

# **‘Bayesian’ theories of perception, cognition and mental illness**

CCN Lecture 13

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## The challenge faced by the brain: uncertainty

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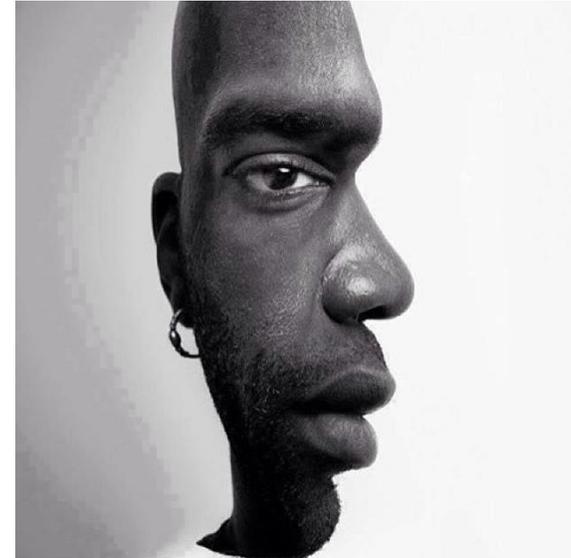


**is the cat going up or down?**

# Uncertainty everywhere

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- Humans & animals operate in a world of sensory **uncertainty and ambiguity**:
  - e.g. mapping of 3D objects to 2D image
  - intrinsic limitations of the sensory systems
  - > **multiple interpretations** about the world are possible;

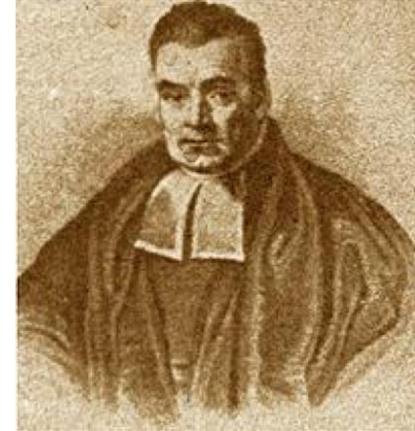


- The brain must **deal with this uncertainty** to generate perceptual representations and guide actions.
- Perception must work *backwards to extract underlying cause of noisy inputs* :  
**unconscious, probabilistic inference**
- The brain as a **guessing machine**.

# The Uncertain History of the Bayesian Brain

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- **Bayesian Statistics (mathematics)**: Thomas Bayes (1702-1761), Richard Price (1723-1791), Pierre-Simon Laplace (1749-1827), Harold Jeffreys (1891-1989), Richard Cox (1898-1991), Edwin Jaynes (1922-1998)
- 1860s: **Helmholtz** : perception as unconscious inference, making assumptions and conclusions from incomplete data, based on previous experiences.
- 1990s : **Geoff Hinton**, **Peter Dayan** - brain as generative model.
- 2000s --> enters experimental (psychophysics) world, spreads in theoretical world, now physiology?



# What is Bayes' theorem about ?

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Q: What is the chance  
that it will rain today?

Compute  $P(h|e)$ :

- probability that it is going to rain given the evidence (e.g. the clouds look dark)  
you use
- $P(e|h)$  : probability of the evidence (that the clouds look dark) when it is actually going to rain (from previous measurements - model of the world).
- $P(h)$ : prior knowledge or bias about the probability of rain (before observing any data)



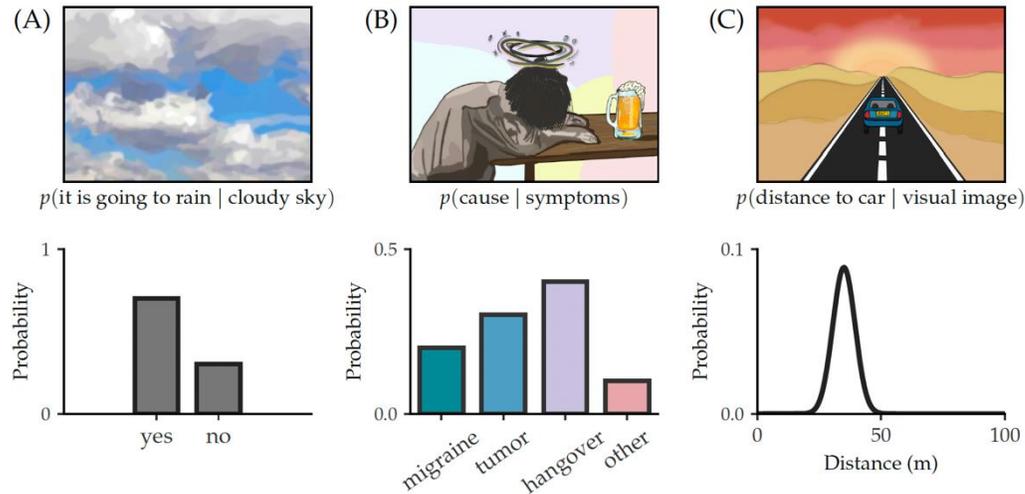
## Bayes' theorem

$$P(h_1|e) = \frac{P(e|h_1)P(h_1)}{P(e)}$$

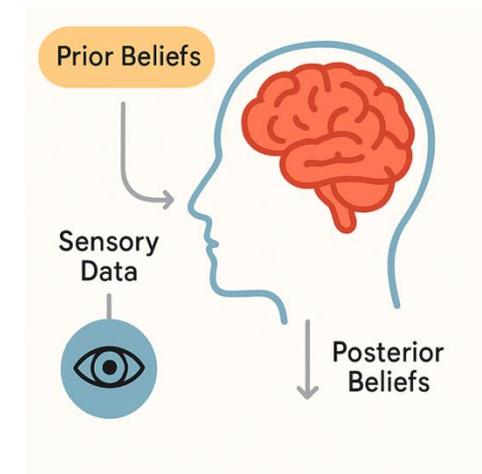
$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalizing constant}}$$

# A Bayesian theory of the Brain

- **Purpose of the brain**: infer state of the world from noisy and incomplete data.
- Information has the form of a **conditional prob. density function**



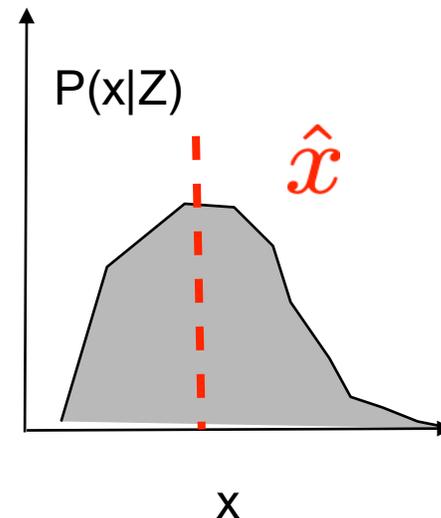
- Brain learns & stores and combines **likelihoods**,  $P(Z|x)$ , and **prior** knowledge  $P(x)$ .
- Computes (approximations of?) **posteriors** using Bayes Rule.



# A Bayesian theory of the Brain

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- Benefits:
  - **integrate** information optimally over **space & time**
  - and from **different sensory cues and modalities**
  - **propagate** information without committing too early to particular interpretations.
  
- **Commit as late as possible**, then collapsing the distribution into a single number = decision, or action taken.  
e.g. take the max of the posterior



# Cost functions - Bayesian Decision Theory

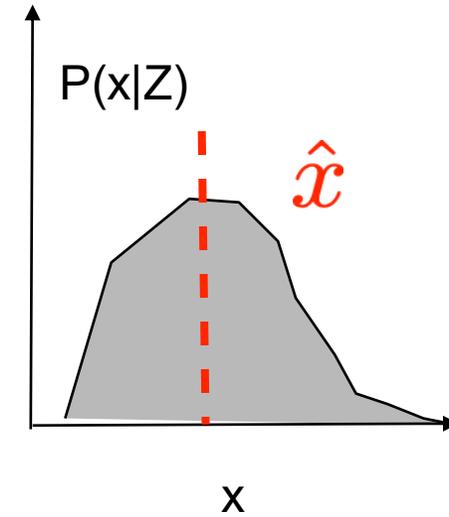
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Best option depends on **cost function** :

- Taking the **max of the posterior**

$$\hat{x} = \operatorname{argmax}_x P(x|Z)$$

optimizes a cost function that is 0 when  $\hat{x} = x$   
and  $e=cst$  otherwise.



max of the posterior

- another option is to take the **mean of the posterior**:

$$\hat{s} = \int xp(x|Z)dx$$

minimizes the mean squared error  $(\hat{x} - x)^2$

- another option : **samples** from the posterior.

# This series of lectures

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- 1) Do people behave as Bayesian Observers?**
  - a - Evidence from multi-sensory integration (today)
  - b - What priors does the brain use? (lecture 14)
  
- 2) A new way to understand Mental Illness?**

(lecture 15)

  - 1) What does this tell us about the Brain?**

(lecture 16)

Controversies and possible implementation ideas

# 1) Do People behave as Bayesian Observers?

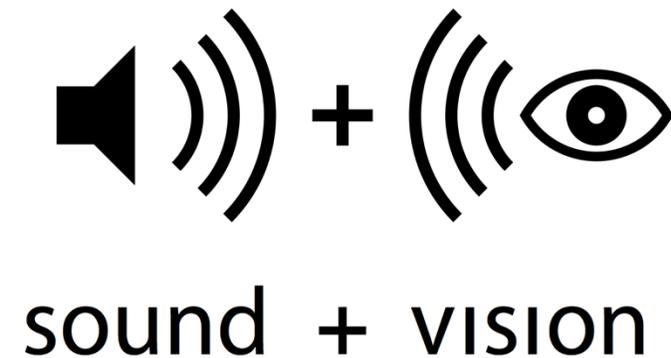
- Bayesian hypothesis as a benchmark for performance.

# Is the Human Brain “Bayesian-optimal”?

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- Humans **not optimal** / achieving the level of performance afforded by the uncertainty in the physical stimulus (e.g. movies)
- The question is:
  - 1 - Do neural computations take into account the **uncertainty** of measurements **at each stage of processing**?
  - 2 - Combine it optimally with **previous experience**?
- **Testable predictions** at the behavioural level
- (distinguish Probabilistic vs Bayesian vs Optimal. WJ Ma 2012)

**a) - Do brains take into account measurement uncertainty  
when combining different (simultaneous) information?  
Combine different sources optimally?**



## Example: integrating vision and audition

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- We unconsciously combine information all the time, and visual information can greatly influence auditory information
- Examples: McGurk effect, Ventriloquism

<https://www.youtube.com/watch?v=G-IN8vWm3m0&t=33s>

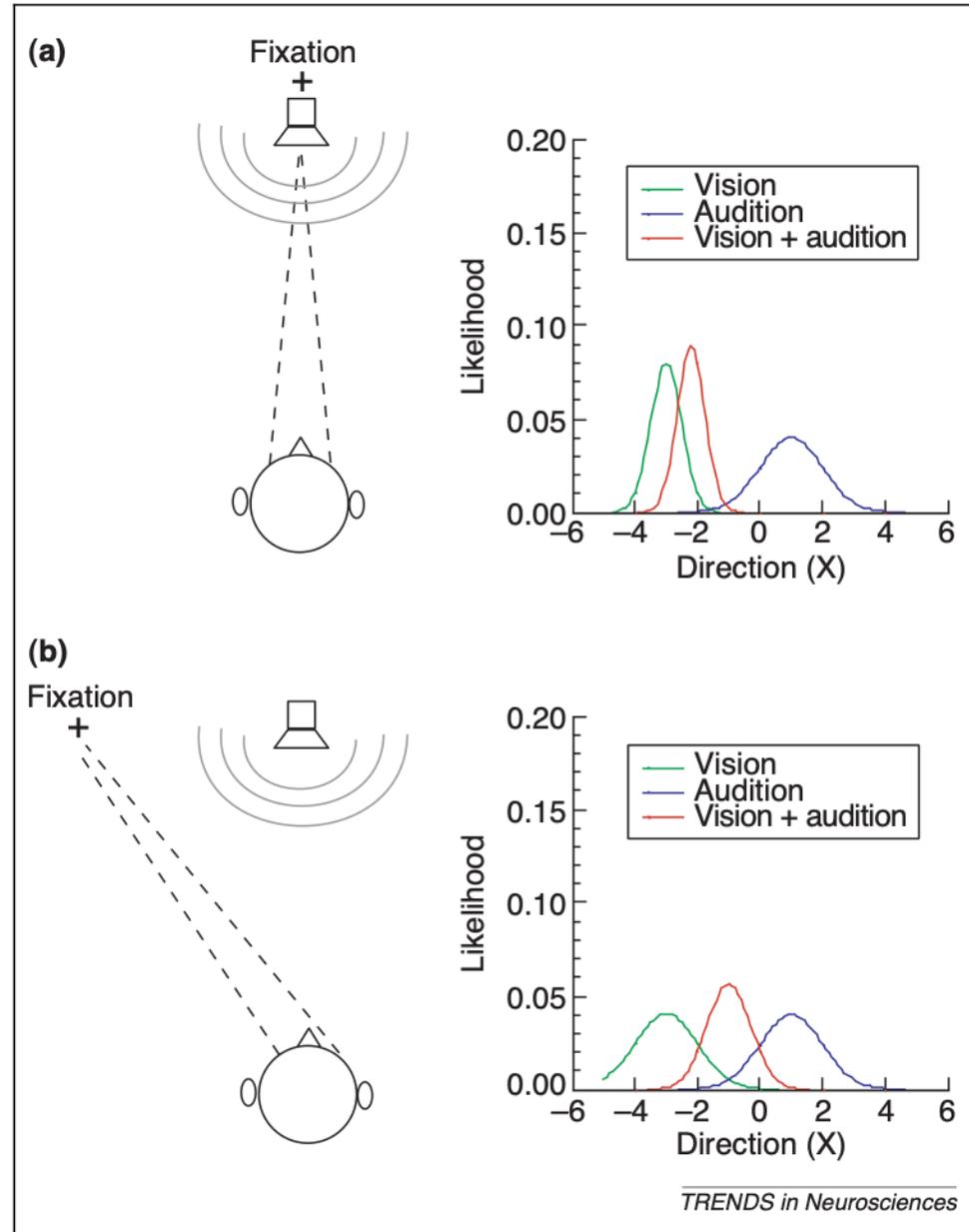




<https://www.youtube.com/watch?v=G-IN8vWm3m0&t=33s>

# Bayesian Cue Integration (1): Predictions

- e.g. **integration** between visual and auditory information for localisation
- prediction 1 (position): if visual cue is more reliable, then final estimate is shifted towards visual cue.
- prediction 2 (variance or discrimination threshold): Final discrimination threshold lower than that for each modality ; varies if reliability of one modality varies.



## Bayesian Cue Integration (2): Theory

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- Theory tells us how **posterior** depends on individual likelihoods:

$$\hat{x} = \operatorname{argmax}_x P(x|d_1, d_2)$$

$$P(x|d_1, d_2) = \frac{P(d_1, d_2|x)P(x)}{P(d_1, d_2)} \propto P(d_1|x)P(d_2|x)P(x)$$

- Assuming that the **likelihood are gaussian**, i.e.

$$P(d_1|x) \propto \exp\left(-\frac{(d_1 - x)^2}{2\sigma_1^2}\right)$$

- And that the prior is flat, we can determine **mean** and **width of posterior** (gaussian):

$$P(d_1|x)P(d_2|x) \propto \exp\left(-\frac{(d_1 - x)^2}{2\sigma_1^2} - \frac{(d_2 - x)^2}{2\sigma_2^2}\right) \propto \exp\left[-\frac{\left[x - \frac{\sigma_2^2 d_1 + \sigma_1^2 d_2}{\sigma_1^2 + \sigma_2^2}\right]^2}{2\sigma_1^2 \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)}\right]$$

## Bayesian Cue Integration (2): Theory

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- If we know mean estimate and variance for each modality in isolation, we can deduce **mean of bimodal estimate**:

$$\mu = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} d_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} d_2$$

weighted linear  
combination

pushed towards  
more reliable cue

- and **discrimination threshold**

$$T_{1,2}^2 \propto \sigma_{1,2}^2 = \sigma_1^2 \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)$$

smaller than  
1 or 2 alone

- visual + haptic cues
- vary noise level / visual cue
- compute discrimination threshold for each cue alone, or when both are present.

$$T_{1,2}^2 \propto \sigma_{1,2}^2 = \sigma_1^2 \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)$$

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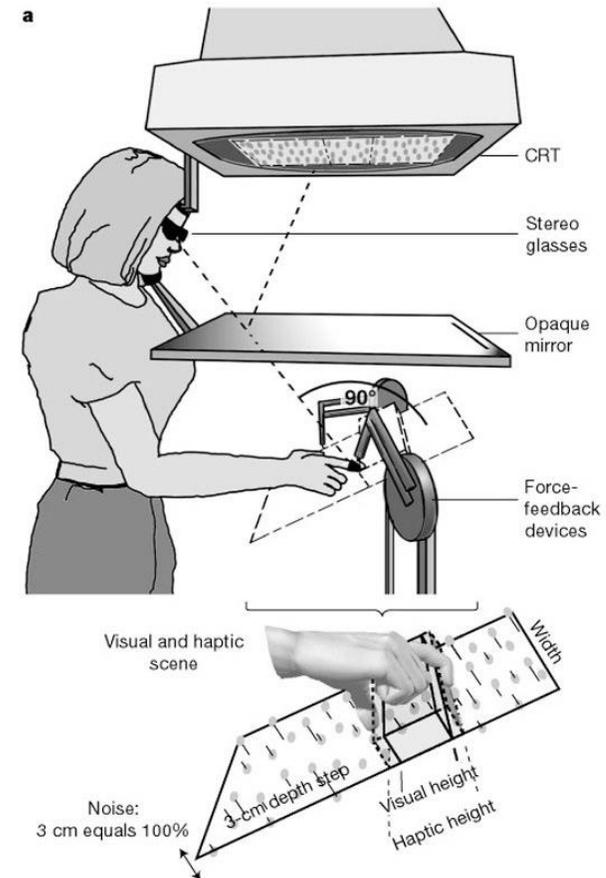
## Humans integrate visual and haptic information in a statistically optimal fashion

Marc O. Ernst\* & Martin S. Banks

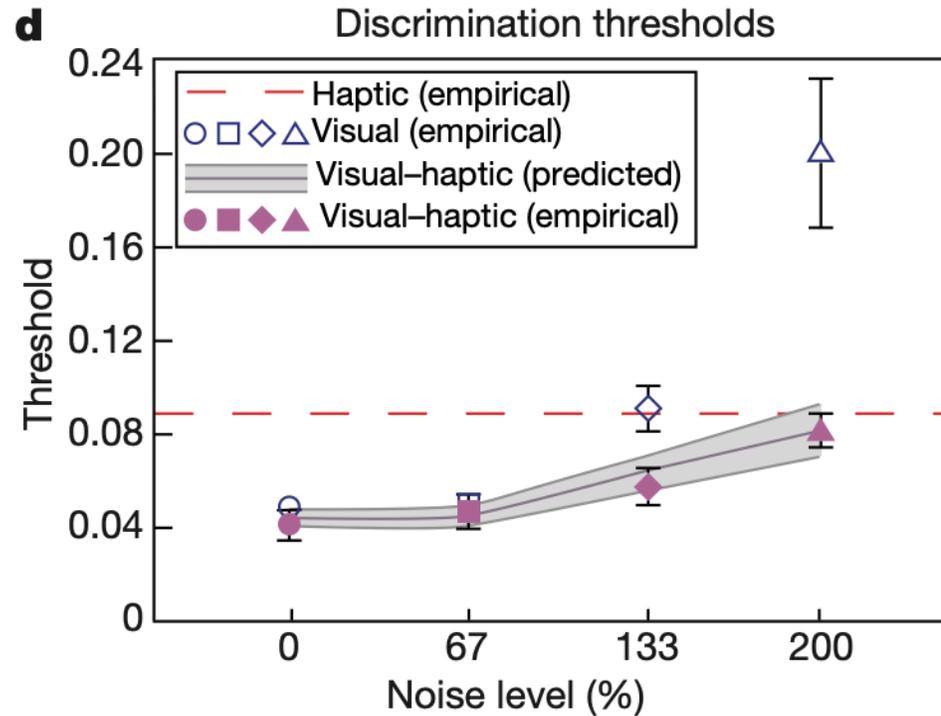
Vision Science Program/School of Optometry, University of California, Berkeley  
94720-2020, USA

.....

When a person looks at an object while exploring it with their hand, vision and touch both provide information for estimating the properties of the object. Vision frequently dominates the



# Bayesian Cue Integration (5): Ernst & Banks, *Nature*, 2002



- **optimal integration** of visual and haptic cues.
- '**visual capture**' for low visual noise, '**haptic capture**' for high visual noise
- instantaneous '**switch**'
- numerous studies replicate this result in a variety of paradigms (e.g. Alais & Burr, 2004).

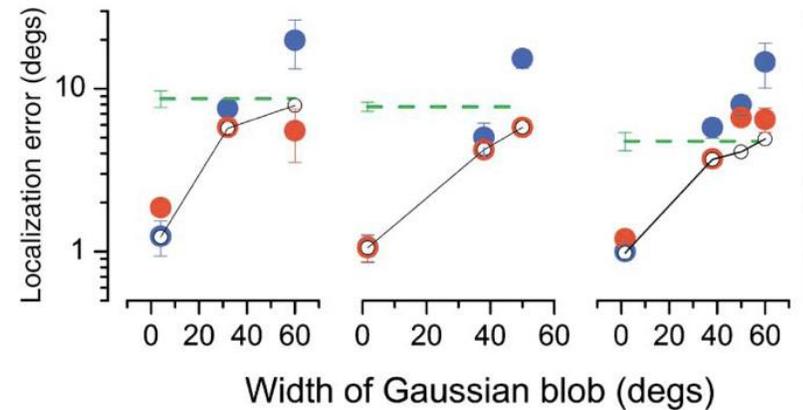
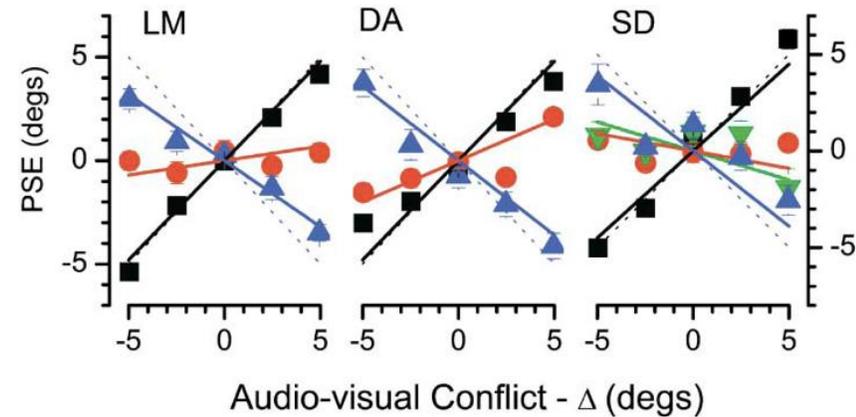
# The Ventriloquist Effect Results from Near-Optimal Bimodal Integration

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---: auditory alone  
○ : visual alone  
○ : prediction

# Bayesian Cue Integration (7)

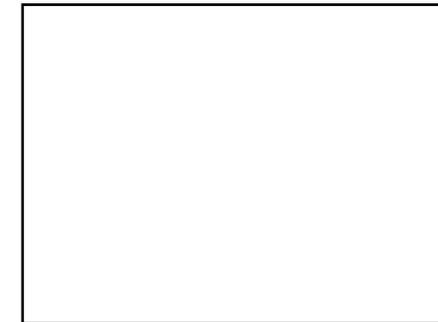
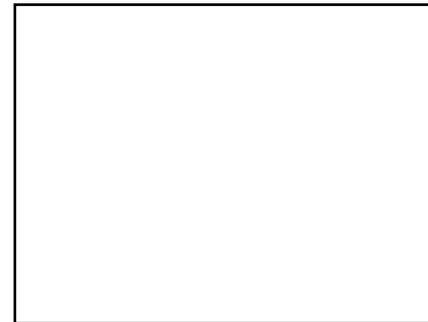
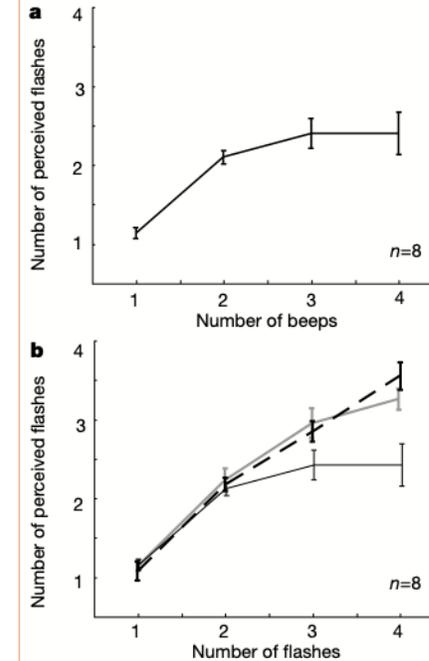
- Vision often dominates other modalities but .. capture of vision by sound also exists.
  - Double flash illusion
- Shams et al, *Nature*, 2000.

## Illusions

### What you see is what you hear

Vision is believed to dominate our multisensory perception of the world. Here we overturn this established view by showing that auditory information can qualitatively alter the perception of an unambiguous visual stimulus to create a striking visual illusion. Our findings indicate that visual perception can be manipulated by other sensory modalities.

We have discovered a visual illusion that is induced by sound: when a single visual flash is accompanied by multiple auditory beeps, the single flash is incorrectly perceived as multiple flashes. These results were obtained by flashing a uniform white disk (subtending 2 degrees at 5 degrees eccentricity) for a variable number of times (50 milliseconds apart) on a black background. Flashes were accompanied by a



# Cue Integration (8): when not to integrate?

- If spatial or temporal disparity is too large: integration no longer appropriate

-> **segmentation**.

- A problem of **causal inference: humans infer the causal structure (i.e. presence of one cause or several causes) as well as the location of causes**

[Körding et al 2007; Shams & Beierholm 2011]

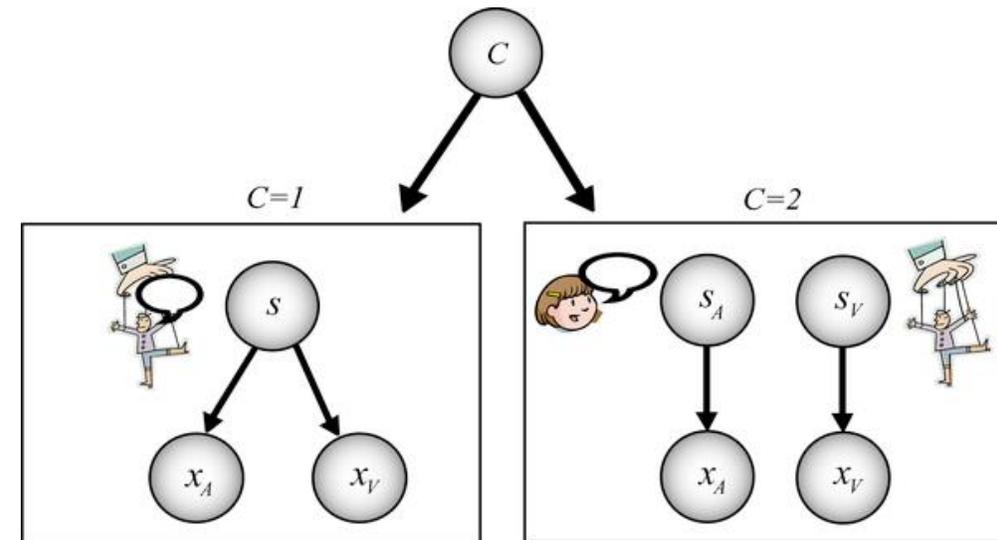
In this case, we want to compare two hypotheses:

$C = 1$ : common cause

Vs

$C = 2$ : separate causes

Compute: 
$$P(C = 1 \mid \mathbf{x}_v, \mathbf{x}_a) = \frac{P(\mathbf{x}_v, \mathbf{x}_a \mid C = 1) P(C = 1)}{P(\mathbf{x}_v, \mathbf{x}_a)}$$



## Cue Integration (8): when not to integrate?

This posterior depends on:

- How close the signals are in space/time
- How noisy each modality is
- Prior expectations that cues tend to belong together  $P(C=1)$

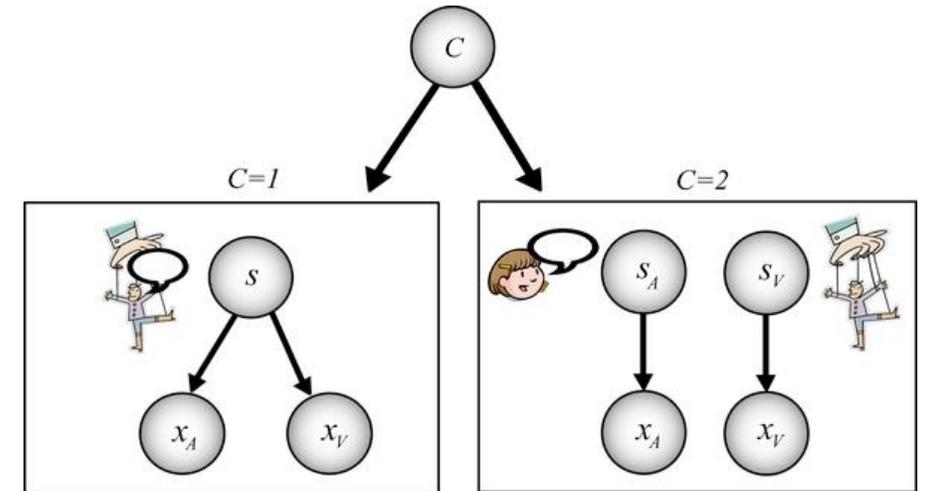
The brain will **fuse** when  $P(C = 1)$  is high and **segregate** when  $P(C = 1)$  is low (or do something in between).

- This can be assessed by asking participants “Did the sound and flash come from the **same source** or **different sources?**”

- If participants are asked to localise one source (whether they fuse or segregate), estimate can be **a weighted average of the common-cause and separate-cause estimates**:

$$\hat{s} = p(C = 1 | x_V, x_A) \hat{s}_{C=1} + p(C = 2 | x_V, x_A) \hat{s}_{C=2} \quad .$$

This often matches human behavior well.



# Cue Integration (9) : When is cue combination sub-optimal?

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Report

## Young Children Do Not Integrate Visual and Haptic Form Information

Monica Gori,<sup>1,2</sup> Michela Del Viva,<sup>3,4</sup> Giulio Sandini,<sup>1,2</sup> and David C. Burr<sup>3,5,\*</sup>

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Several studies have shown that adults integrate visual and haptic information (and information from other modalities) in a statistically optimal fashion, weighting each sense according to its reliability [1, 2]. When does this capacity for crossmodal integration develop? Here, we show that prior to 8 years of age, integration of visual and haptic spatial information is far from optimal, with either vision or touch dominating totally, even in conditions in which the dominant sense is far less precise than the other (assessed by discrimination thresholds). For size discrimination, haptic information dominates in determining both perceived size and discrimination thresholds, whereas for orientation discrimination, vision dominates. By 8–10 years, the integration becomes statistically optimal, like adults. We suggest that during development, perceptual systems require constant recalibration, for which cross-sensory comparison is important. Using one sense to calibrate the other precludes useful combination of the two sources.

- **Children**, up to 8-10 y.o., are less efficient at integrating multi sensory integration.

Often rely on one dominant modality (often vision) or show suboptimal weighting, or trial-by-trial switching.

# Cue Integration (9) : When is cue combination sub-optimal?

Neuroscience and Biobehavioral Reviews 95 (2018) 220–234



## Audiovisual multisensory integration in individuals with autism spectrum disorder: A systematic review and meta-analysis

Jacob I. Feldman<sup>a,\*</sup>, Kacie Dunham<sup>b</sup>, Margaret Cassidy<sup>b</sup>, Mark T. Wallace<sup>c,d,e,f,g,h</sup>, Yupeng Liu<sup>b</sup>, Tiffany G. Woynaroski<sup>f,g,h</sup>

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RESEARCH ARTICLE



## Aberrant causal inference and presence of a compensatory mechanism in autism spectrum disorder

Jean-Paul Noel<sup>1†</sup>, Sabyasachi Shivkumar<sup>2†</sup>, Kalpana Dokka<sup>3†</sup>, Ralf M Haefner<sup>2‡</sup>, Dora E Angelaki<sup>1,3\*\*</sup>

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<sup>3</sup>Department of Neuroscience, Baylor College of Medicine, Houston, United States

**Abstract** Autism spectrum disorder (ASD) is characterized by a panoply of social, communicative, and sensory anomalies. As such, a central goal of computational psychiatry is to ascribe the heterogeneous phenotypes observed in ASD to a limited set of canonical computations that may have gone awry in the disorder. Here, we posit causal inference – the process of inferring a causal structure linking sensory signals to hidden world causes – as one such computation. We show that audio-visual integration is intact in ASD and in line with optimal models of cue combination, yet multisensory

- wider temporal binding windows also reported in autism (i.e. more likely to think the two signals belong together)” as well as diminished audiovisual integration (i.e. the cues are not weighted by reliability properly).

## Intermediate conclusions

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- The world we navigate is characterised by **uncertainty** and ambiguity.
- According to Bayesian Brain theory, our brain automatically learns and uses **probability distributions** to model our environment, infer what is around us, and compute actions.
- Psychophysical studies investigating **multi-sensory integration and causal inference** show that our brain takes into account uncertainty of measurements, in a way compatible with Bayesian models.
- **Deviations from Bayesian optimal** can be measured in individual participants and give insights into biological/ cognitive constraints, and/or psychopathology.