Lecture 0: Basic Physics
Goals

• Synchronize basic materials for the CGGS course
• Target audience: students without any high-school (or basic university) background in mechanics.

• Topics
  • Basic elements of kinematics: position, velocity, acceleration
  • Basic elements of dynamics: forces, momentum, energy
    • Gravity and mass
  • Newton’s three laws
Position

• A system: a collection of objects in space and time

• Given a coordinate system
  • **Static** objects: “sit” in given coordinates \((x, y)\)
  • **Dynamic** objects: move through a trajectory \((x(t), y(t))\)
    • Tracing a curve in space

• How does the choice of coordinates matter?

• **Initial assumption**: objects are pointwise
  • Neglecting their shapes

![Diagram showing a point moving through a trajectory in a coordinate system.](image)
Velocity

- Velocity: Instantaneous change in position $\vec{v} = \frac{d\vec{x}}{dt}$
- Suppose object position $\vec{x}_0$ and constant velocity $\vec{v}$ after time step $\Delta t$:
  - $\vec{x}_0(t + \Delta t) = \vec{x}_0(t) + \vec{v}\Delta t$
  - $\Delta \vec{x}_0 = \vec{x}_0(t + \Delta t) - \vec{x}_0(t) = \vec{v}\Delta t$.
- …$\vec{v}$ is never constant in practice
  - A function of time $\dot{\vec{v}}(t)$
  - Position is integrated in time: $\vec{x}_0(t) = \vec{x}_0 + \int_0^t \dot{\vec{v}}(s) \, ds$
- Velocity SI (standard international) units: $m/sec$
Acceleration

- **Instantaneous** change in velocity: $\vec{a} = \frac{d\vec{v}}{dt}$
- Constant acceleration: $\Delta \vec{v} = \vec{a} \Delta t$
- Otherwise, integrate: $\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(s) \, ds$

- **Note vector quantities!**
  - Position: trajectory of a point
  - Velocity: tangent to trajectory curve
  - Speed: absolute value of velocity
  - Acceleration: the **change** in the tangent
Relative Quantities

• Using coordinates, our vector quantities are relative to the chosen axis system (origin + xyz direction)

• They are viewpoint dependent

• The derivation/integration relations are equivariant!
Forces

• Acceleration is induced by a force $\vec{F}$

• Direction of force = direction of associated acceleration

• Net force (and net acceleration): the sum of all acting forces: $\vec{F} = \sum \vec{F_i}$
Newton’s laws of motion

• In the late 17\textsuperscript{th} century, Sir Isaac Newton described 3 laws that govern all motion on Earth

• ...ultimately, an approximation
  • Small scale: quantum mechanics
  • Big scale: theories of relativity
1st Law of Motion

- Sum of forces on an object is null $\iff$ there is no change in the motion

\[
\text{If } F_{\text{net}} = 0, \text{ there is no change in motion}
\]

- With zero force sum:
  - An object at rest stays at rest
  - A moving object perpetuates in the same velocity
- Behavior of objects in the outer space
- …relative to a given system!
2nd Law of Motion

• Each force induces a co-directional acceleration in linear to the mass of the object:

\[ \vec{F}_{net} = m \cdot \vec{a} \]

\( m \) is the mass and \( \vec{a} \) the acceleration

• Consequently:
  • More force \( \Leftrightarrow \) faster speed-up
  • Same force \( \Leftrightarrow \) lighter objects accelerate faster than heavy objects

• This is the most important physical rule of the course!
  • The dynamics-kinematics relation
3rd Law of Motion

• Forces have consequences:

When two objects come into contact, they exert equal and opposite forces upon each other.

• All forces are actually interactions between bodies!

What happens here?

Gravity pulls book down (book-earth relation)
Table constrains book from passing through it (book-table relation; both solid objects!)
Book presses on table (table-book relation; load!)
**Gravity**

- **Newton’s Law of Gravitation:** the gravitation force between two masses $A$ and $B$ is:

$$
\vec{F}_g = \vec{F}_{A\rightarrow B} = -\vec{F}_{B\rightarrow A} = G \frac{m_A m_B}{r^2} \vec{u}_{AB}
$$

- $G$: gravitational constant $6.673 \times 10^{-11}$ $[m^3 \text{kg}^{-1} \text{s}^{-2}]$

- $r = |\vec{p}_A - \vec{p}_B|$: the distance between the objects

- $\vec{u}_{AB} = \frac{\vec{p}_A - \vec{p}_B}{|\vec{p}_A - \vec{p}_B|}$: the unit direction between them
Gravity on Earth

By applying Newton’s 2\textsuperscript{nd} law to an object with mass \( m \) on the surface of the Earth, we obtain:

\[
\vec{F}_{net} = \vec{F}_g = m \cdot \vec{a}
\]

\[
G \frac{m \cdot m_{Earth}}{r_{Earth}^2} = m \cdot a
\]

\[
G \frac{m_{Earth}}{r_{Earth}^2} = a \quad \text{Mass of object is canceled out!}
\]

\[
\alpha = g_{Earth} = 6.673 \times 10^{-11} \frac{5.98 \times 10^{24}}{(6.377 \times 10^{6})^2} \approx 9.81 \text{ m/s}^2
\]

http://lannyland.blogspot.co.at/2012/12/10-famous-thought-experiments-that-just.html
Gravity on Other Planets

- On Earth at altitude $h$: $a = G \frac{m_{Earth}}{(r_{Earth} + h)^2}$

- On the Moon
  - $m_{moon} = 7.35 \times 10^{22}$ kg
  - $r_{moon} = 1738$ km
  - $g_{moon} = 1.62 \text{ m/s}^2$

- On Mars
  - $m_{mars} = 6.42 \times 10^{23}$ kg
  - $r_{mars} = 3403$ km
  - $g_{mars} = 3.69 \text{ m/s}^2$
Weight

• Weight $\Leftrightarrow$ gravitational force

$$\vec{W} = m \cdot \vec{g}$$

• We weigh different on the moon (but have the same mass...)

• Force units: $[kg \cdot \frac{m}{sec^2}]$
  - Denoted as Newtons $[N]$
Free-Body Diagram

- To get acceleration: sum forces and divide by mass (D'Alembert's principle):
  \[ \vec{F}_{net} = \sum \vec{F}_i = m \cdot \vec{a} \]

- Forces add up linearly as vectors.
  - Important: when all are represented in the same axis system!

- The Free-Body Diagram includes:
  - Object shape: center of mass, contact points
  - Applied forces: direction, magnitude, and point of application

https://www2.southeastern.edu/Academics/Faculty/rallain/plab193/files/a8312fbf3bde480430906169ad22bd5-4.html
Normal force

- Force acting as a **reaction** to contact
  - Direction is **normal to the surface** of contact
  - **Magnitude** enough to cancel the weight so object doesn’t go through the plane

Here, \( \vec{F}_N = \vec{W} \cos(\alpha) = m \vec{g} \cos(\theta) \)

- Related to **constraints and collision handling** (more later)
- Object slides down plane with the remainder force: \( \vec{W} \sin(\theta) \)
Friction

• Can the object stay in total equilibrium?
  • An extra tangential friction force must cancel $\vec{W} \sin(\theta)$
• Ability to resist movement
  • Static friction keeps an object on a surface from moving
  • Kinetic friction slows down an object in contact
Friction

- **Static friction**: a threshold force
  - object will not move unless tangential force is stronger

- **Kinetic friction**: when the object is moving

- Depends on the materials in contact
  - smoother ⇔ less friction

- Coefficient of friction $\mu$ determines friction forces:
  - Static friction: $F_s = \mu_s F_N$
  - Kinetic friction: $F_k = \mu_k F_N$
Friction

• The kinetic coefficient of friction is always smaller than the static friction.

• If the tangential force is larger than the static friction, the object moves.

• If the object moves while in contact, the kinetic friction is applied to the object.
# Friction

<table>
<thead>
<tr>
<th>Surface Friction</th>
<th>Static ($\mu_s$)</th>
<th>Kinetic ($\mu_k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel on steel (dry)</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Steel on steel (greasy)</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Teflon on steel</td>
<td>0.041</td>
<td>0.04</td>
</tr>
<tr>
<td>Brake lining on cast iron</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Rubber on concrete (dry)</td>
<td>1.0</td>
<td>0.9</td>
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<tr>
<td>Rubber on concrete (wet)</td>
<td>0.30</td>
<td>0.25</td>
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<tr>
<td>Metal on ice</td>
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<td>0.02</td>
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<tr>
<td>Steel on steel</td>
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<tr>
<td>Aluminum on steel</td>
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<tr>
<td>Copper on steel</td>
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<td>0.36</td>
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<tr>
<td>Nickel on nickel</td>
<td>1.1</td>
<td>0.53</td>
</tr>
<tr>
<td>Glass on glass</td>
<td>0.94</td>
<td>0.40</td>
</tr>
<tr>
<td>Copper on glass</td>
<td>0.68</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Springs

• React according to *Hook’s Law* on extension and compression, *i.e.* on the relative displacement

• The relative length $l$ to the rest length $l_0$ determines the applied force:

\[
F_K = -K(l - l_0)
\]

• $K$ is the *spring constant* (in $N/m$)

• Scalar spring: two directions
Work

• A force $\vec{F}$ does work $W$ (in Joule = N \cdot m), if it achieves a displacement $\Delta \vec{x}$ in the direction of the displacement:

$$W = \vec{F} \cdot \Delta \vec{x}$$

• Note dot product between vectors
• Scalar quantity
Kinetic energy

- The kinetic energy $E_K$ is the energy of an object in velocity:

$$E_K = \frac{1}{2} m |\vec{v}|^2$$

- The faster the object is moving, the more energy it has.
- The energy is a scalar (relative to speed $\nu = |\vec{v}|$, regardless of direction).
- Unit is also Joule:

$$kg (m/sec)^2 = \left( kg \times \frac{m}{sec^2}\right) m = N \times m = J$$
Work-Energy theorem

• The **Work-Energy theorem**: net work $\Leftrightarrow$ change in kinetic energy:

\[
W = \Delta E_K = E_K(t + \Delta t) - E_K(t)
\]

i.e.

\[
\vec{F} \cdot \Delta \vec{x} = \frac{1}{2}m(v(t + \Delta t)^2 - v(t)^2)
\]

• Very similar to Newton’s second law...
  • In fact, just the integrated/variational version
Potential energy

• (Gravitational) **Potential energy** is the energy ‘stored’ in an object due to relative height difference
  • The amount of work that would be done if we were to set it free
    \[ E_P = m \cdot g \cdot h \]
  • Simple product of the weight \( W = m \cdot g \) and height \( h \)
  • Also measured in Joules (as here \( kg \cdot \frac{m}{sec^2} \cdot m \))

• Other potential energies exist (like a compressed spring)
Conservation of mechanical energy

• **Law of conservation:** in a closed system, energy **cannot** be created or destroyed.
  • Energy may switch form
  • May transfer between objects
  • Classical example: falling trades potential and kinetic energies

\[
E_K(t + \Delta t) + E_P(t + \Delta t) = E_K(t) + E_P(t)
\]

i.e.

\[
\frac{1}{2}mv(t + \Delta t)^2 + mgh(t + \Delta t) = \frac{1}{2}mv(t)^2 + mgh(t)
\]
Conservation: Example

- A roller-coaster cart at the top of the first hill
  - Much potential energy, but only a little kinetic energy
  - Going down the drop: losing height, picking up speed
  - At the bottom: almost all potential energy switched to kinetic, cart is at its maximum speed
Conservation of Mechanical Energy

• External forces are usually applied:
  • Friction and air resistance
  • Where does the “reduced” energy go?
  • Converted into heat and air displacements (sound waves, wind)

• We compensate by adding an extra term $E_O$ to the conservation equation:
  $$E_K(t + \Delta t) + E_P(t + \Delta t) + E_O = E_K(t) + E_P(t)$$
  • if $E_O > 0$, some energy is ‘lost’
Momentum

- The linear momentum $\vec{p}$: the mass of an object multiplied by its velocity:

  \[ \vec{p} = m \cdot \vec{v} \]

- Heavier object/higher velocity $\Leftrightarrow$ more momentum (more difficult to stop)

- Unit is \([\text{kg} \cdot \text{m/sec}]\)

- Vector quantity (velocity)
Impulse

• A change of momentum:
  \[ \vec{j} = \Delta \vec{p} \]

• Compare:
  • Impulse is change in momentum
  • Work is change in energy

• Unit is also \([kg \cdot \frac{m}{sec}]\) (like momentum)

• Impulse \(\Leftrightarrow\) force integrated over time:
  \[ \vec{j} = \int_0^t \vec{F} dt = m \int_0^t \vec{a} dt = m \Delta \vec{v} \]
Conservation of Momentum

- **Law of conservation**: in a closed system (no external forces/impulses), momentum **cannot** be created or destroyed.
- **Compare**: conservation of energy.
- Implied from 3rd law.
  - Objects react with the same force exerted on them.
- Special case of Noether’s theorem: every physical system (With a symmetric action) has a conservation law.