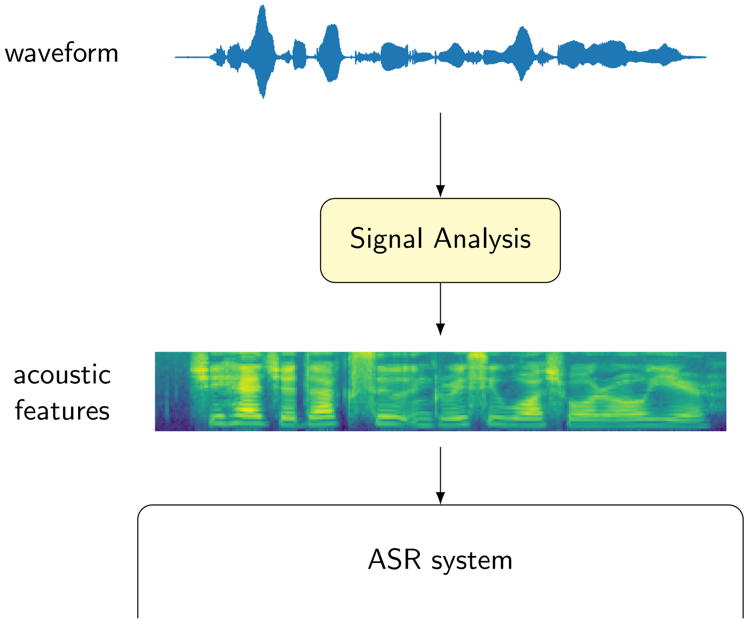
Speech Signal Analysis 1

Hao Tang

Automatic Speech Recognition—ASR Lecture 2
15 January 2026


waveform
Signal Analysis
acoustic
features
ASR system

Outline

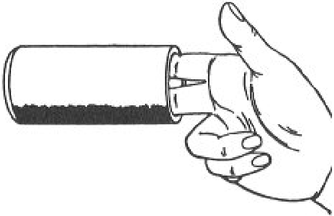
- Waveforms
  - Dithering
  - Removing DC offset
  - Pre-emphasis
- Spectrograms
  - Discrete Fourier transform (DFT)
  - Linearity and the shift theorem
  - Short-time Fourier transform
  - Windowing

waveform 

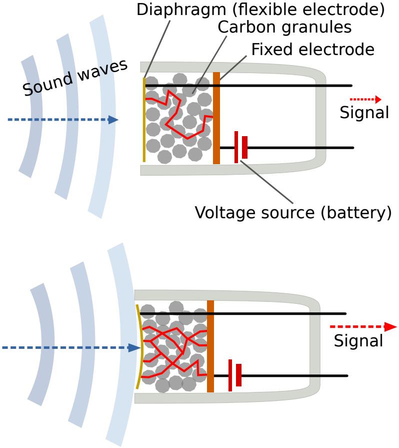
- Speech is part of sound waves.
- If we want to study speech, we need to be able to record, replay, and visualize speech.

Phonautograph (1857)

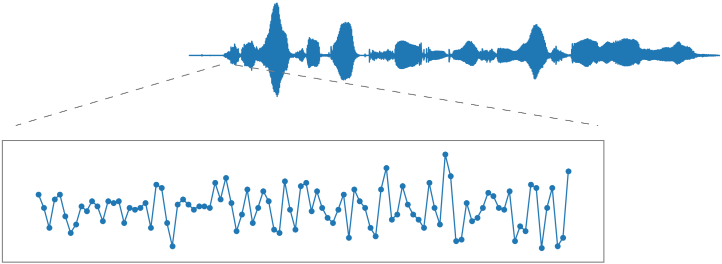
Phonograph (1877)



Carbon Microphone (1877)

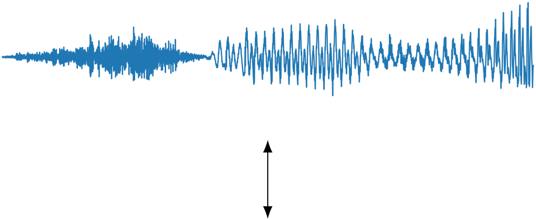


Wave Samples



- Sound waves are sampled and quantized.
- The typical sampling rate is 16,000 Hz. Each sample is typically a 16-bit integer.
- We will use $x[t]$ to denote the t-th sample in the signal x.

Line Plots and Vectors



$$\begin{bmatrix} -0.53 & -0.32 & 0.02 & 0.44 & \cdots & 0.18 \end{bmatrix}$$

Common Preprocessing in the Time Domain

- Dithering

$$y[t] = x[t] + \epsilon \qquad \epsilon \sim \mathcal{N}(0, 1)$$

  - Add a little Gaussian noise to the signal.
  - Avoid the signal being zeros, since we will be taking logarithm at some point.

- Removing DC offset

$$y[t] = x[t] - \frac{1}{T} \sum_{i=1}^{T} x[i]$$

  - Ensure that the signal has mean zero.
  - Most processing assumes the signal to have zero mean.

Common Preprocessing in the Time Domain

- Pre-emphasis

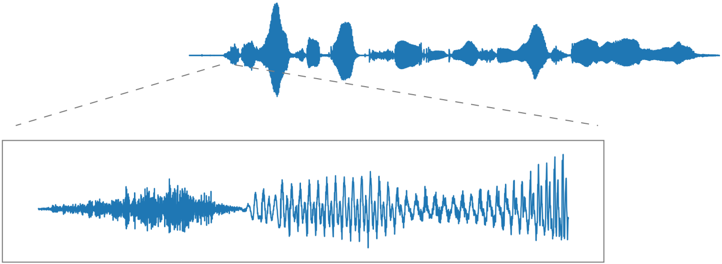
$$y[t] = x[t] - 0.97 \cdot x[t-1]$$

  - Emphasize the high-frequency components.
  - We will come back to this after we talked about frequency analysis.

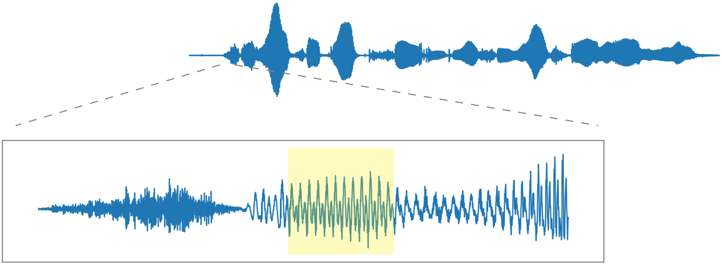
Ohm's Acoustic Law (1843)

If you hear a pitch of a certain frequency, then there must be energy of that frequency present in the sound wave.

Periodicity in Speech



Periodicity in Speech



Discrete Fourier Transform

$$X[k] = \sum_{t=0}^{T-1} x[t] e^{-i2\pi tk/T} \qquad \text{for } k = 0, \dots, T-1, \text{ and } i = \sqrt{-1}$$

Discrete Fourier Transform

$$X[k] = \sum_{t=0}^{T-1} x[t] e^{-i 2\pi t k / T} \qquad \text{for } k = 0, \dots, T-1, \text{ and } i = \sqrt{-1}$$

$$X[k] = \begin{bmatrix} e^{i 2\pi k \cdot 0 / T} & e^{i 2\pi k \cdot 1 / T} & \cdots & e^{i 2\pi k \cdot (T-1) / T} \end{bmatrix}^* \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[T-1] \end{bmatrix}$$

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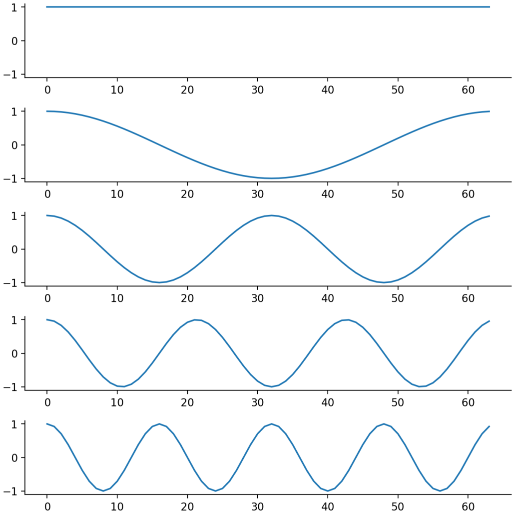
$$v_k \triangleq \begin{bmatrix} e^{i2\pi k \cdot 0/T} & e^{i2\pi k \cdot 1/T} & \dots & e^{i2\pi k \cdot (T-1)/T} \end{bmatrix}$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

Fourier Basis

$\Re\{v_0\}$, $\Re\{v_1\}$, $\Re\{v_2\}$, $\Re\{v_3\}$, $\Re\{v_4\}$



Fourier Basis

- The larger the k, the higher the frequency.

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- The set $\{v_0/T, v_1/T, \dots, v_{T-1}/T\}$ is an orthonormal basis.

$$v_m^* v_n = \begin{cases} 0 & \text{if } m \neq n \\ T & \text{if } m = n \end{cases}$$

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- Fourier transform is a change of coordinates.

Discrete Fourier Transform

$$X[k] = \sum_{t=0}^{T-1} x[t] e^{-i2\pi tk/T} = v_k^* x$$

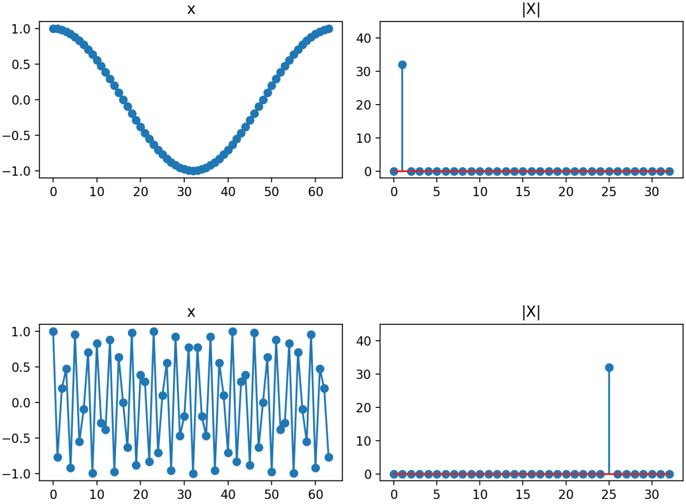
- $X[k]$ is a complex number.
- $X[k]$ is a (complex) dot product of a complex sinusoid v_k and the signal x.
- $X[k]$ tells us how similar x is to v_k.
- The large k's in X are high-frequency components, while the small k's in X are low-frequency components.

Discrete Fourier Transform

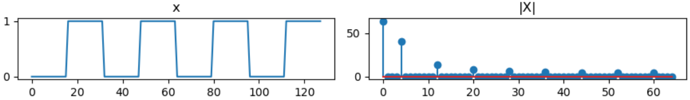
$$X = \mathcal{F}\{x\}$$

- DFT decomposes a signal into frequency components.
- X is also called the spectrum of x.

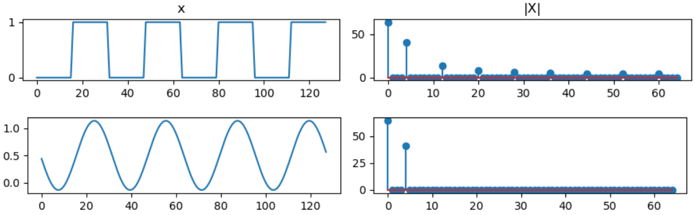
Discrete Fourier Transform



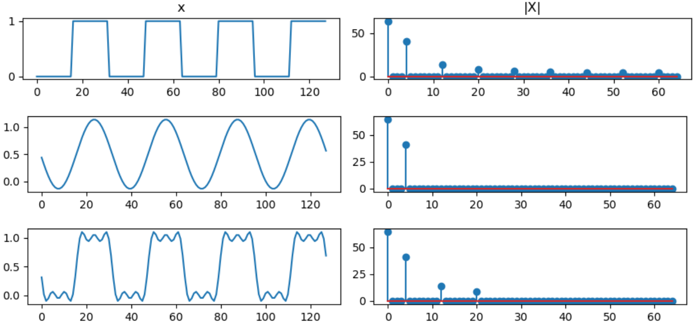
Discrete Fourier Transform



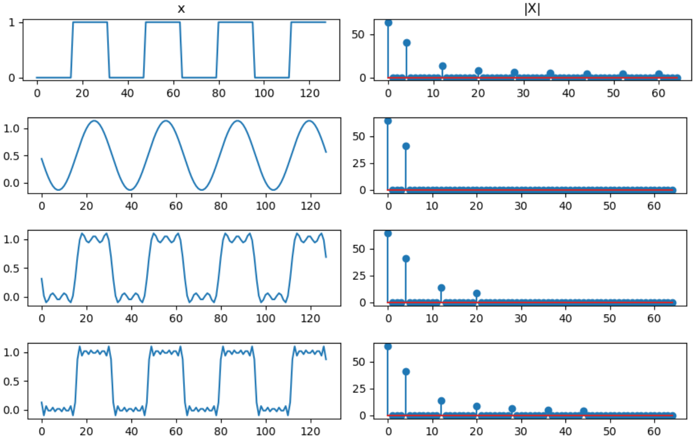
Discrete Fourier Transform



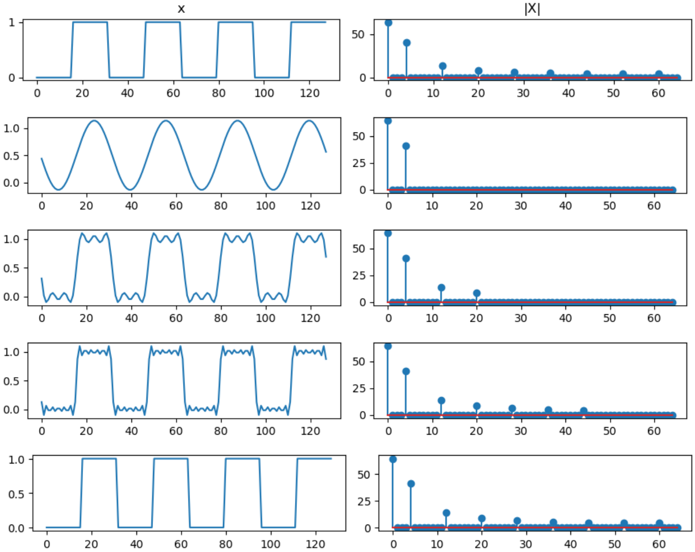
Discrete Fourier Transform



Discrete Fourier Transform



Discrete Fourier Transform



Properties of DFT

- Linearity

$$\mathcal{F}\{a_1 x_1 + a_2 x_2\} = a_1 \mathcal{F}\{x_1\} + a_2 \mathcal{F}\{x_2\}$$

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- Linearity

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- Shift Theorem

$$\text{If } y[t] = x[t-1], \text{ then } Y[k] = e^{-i2\pi k/T} X[k].$$

Proof of the Shift Theorem

$$Y[k] = \sum_{t=0}^{T-1} y[t] e^{-i2\pi tk/T}$$

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$$Y[k] = \sum_{t=0}^{T-1} y[t] e^{-i2\pi tk/T}$$
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Proof of the Shift Theorem

$$
\begin{aligned}
Y[k] &= \sum_{t=0}^{T-1} y[t] e^{-i2\pi tk/T} \\
&= \sum_{t=0}^{T-1} x[t-1] e^{-i2\pi tk/T} \\
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$$

Pre-emphasis

- Definition

$$y[t] = x[t] - 0.97 \cdot x[t-1]$$

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- DFT of pre-emphasis

$$\begin{aligned} Y[k] &= X[k] - 0.97 \cdot e^{-i2\pi k/T} X[k] \\ &= (1 - 0.97 \cdot e^{-i2\pi k/T}) X[k] \end{aligned}$$

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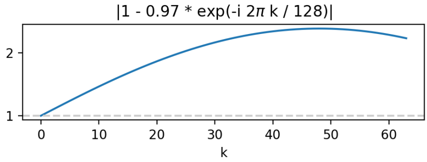

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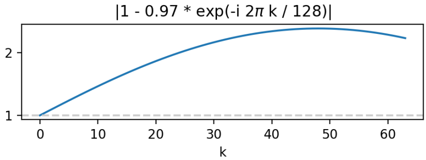
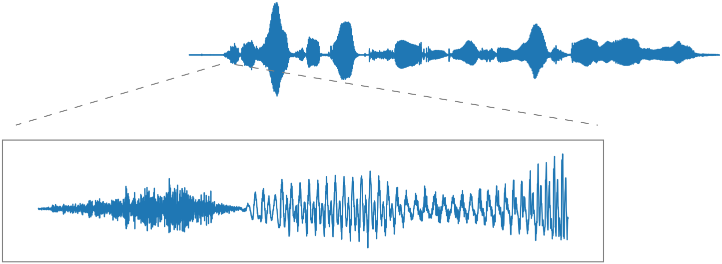
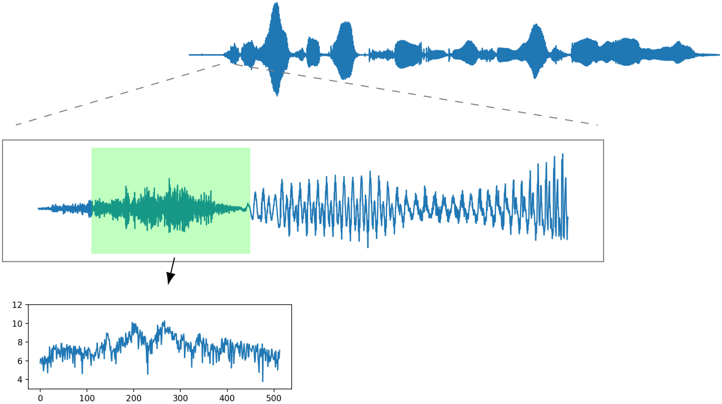


- In other words, pre-emphasis emphsizes the high-frequency region.

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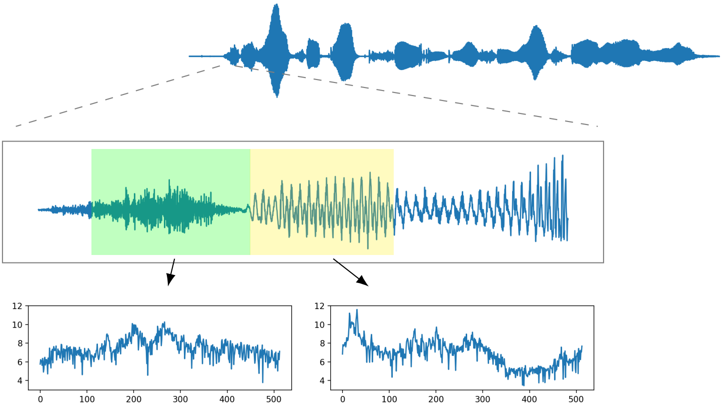

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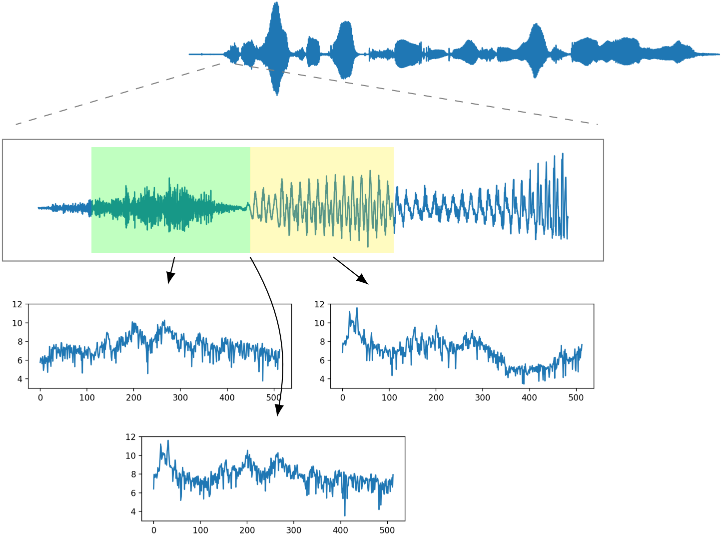
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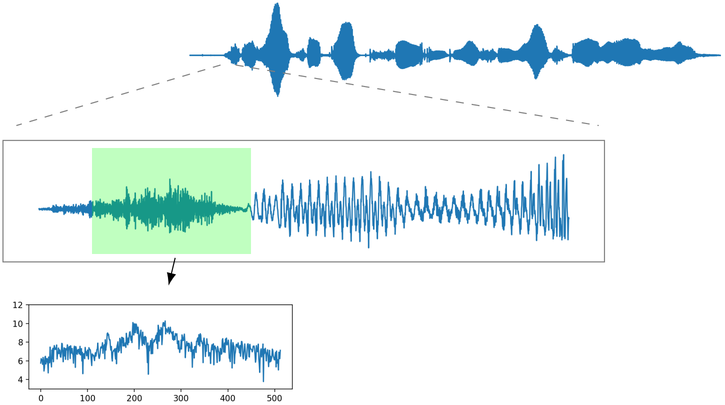
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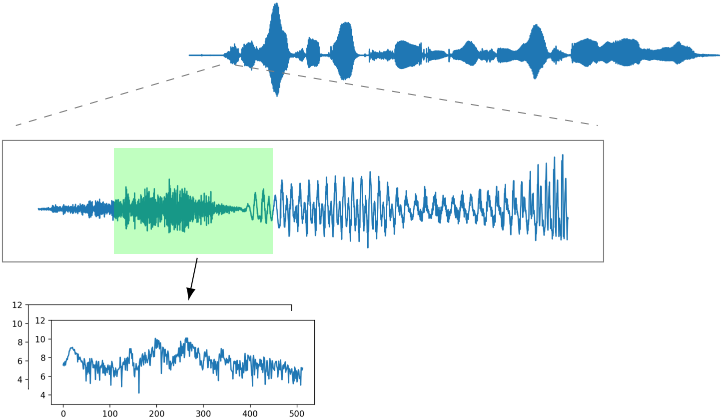
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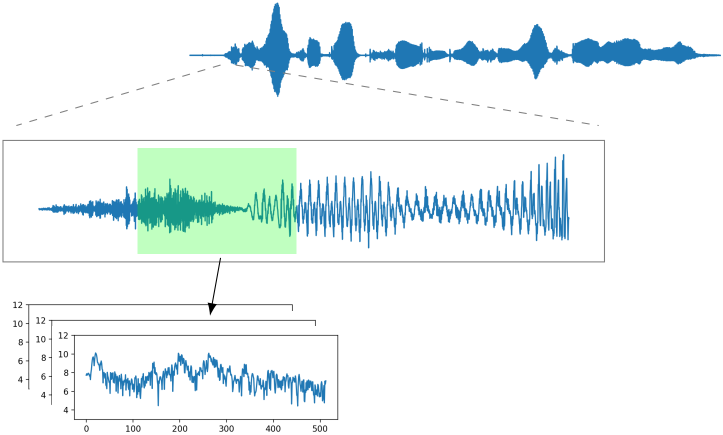
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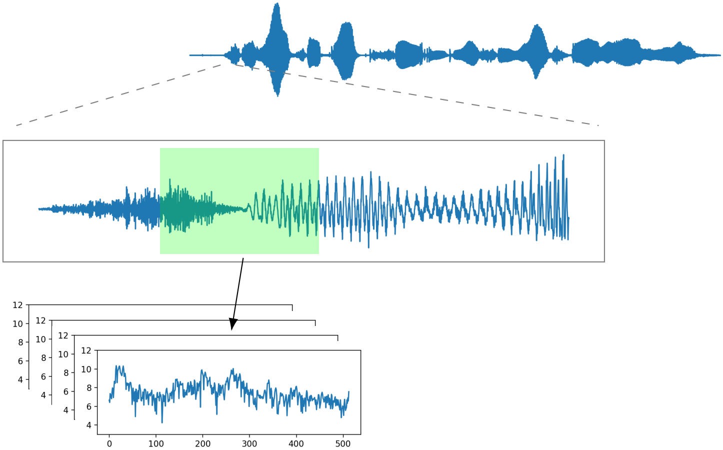
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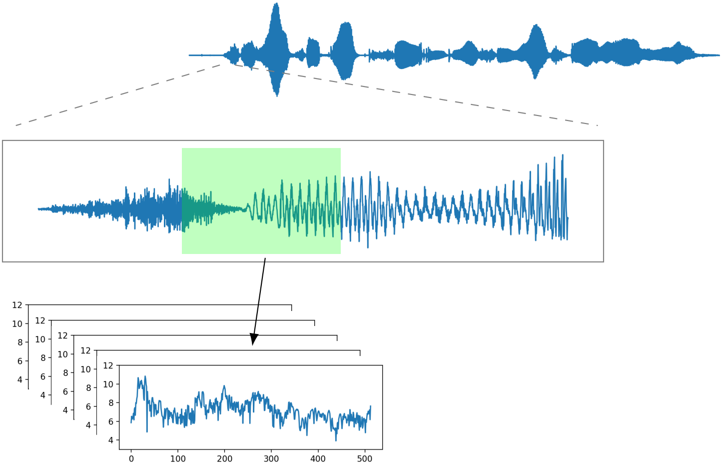
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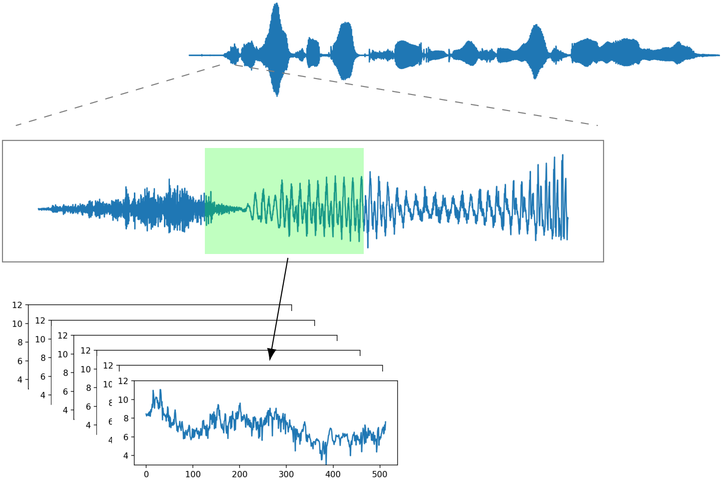
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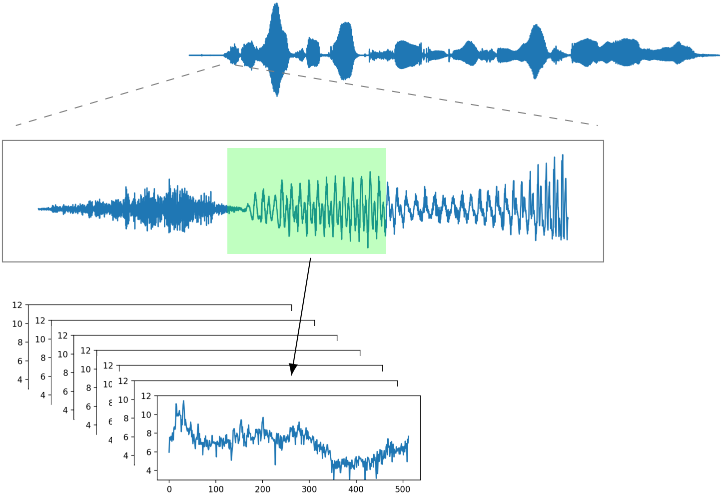
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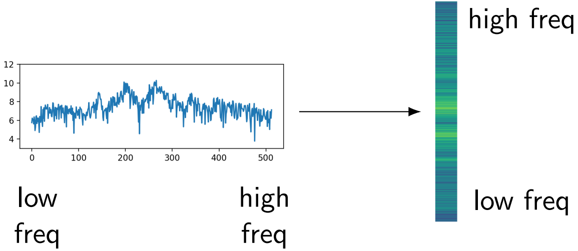
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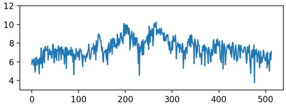
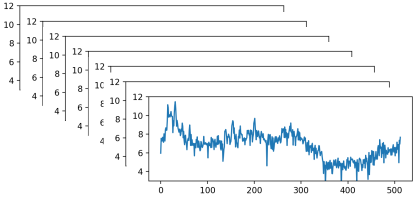


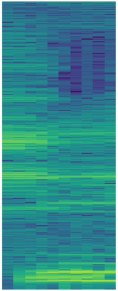
low freq

high freq

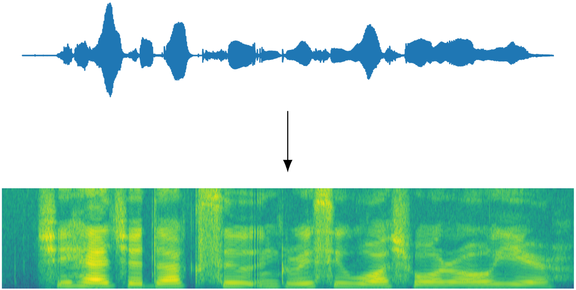
high freq

low freq

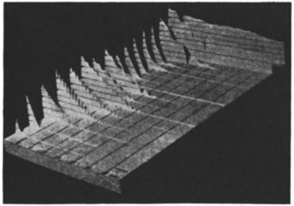


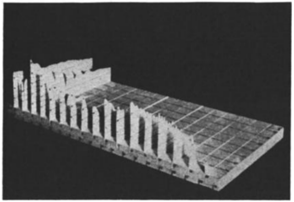


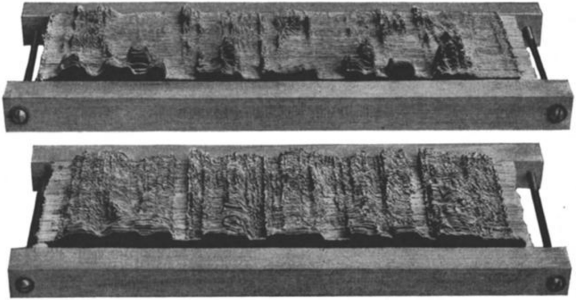
Short-Time Fourier Transform



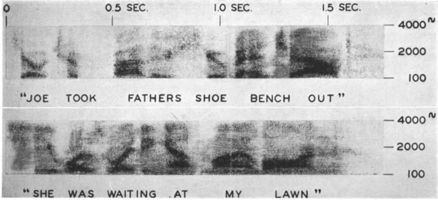
- Speech is non-stationary.
- Extract spectra with a sliding window, typically with a 25ms window size and a 10ms hop.
- Display the spectra as a heat map.

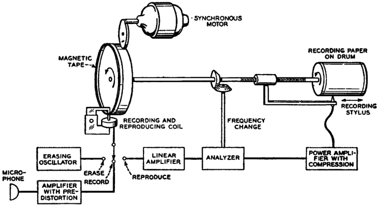


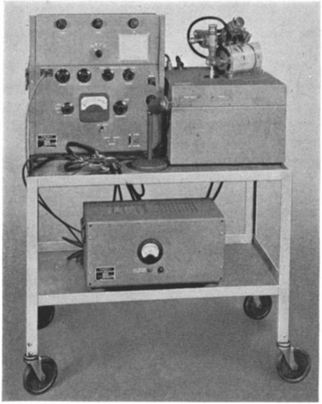




Sound Spectrograph (1946)

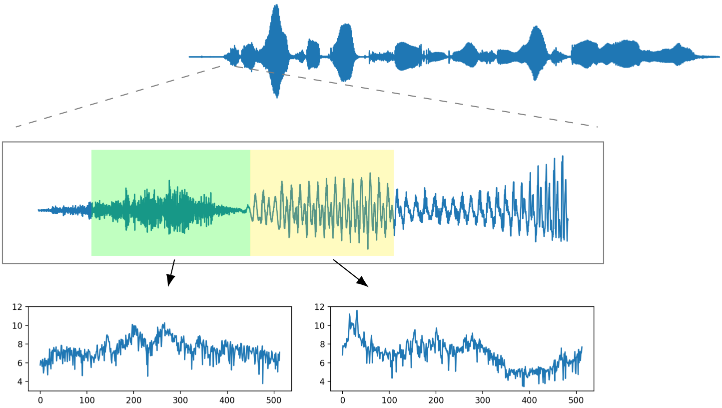






Fast Fourier Transform (1965)

- The algorithm that we know of today was proposed in 1965.
- It was applied to speech on a computer around 1969.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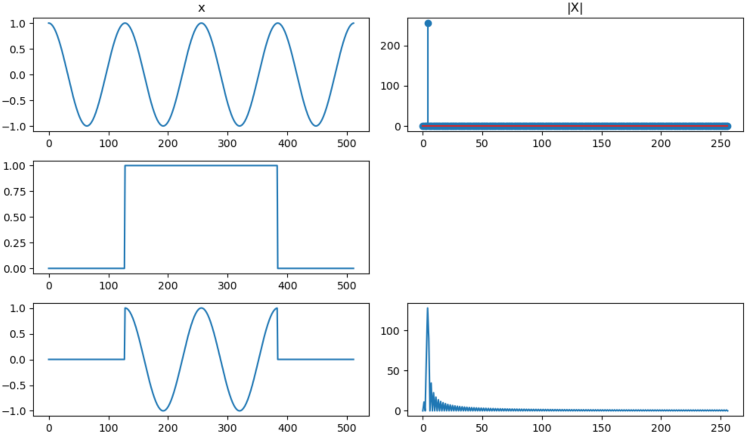
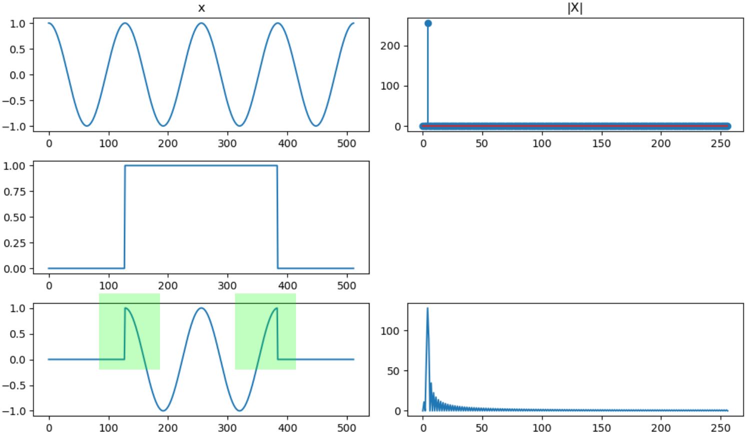
Windowing



Windowing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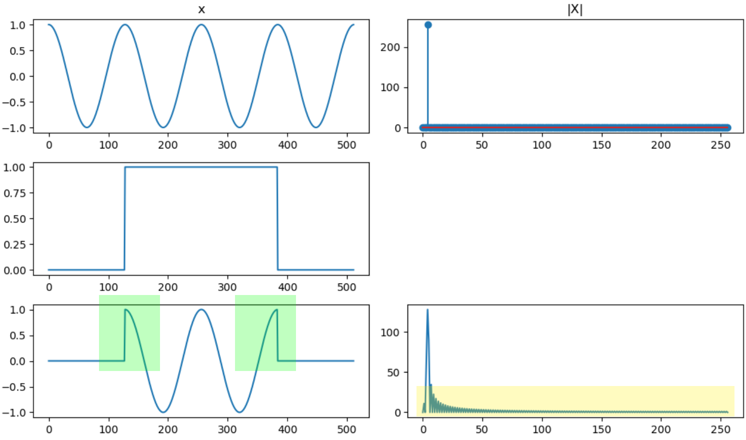

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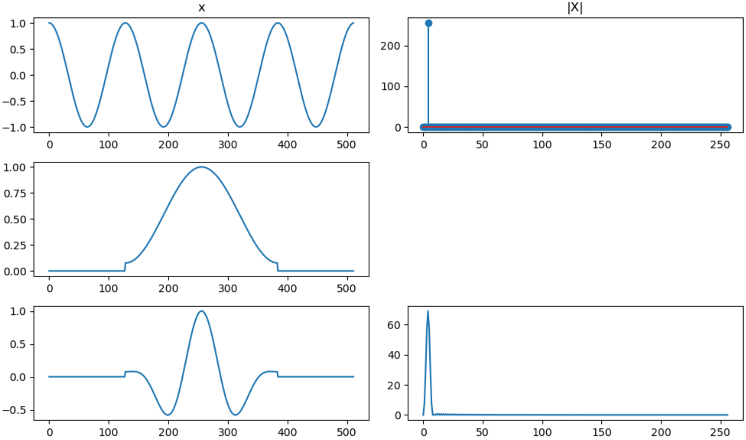
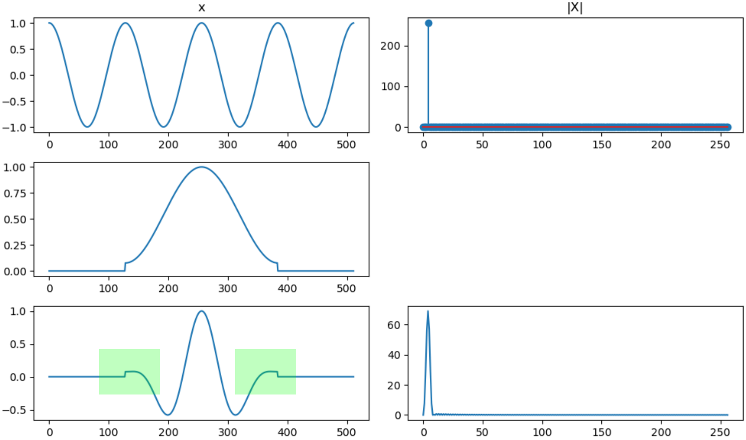
Windowing

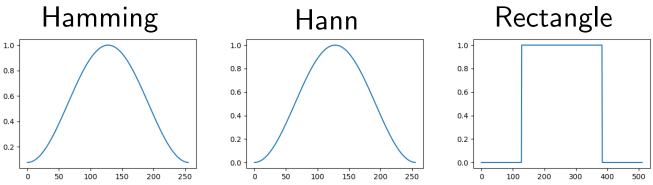


Windowing

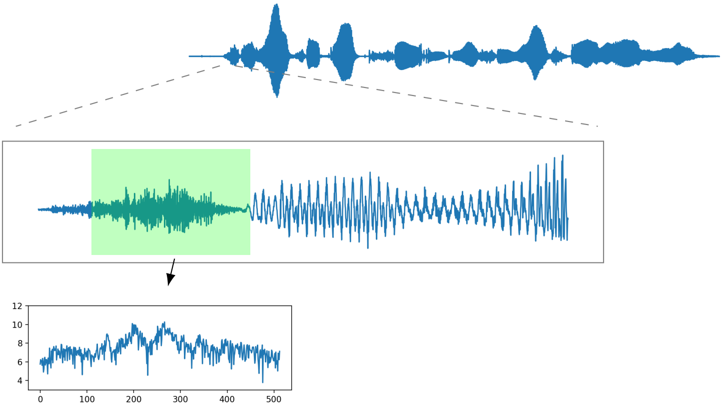


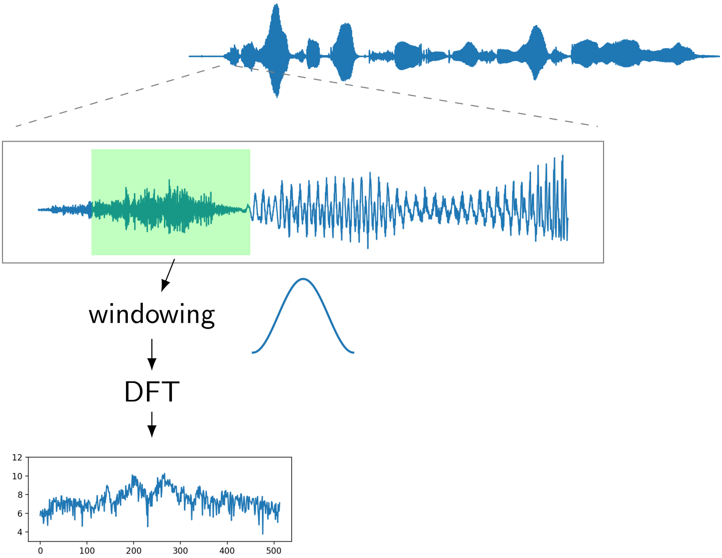
Windowing

$$y[t] = x[t] \cdot w[t]$$

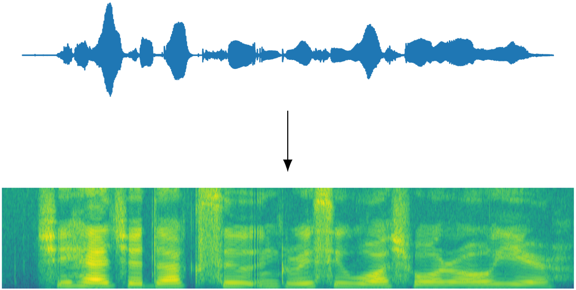


- The signal w is called a window.
- Windowing is elementwise product.


12
10
8
6
4
0
100
200
300
400
500


windowing
DFT
12
10
8
6
4
0
100
200
300
400
500

Spectrogram



- dithering, removing DC offset, pre-emphasis
- windowing
- Discrete Fourier transform (DFT)
- Short-time Fourier transform (STFT)

Further Reading

- Chapter 1–5, Oppenheim, Willsky, and Nawab, "Signals and Systems," 1997
- Chapter 2, O'Shaughnessy, "Speech Communications: Human and Machine," 2000
- Oldest recorded voices sing again
  http://news.bbc.co.uk/1/hi/technology/7318180.stm
- The ear does not do a Fourier transform
  https://www.dissonances.blog/p/the-ear-does-not-do-a-fourier-transform