Compiling Techniques

Lecture 16: Dataflow Analysis

Idea: Change Representation that makes def-use chains explicit

As a first step, we translate the nested AST representation into a graph representation:

AST

```
assign() {
  id_expr() ["id" = "x"] } {
  literal() ["value" = 1 : !i32]  }
assign() {
  id_expr() ["id" = "y"] } {
  binary_expr() ["op" = "+"] {
    id_expr() ["id" = "x"] } {
    literal() ["value" = 1 : !i32] } }
assign() {
  id_expr() ["id" = "x"] } {
  literal() ["value" = 2 : !i32] }
assign() {
  id_expr() ["id" = "z"] } {
  binary_expr() ["op" = "+"] {
    id_expr() ["id" = "x"] } {
       literal() ["value" = 1 : !i32] } }
```

→

Graph-based IR

```
%l0 : !int = literal() ["value" = 1 : !i32]
assign(%x : !int, %l0 : !int)
%l1 : !int = literal() ["value" = 1 : !i32]
%t0 : !int = binary_expr(%x : !int, %l1 : !int) ["op" = "+"]
assign(%y : !int, %t0 : !int)
%l2 : !int = literal() ["value" = 2 : !i32]
assign(%x : !int, %l2 : !int)
%l3 : !int = literal() ["value" = 1 : !i32]
%t1 : !int = binary_expr(%x : !int, %l3 : !int) ["op" = "+"]
assign(%z : !int, %t1 : !int)
```

AST to Graph IR Translation Overview

- Recursively visit the AST nodes
- For each AST node without children create corresponding Graph node
- For each AST node with children create list of Graph nodes
- Replace nested regions representing AST children with Names (*%x*)
- Maintain context during translation that relates
  variable names in the AST with Names in the Graph (*%x*)

Types in the Translation

- In the Graph IR operations (can) have a result type

- To simplify the translation it helps to change the type checking to add the type of every expression as an attribute to the AST node

- Then the type of an expression in the AST is directly available when translating the AST node

Control-flow and Data-flow Analysis

Control-flow / data-flow analysis aim to understand the program's behaviour without executing it by analysing the possible different branches a program can take and where variables are accessed.

Analysis enables beneficial program transformations:

Optimizations = Analysis + Transformation

Data-flow Analysis

Data-flow analysis gathers information for *each program point*
by analysing the static code approximating its **dynamic** behaviour

Examples:

- Reaching Definitions
- Initialised Variables
- Constant Propagation
- Sign Analysis
- Liveness of variables

```
int foo(int input) {
  int x,y,z;
  x = input;
  while (x > 1) {
    y = x / 2;
    if (y > 3) x = x - y;
    z = x - 4;
    if (z > 0) x = x / 2;
    z = z-1;
  }
  return x;
}
```

Is z ever initialised?

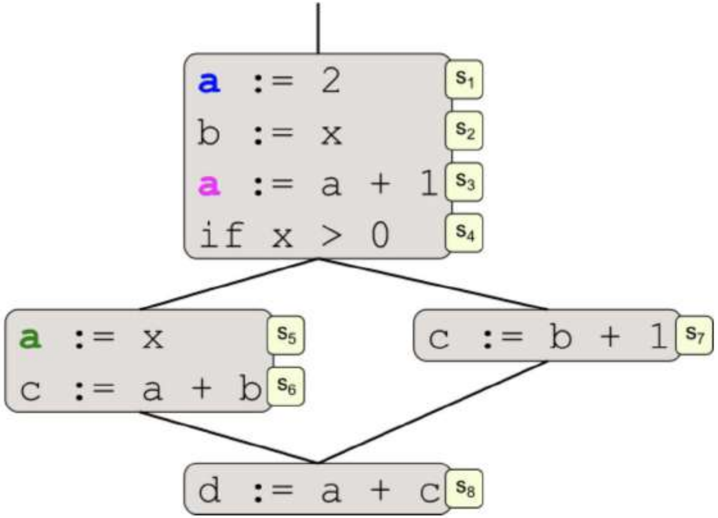
What values are possible for y here?

Is this computation ever used?

Reaching Analysis: Reaching definitions

*Definition of variable **v** at program point **d** reaches point **u** if there exists a control-flow path **p** from **d** to **u** such that no definition of **v** appears on that path.*

Reaching definitions of **a**?



Reaching Analysis: Reaching definitions

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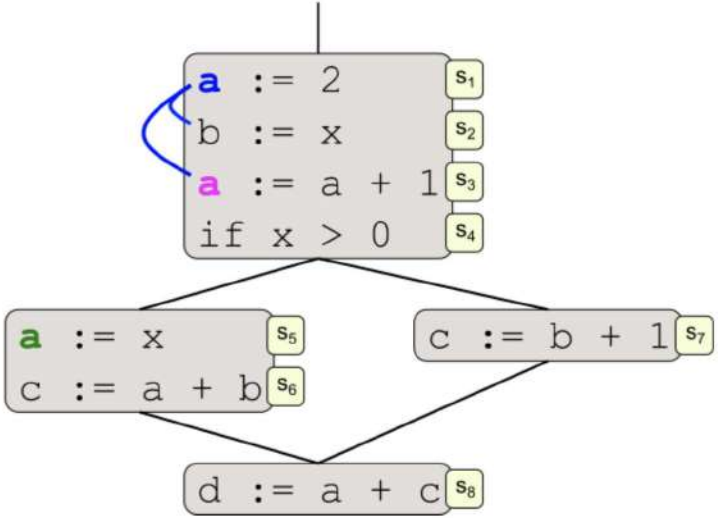

Reaching Analysis: Reaching definitions

Definition of variable ***v*** *at program point* ***d*** ***reaches*** *point* ***u*** *if there exists a control-flow path* ***p*** *from* ***d*** *to* ***u*** *such that no definition of* ***v*** *appears on that path.*

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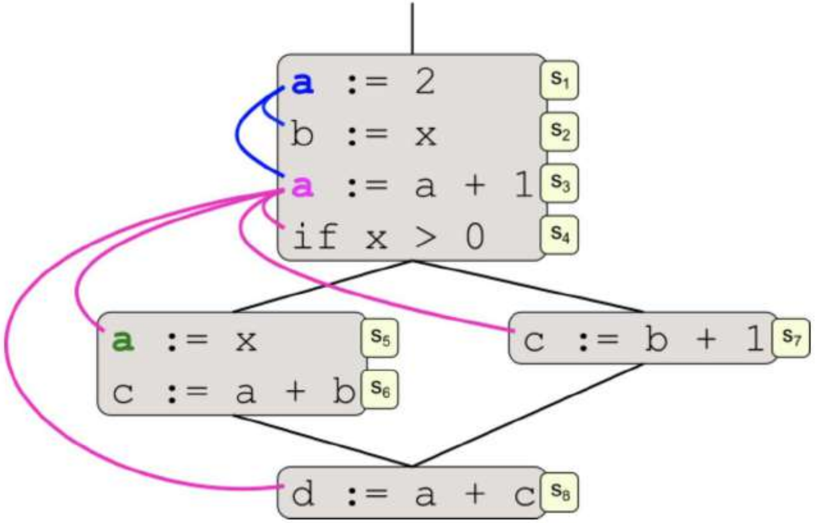

Reaching Analysis: Reaching definitions

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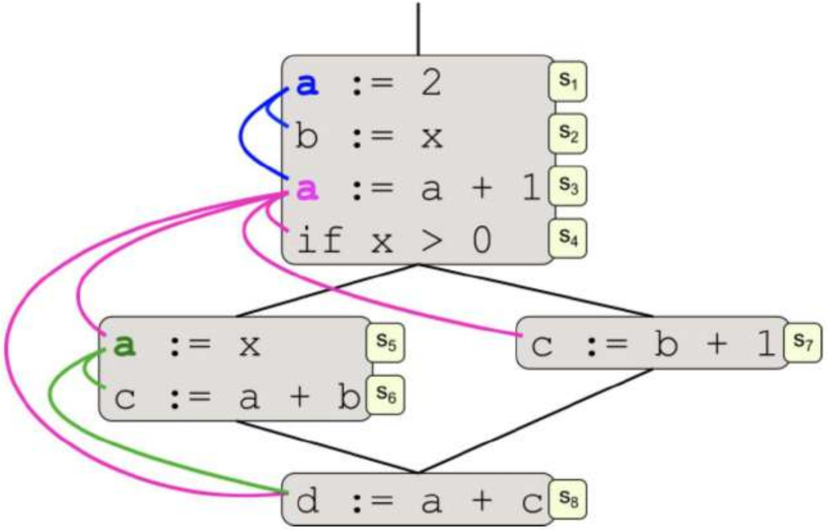


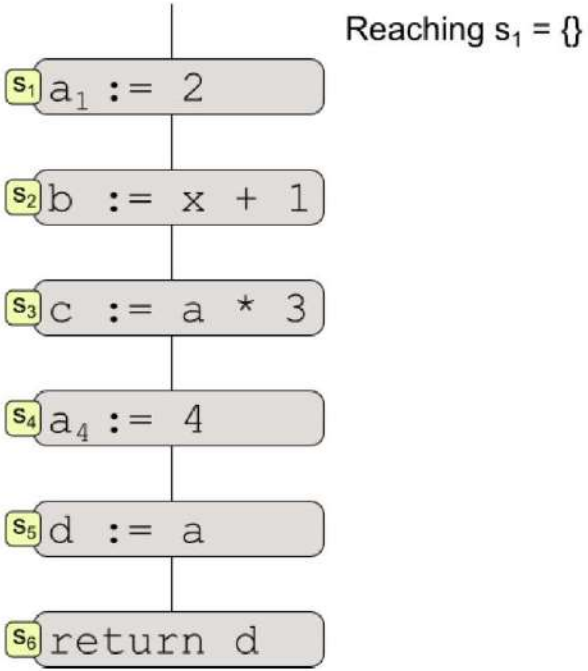
Local Reaching Analysis

A *local analysis* works only on a single basic block

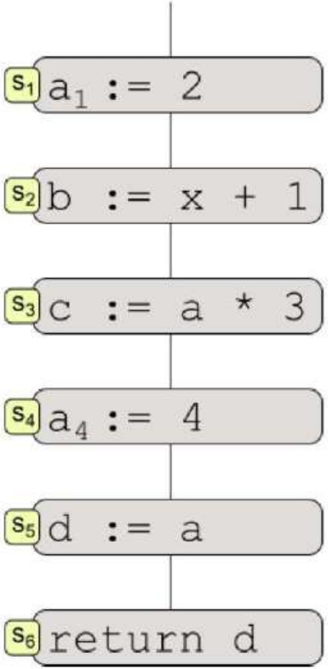
Local Reaching Analysis:

- Maintain a set of current reaching definitions
- Add subscripts to all variable definitions
- Go through all statements from start to end
- If assignment statement x_i **:= …**
  - For all j remove (kill) x_j
  - Add x_i to the set
- Otherwise, the set remains unchanged

Local Reaching Analysis



Local Reaching Analysis

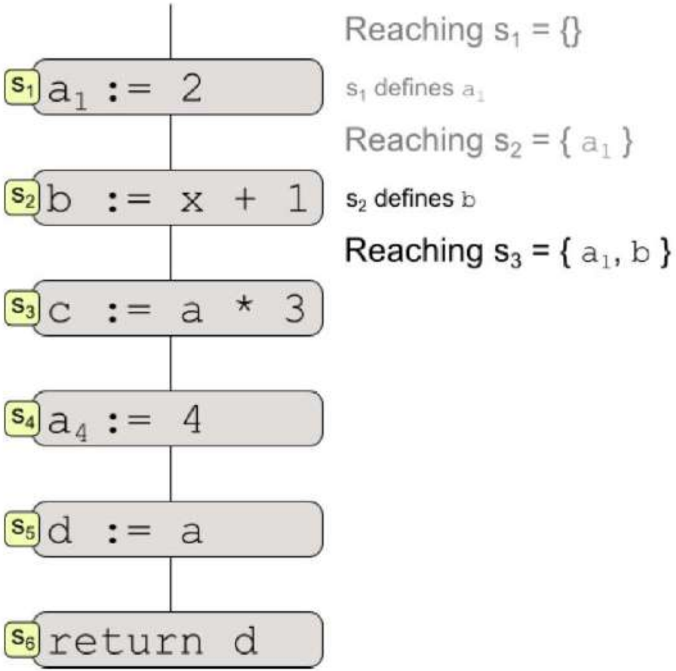


Reaching s_1 = {}

s_1 defines a_1

Reaching s_2 = { a_1 }

Local Reaching Analysis



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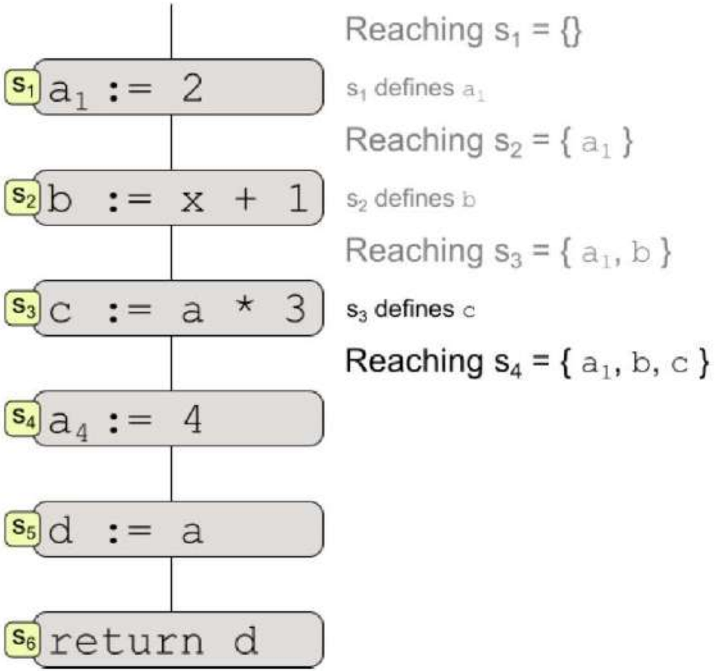
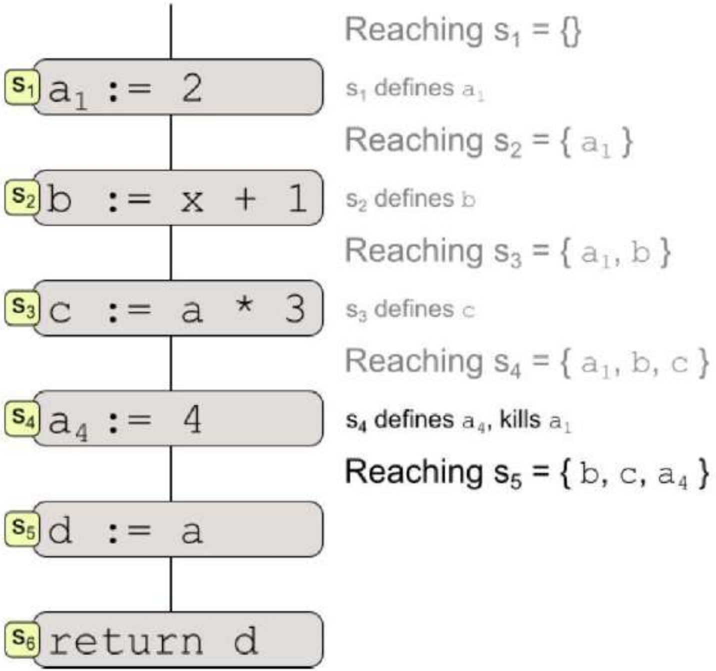
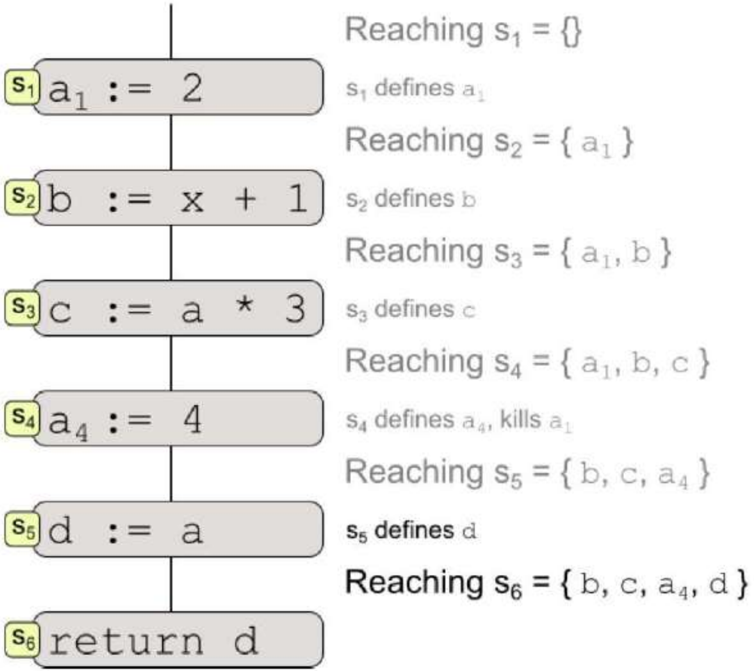


Local Reaching Analysis



Local Reaching Analysis



Global Reaching Analysis

Local Analysis is not enough, we must think about control flow!

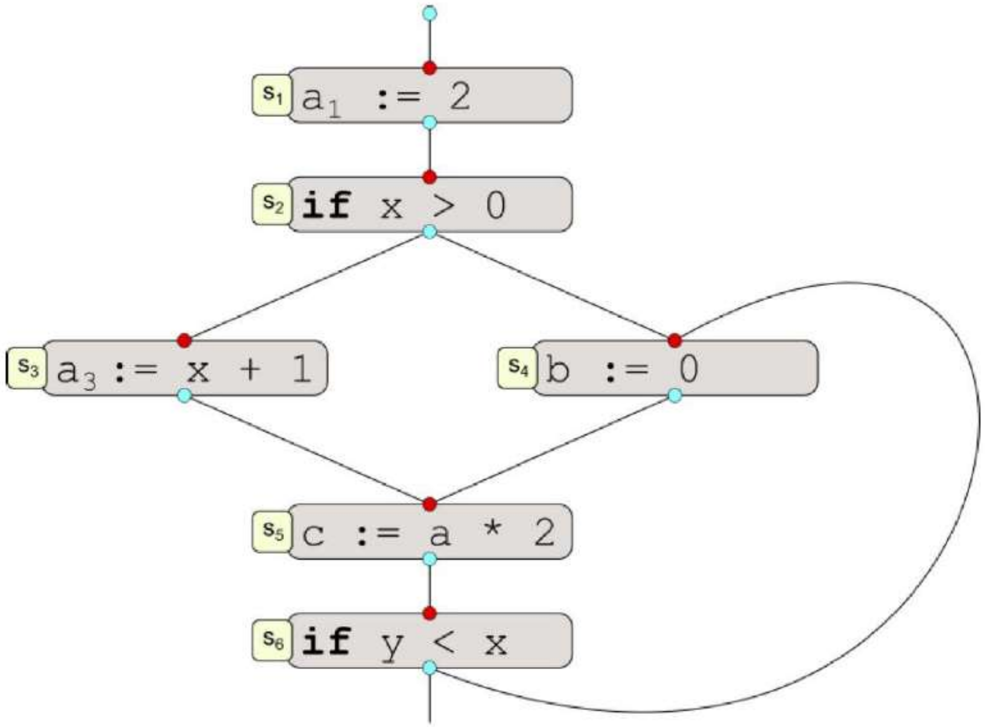
- Control flow complicates matters

- Refine definition of program point:
    - *In* program point for a statement: Entering the statement
    - *Out* program point for a statement: Leaving the statement

- We will try the previous approach and see where it fails

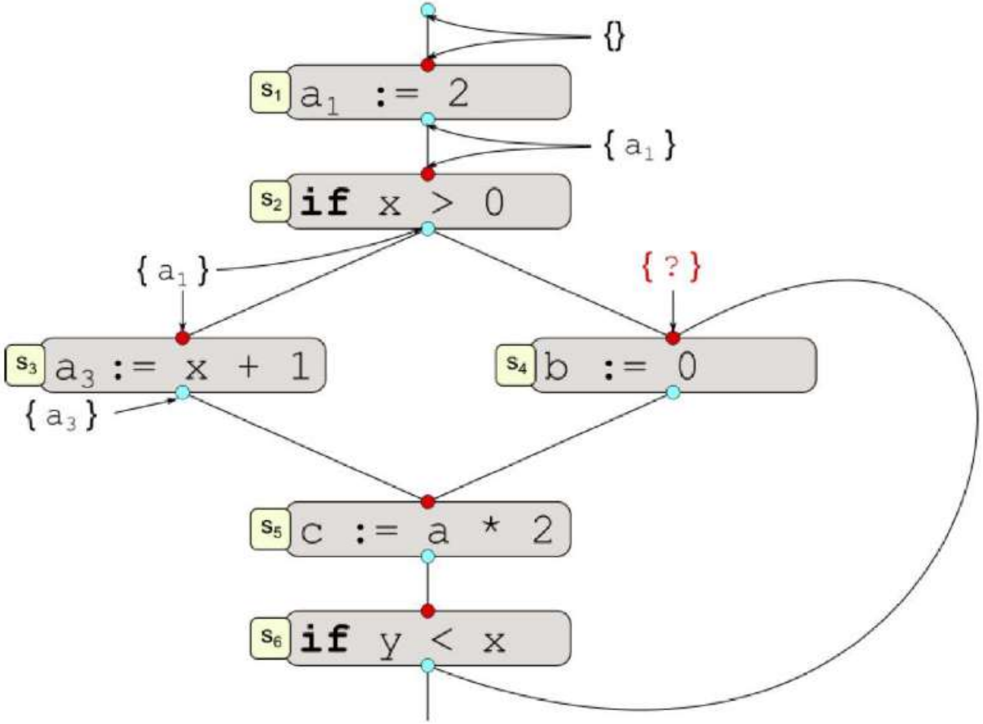
Global Reaching Analysis

Control flow example; try the previous approach



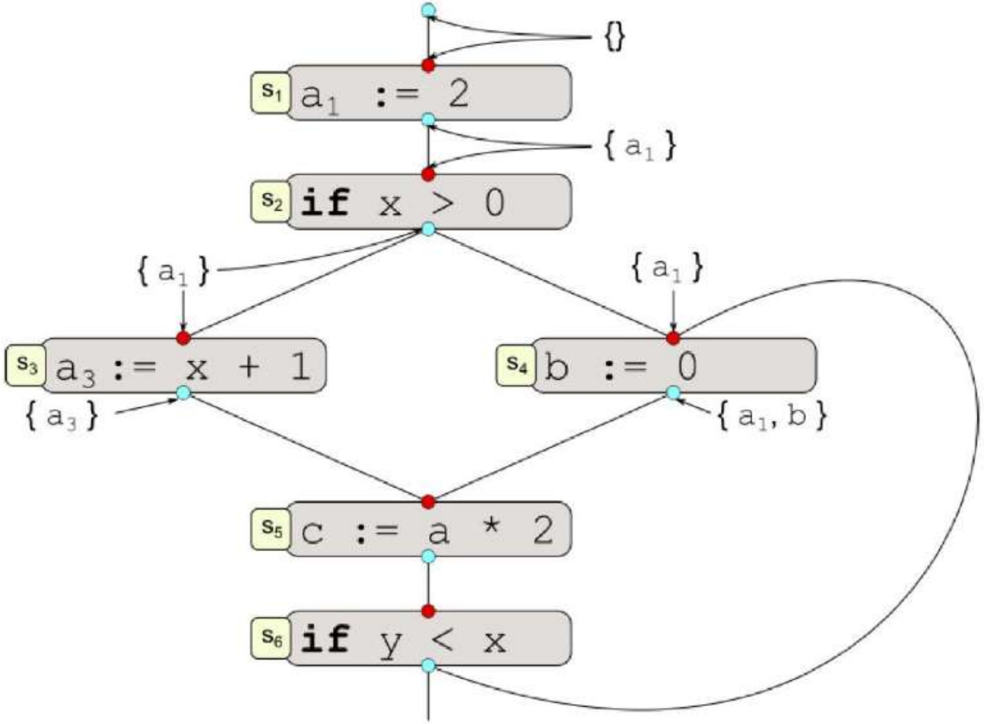
Global Reaching Analysis

s_4 has 2 predecessors; and we don't know $Out(s_6)$ yet



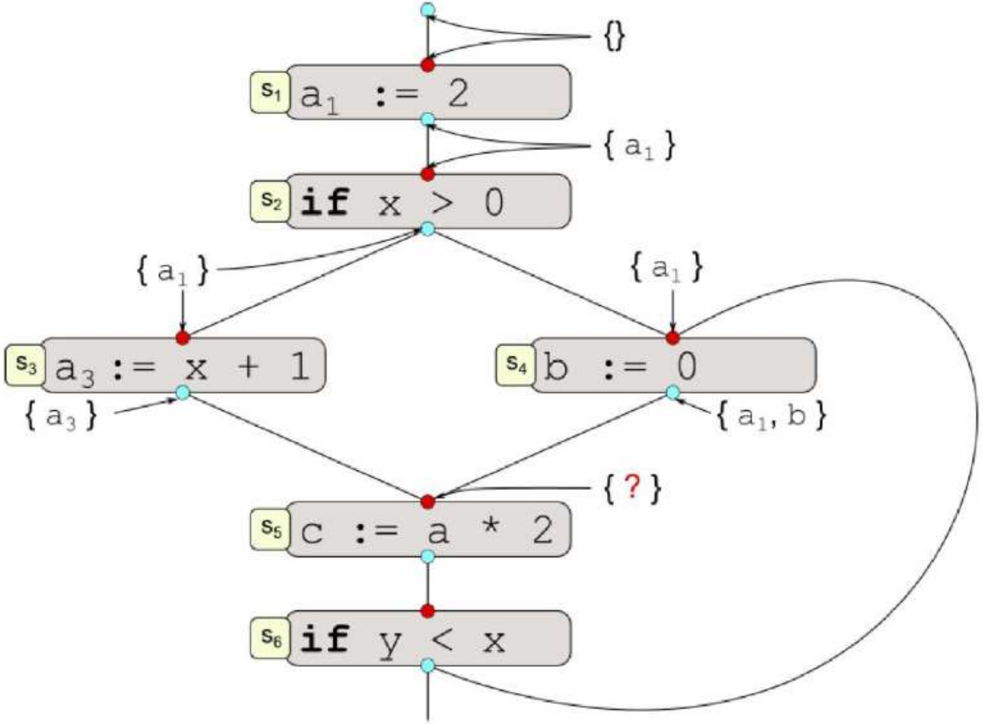
Global Reaching Analysis

But, we know at least that a_1 reaches s_4



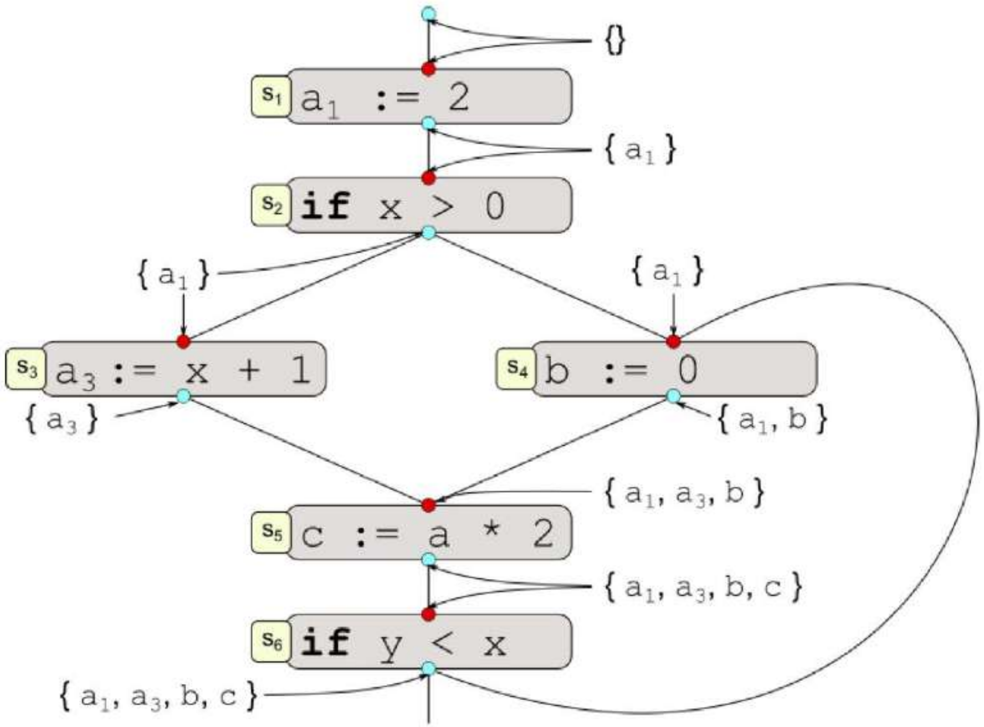
Global Reaching Analysis

s_5 has 2 predecessors



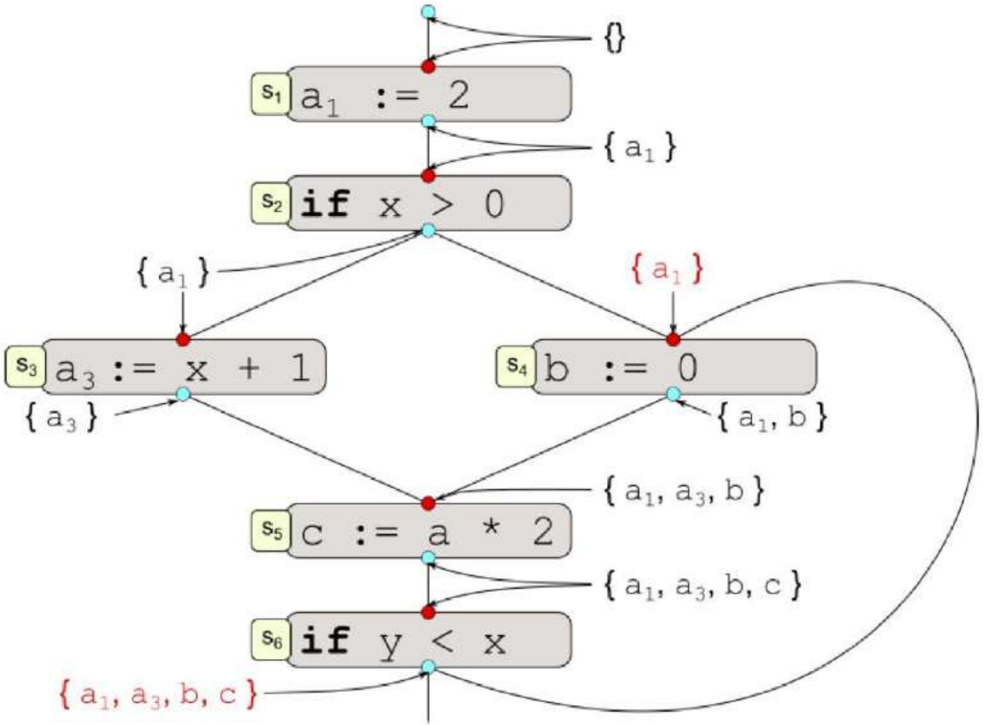
Global Reaching Analysis

All incoming definitions reach ⇒ compute union of the two sets



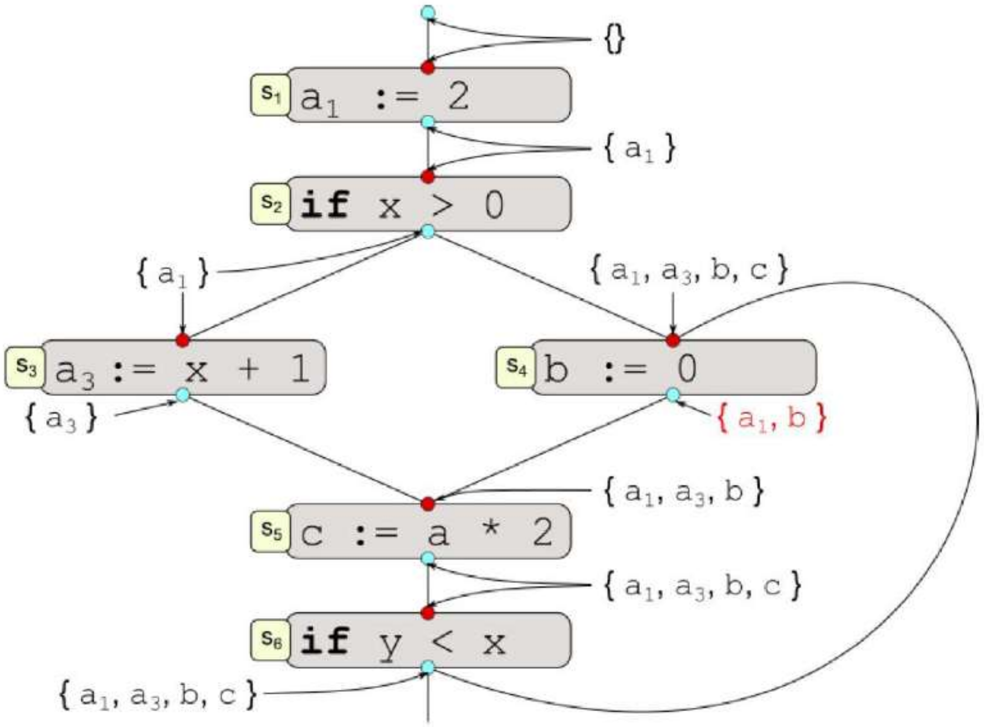
Global Reaching Analysis

Inconsistency, as we now know more about *Out*(s_6)



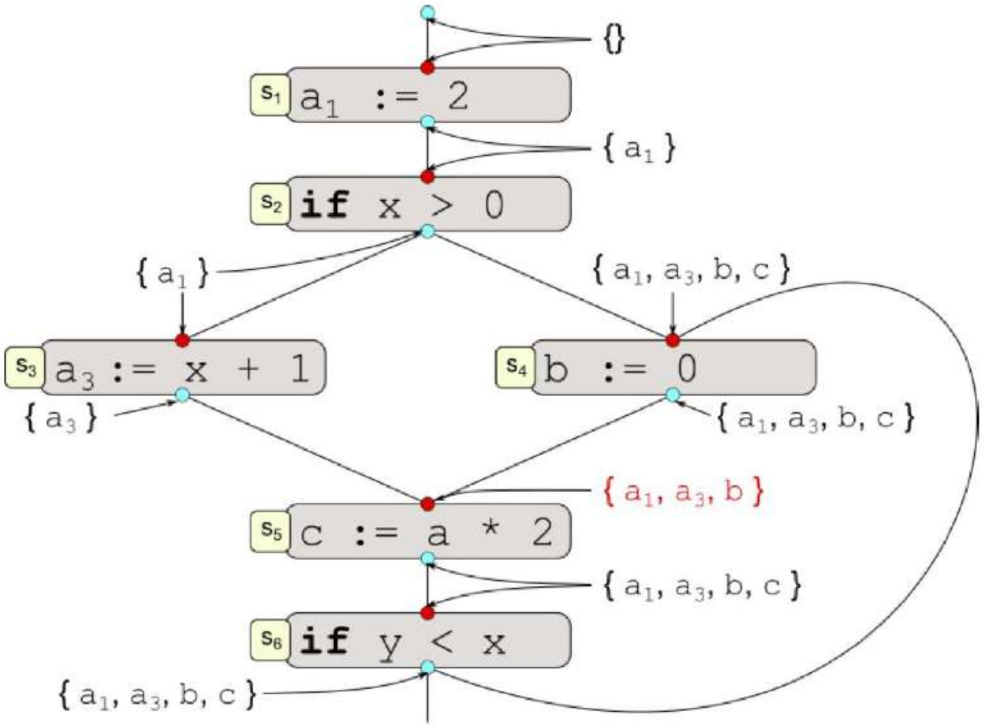
Global Reaching Analysis

All incoming definitions reach ⇒ union ⇒ inconsistency



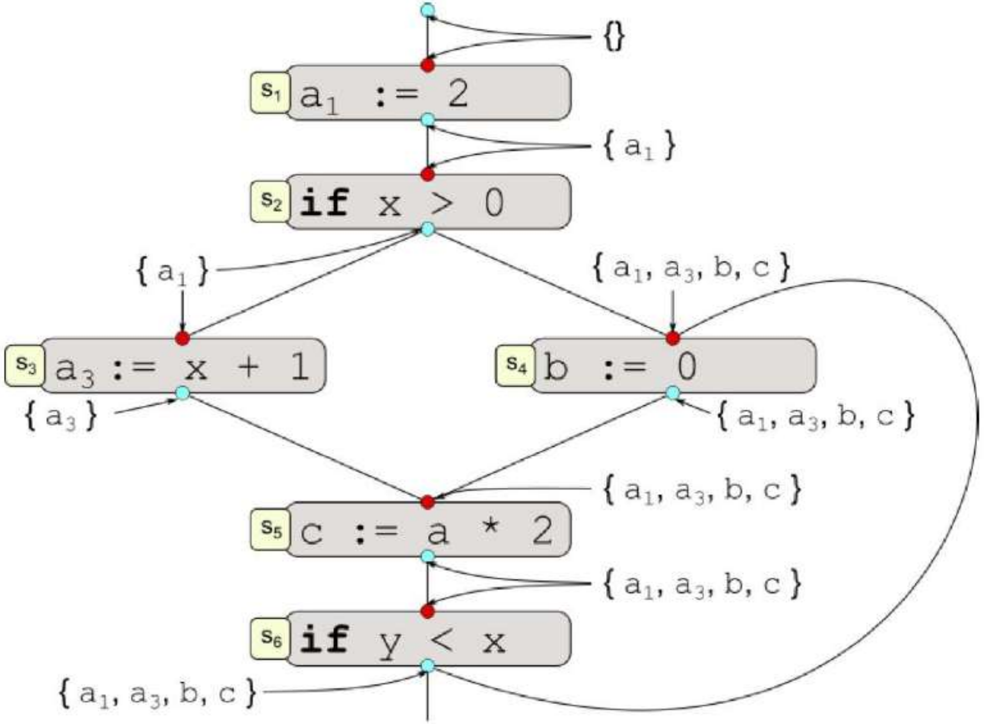
Global Reaching Analysis

Inconsistency



Global Reaching Analysis

Consistent state



Reaching Analysis: *Dataflow equations*

Let us formalise our intuition

- For each statement s, compute $Out(s)$ from $In(s)$
  If s is an assignment to x, delete all definitions of x, and add new definitions:
  $$Out(s : x_i := \ldots) = (In(s) - \{ x_j ; \ \forall j \}) \cup \{x_i\}$$

- Multiple incoming edges must merge to compute $In(s)$
  $$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

- We start with an empty set
  $$Init(s) = \emptyset$$

Reaching Analysis: *Observations*

- Analysis assumes a control flow graph
- Start with a conservative approximation
- Refine the approximations
- Stop when consistent (there are no further changes)
- Information flows *forward* from a statement to its successors

General Dataflow Analysis

- **Direction** – *forward* or *backward*
- **Transfer function** – computes effect of statement

  e.g. $Out(s) = (In(s) - Kill(s)) \cup Gen(s)$
- **Meet operator** – merges values from multiple incoming edges

  e.g. $In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$
- **Value set** – the information being passed around

  e.g. Sets of definitions
- **Initial values**

  Should be most conservative value; Start node often a special case

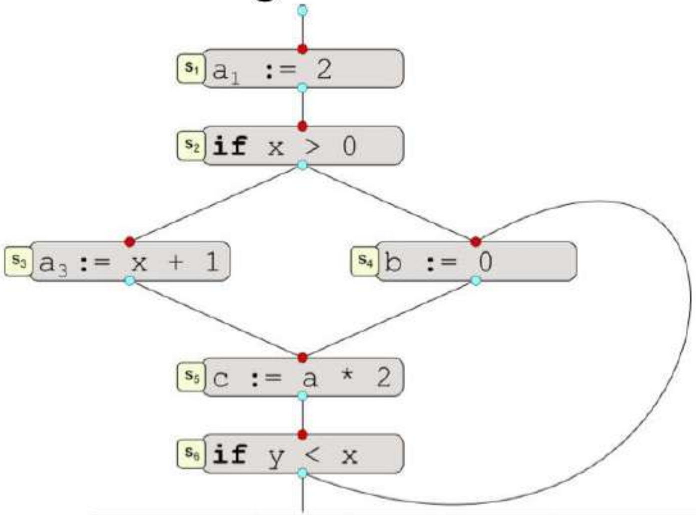
Iterative Round-Robin Algorithm

```
for each node, start_node do
  Initialise start_node

while values changing do
  for each node do
    Apply meet function        // compute In(s)
    Apply transfer function    // compute Out(s)
```

Reaching Analysis with Dataflow Iterative Algorithm

Reaching definitions control flow example - Calculate RD sets?



$$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

$$Out(s : d_i := \ldots) = (In(s) - \{d_j; \forall j\}) \cup d_i$$

$$\Downarrow$$

$$RD(s) = \bigcup_{\forall p: d_i = \ldots \in Pred(s)} (RD(p) - \{d_j; \forall j\}) \cup \{d_i\}$$

Node	s_1	s_2	s_3	s_4	s_5	s_6
RD^4	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Reaching Analysis with Dataflow Iterative Algorithm

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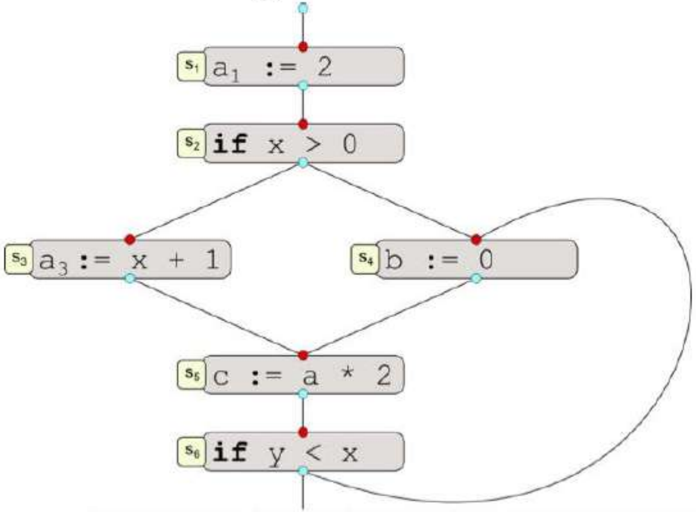

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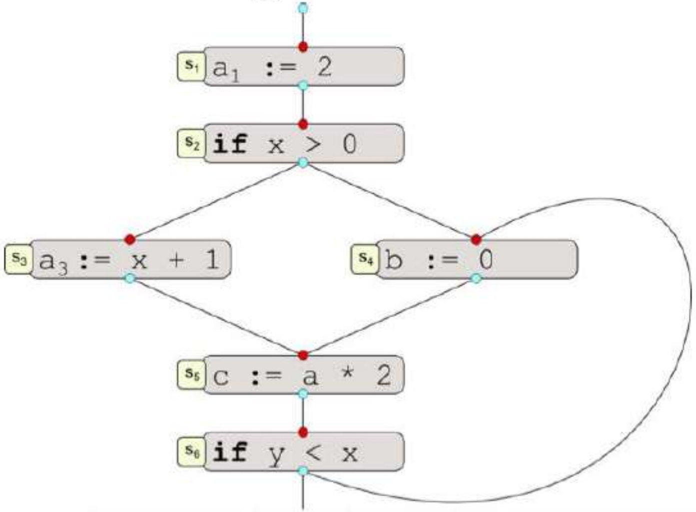

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RD^4	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
	$\varnothing$	a_1				

Reaching Analysis with Dataflow Iterative Algorithm

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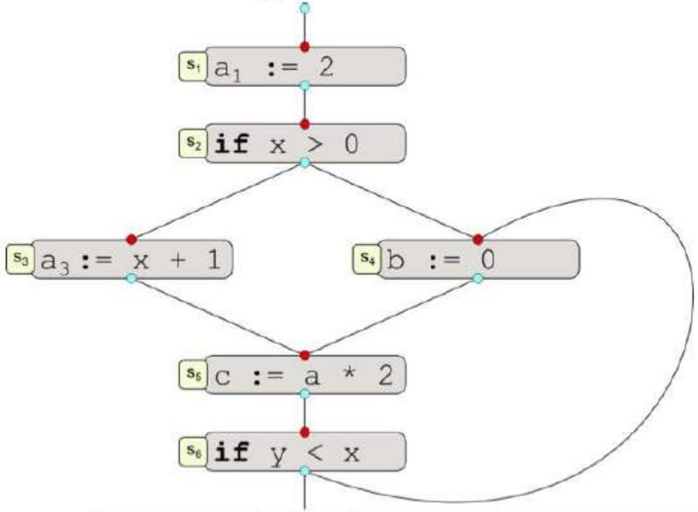

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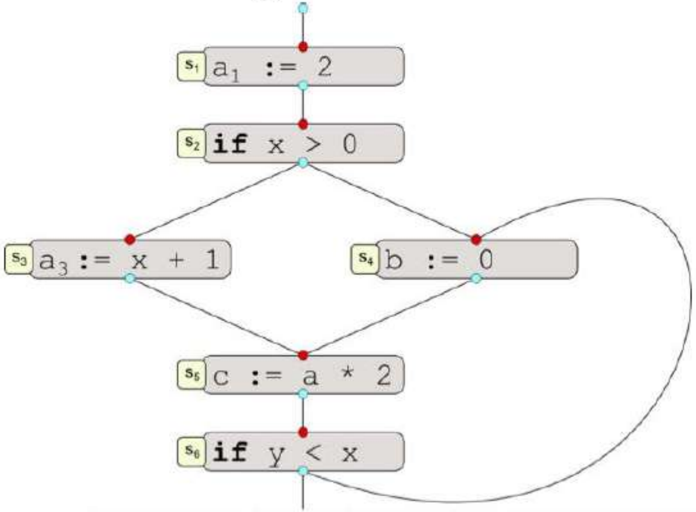
Node	s_1	s_2	s_3	s_4	s_5	s_6
RD^4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	a_1	a_1			

Reaching Analysis with Dataflow Iterative Algorithm

Reaching definitions control flow example - Calculate RD sets?



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RD^4	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
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Reaching Analysis with Dataflow Iterative Algorithm

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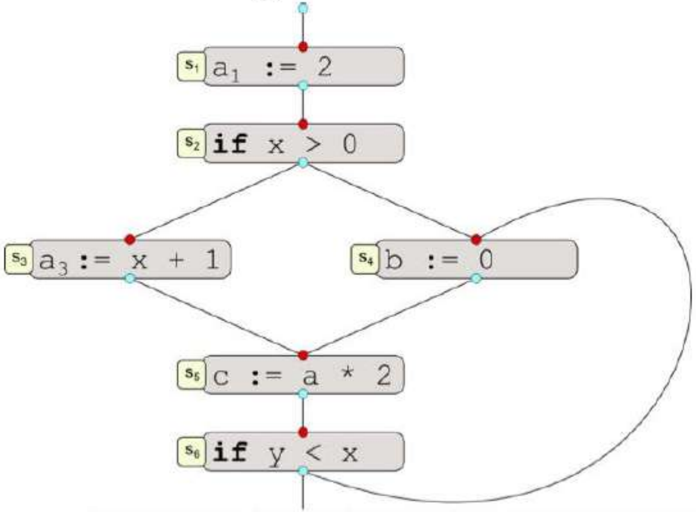

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	$\varnothing$	a_1	a_1	a_1	a_1, a_3, b	

Reaching Analysis with Dataflow Iterative Algorithm

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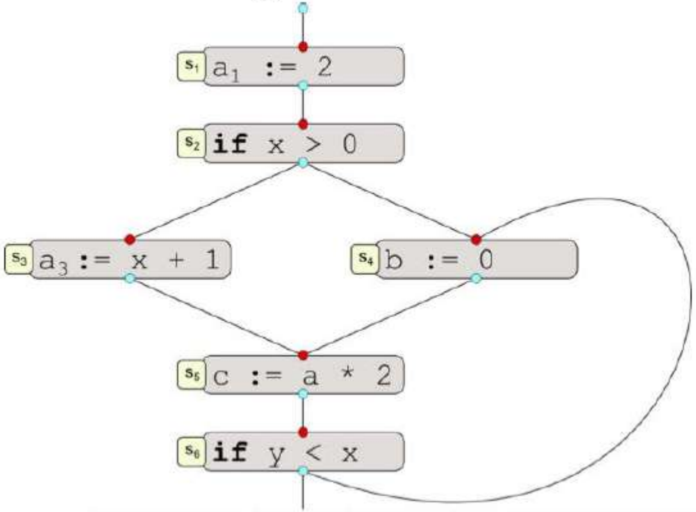

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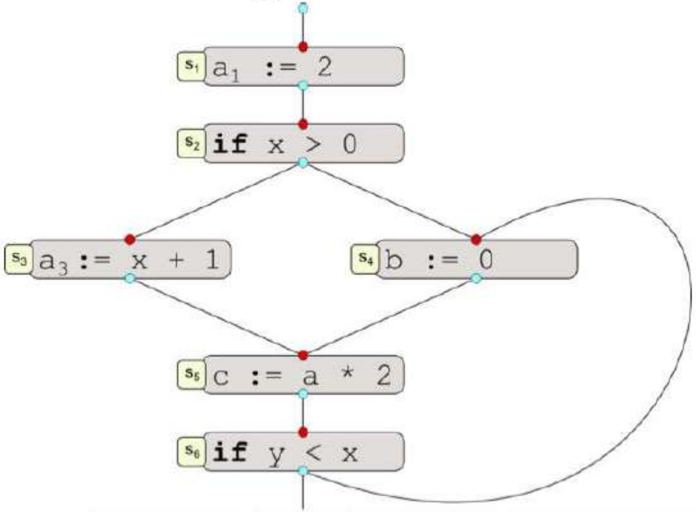
Node	s_1	s_2	s_3	s_4	s_5	s_6
RD[4]	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
	$\varnothing$	a_1	a_1	a_1	a_1, a_3, b	a_1, a_3, b, c

Reaching Analysis with Dataflow Iterative Algorithm

Reaching definitions control flow example - Calculate RD sets?



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	$\varnothing$					

Reaching Analysis with Dataflow Iterative Algorithm

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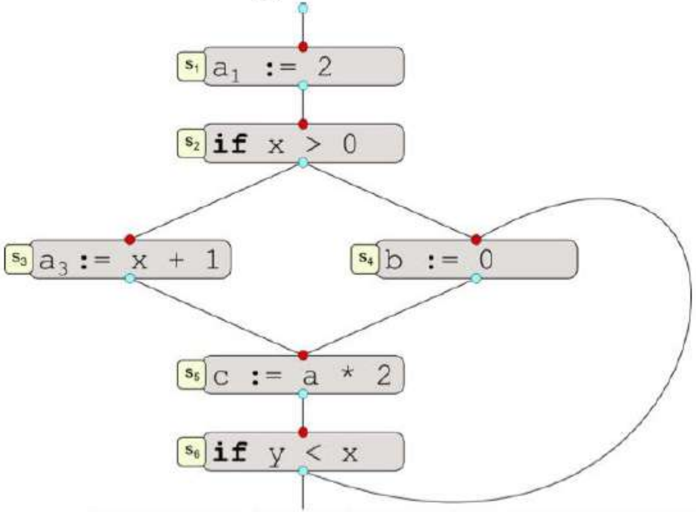

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	$\varnothing$	a_1				

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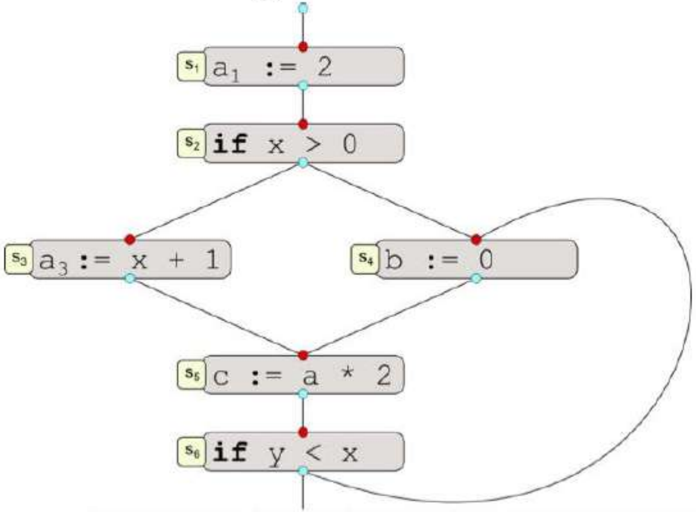

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	$\varnothing$	a_1	a_1			

Reaching Analysis with Dataflow Iterative Algorithm

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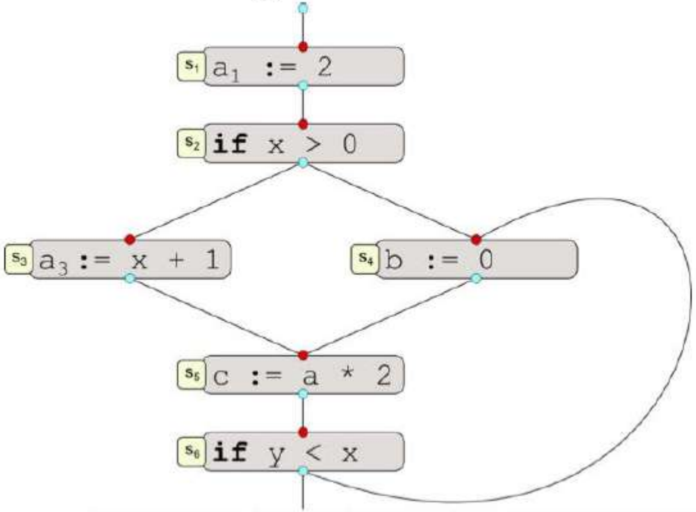

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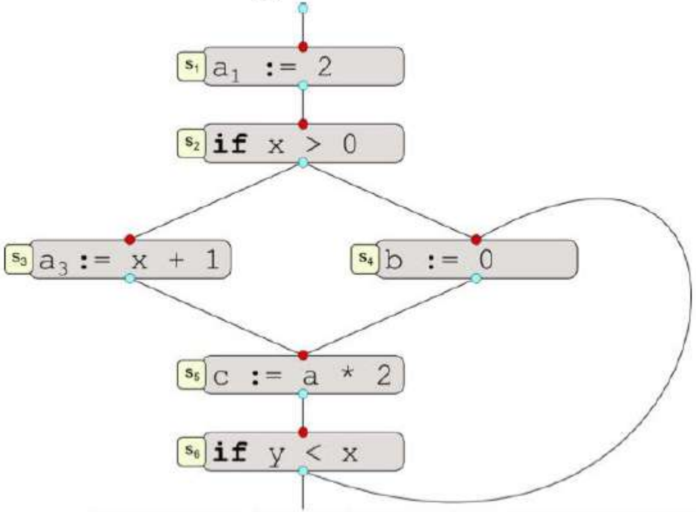

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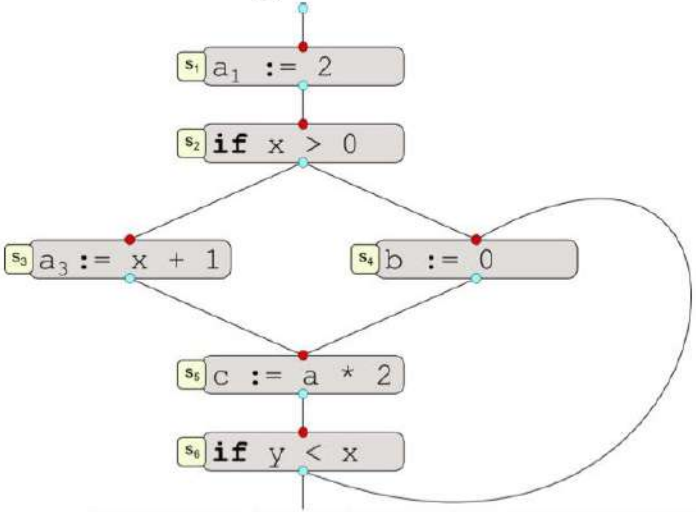
RD^4	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
	$\varnothing$	a_1	a_1	a_1	a_1, a_3, b	a_1, a_3, b, c
	$\varnothing$	a_1	a_1	a_1, a_3, b, c	a_1, a_3, b, c	

Reaching Analysis with Dataflow Iterative Algorithm

Reaching definitions control flow example - Calculate RD sets?



$$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

$$Out(s : d_i := \ldots) = (In(s) - \{d_j; \forall j\}) \cup d_i$$

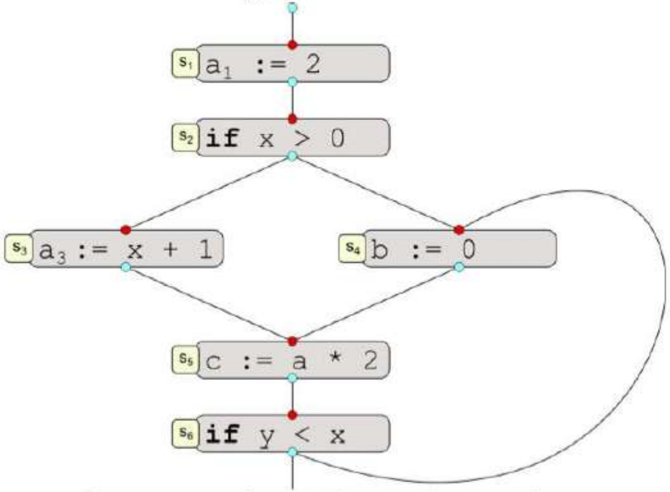
$$\Downarrow$$

$$RD(s) = \bigcup_{\forall p : d_i = \ldots \in Pred(s)} (RD(p) - \{d_j; \forall j\}) \cup \{d_i\}$$

Node	s_1	s_2	s_3	s_4	s_5	s_6
RD[4]	∅	∅	∅	∅	∅	∅
	∅	a_1	a_1	a_1	a_1, a_3, b	a_1, a_3, b, c
	∅	a_1	a_1	a_1, a_3, b, c	a_1, a_3, b, c	a_1, a_3, b, c

Reaching Analysis with Dataflow Iterative Algorithm

Reaching definitions control flow example - Calculate RD sets?



$$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

$$Out(s : d_i := \ldots) = (In(s) - \{d_j; \forall j\}) \cup d_i$$

$$\Downarrow$$

$$RD(s) = \bigcup_{\forall p : d_i = \ldots \in Pred(s)} (RD(p) - \{d_j; \forall j\}) \cup \{d_i\}$$

Node	s_1	s_2	s_3	s_4	s_5	s_6
RD^4	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
	$\varnothing$	a_1	a_1	a_1	a_1, a_3, b	a_1, a_3, b, c
	$\varnothing$	a_1	a_1	a_1, a_3, b, c	a_1, a_3, b, c	a_1, a_3, b, c
	$\varnothing$	a_1	a_1	a_1, a_3, b, c	a_1, a_3, b, c	a_1, a_3, b, c

Iterative Algorithm *Termination*

Does the iterative round-robin is guaranteed to terminate?

Iterative Algorithm *Termination*

Does the iterative round-robin is guaranteed to terminate?

Yes!

- Each step of the iteration can only grow a set or leave unchanged
- Finite number of elements in each set, so finite number of times can change
- Each iteration either has a change or stops
- Therefore, must terminate!

Iterative Algorithm: *Improving Performance*

- Direction (forward vs. backward) can have a big impact on performance
- Round-Robin Algorithm is slow, may require many passes through nodes
- Can speed up by considering basic blocks, rather than individual nodes
- Only nodes which have inputs changed need to be processed,
  keep track with a work list

Liveness Analysis - What & why?

Intuition: A variable is *live* at a program point if its current value may be read during the remaining execution of the program; otherwise, the variable is *dead*.

Useful for *register allocation* and *dead code elimination*

```
int foo(int input) {
  int x,y,z;
  x = input;
  while (x > 1) {
    y = x / 2;
    if (y > 3) x = x - y;
    z = x - 4;
    if (z > 0) x = x / 2;
    z = z-1;
  }
  return x;
}
```

Legal transformation
due to liveness information

```
int foo_opt(int input) {
  int x, yz;
  x = input;
  while (x > 1) {
    yz = x / 2;
    if (yz > 3) x = x - yz;
    yz = x-4;
    if (yz > 0) x = x / 2;

  }
  return x;
}
```

y and z can be stored in the same register

Computation was never used

Definition of Liveness

Definition

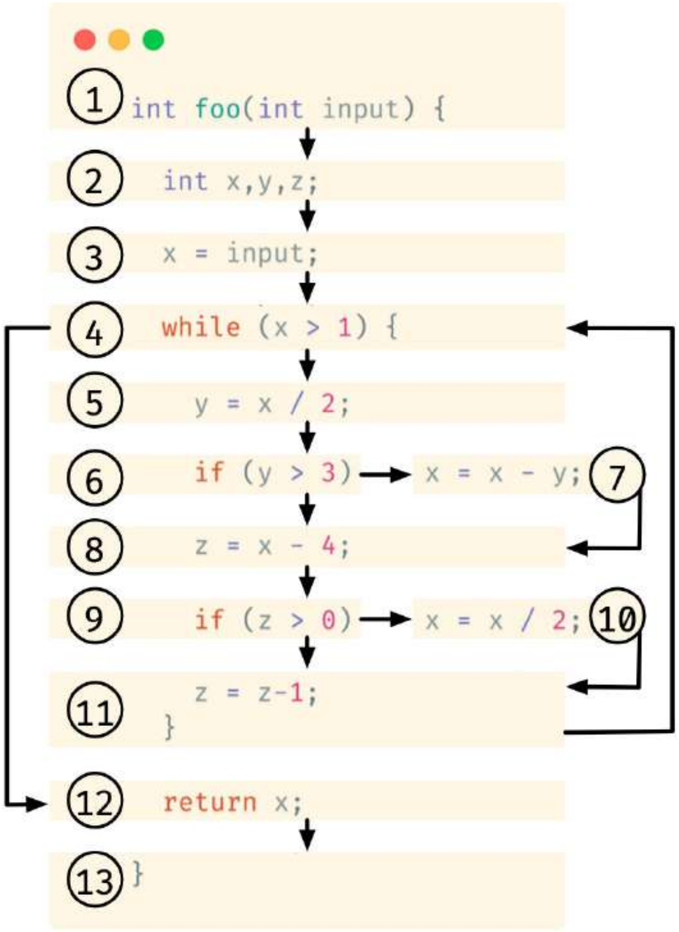
A variable v is live before a CFG node s if

1. $v \in use_{var}(s)$, or
2. $\exists$ a direct path from s to a node that uses v, and that path does not go through a node that defines (overrides) v.

Examples:

Is x life before s = 5? Yes, $x \in \{x\} = use_{var}(5)$

Is z life before s = 5? No, we first hit a def at 8



Backward Dataflow Analysis

- **Direction** – *backward*
- **Transfer function** – computes statement effect

  $In(s) = f_s(Out(s))$
- **Meet operator** – merges values from multiple outcoming edges

  $Out(s) = \bigwedge_{\forall\ b \in Succ(s)} In(b)$
- **Value set** – the information being passed around

  e.g. Sets of variables
- **Initial values**

  Should be most conservative value; Start node often a special case

Liveness as Dataflow Analysis

Out(n) = $candidates$(n)
In(n) = $live$(n)
$f_n(x) = (x - def(n)) \cup use(n)$

- **Direction** – *backward*
- **Transfer function** – computes statement effect

  $$live(n) = (candidates(n) - def_{var}(n)) \cup use_{var}(n)$$

- **Meet operator** – merges values from multiple outcoming edges

  $$candidates(n) = \bigcup_{\forall s \in Succ(n)} live(s)$$

- **Value set** – the information being passed around

  Set of variables + Set of candidates

- **Initial values**

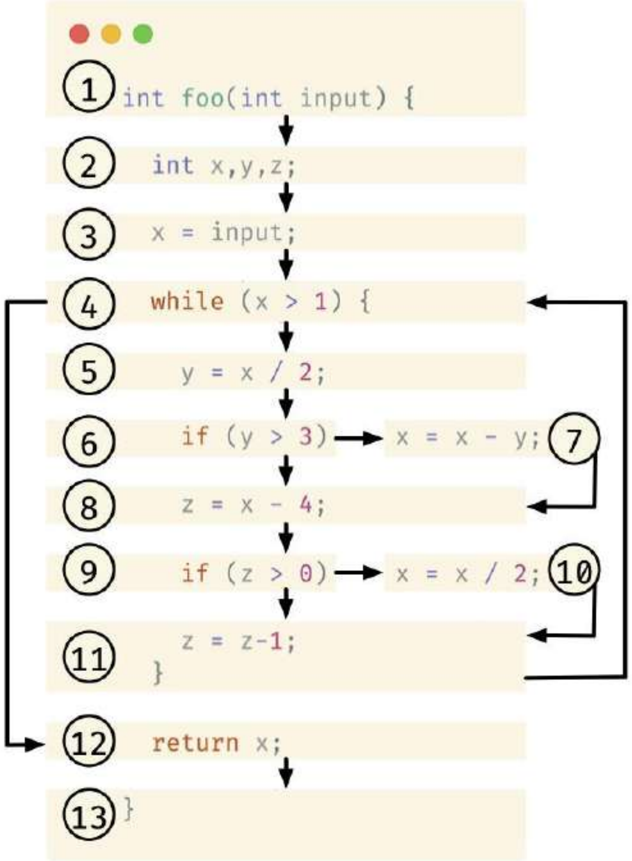
  Empty sets

Liveness Analysis Example

			1st	
Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n]	live[n]
13	{ }	{ }		
12	{x}	{ }		
11	{z}	{z}		
10	{x}	{x}		
9	{z}	{ }		
8	{x}	{z}		
7	{x,y}	{x}		
6	{y}	{ }		
5	{x}	{y}		
4	{x}	{ }		
3	{ }	{x}		
2	{ }	{ }		
1	{ }	{ }		

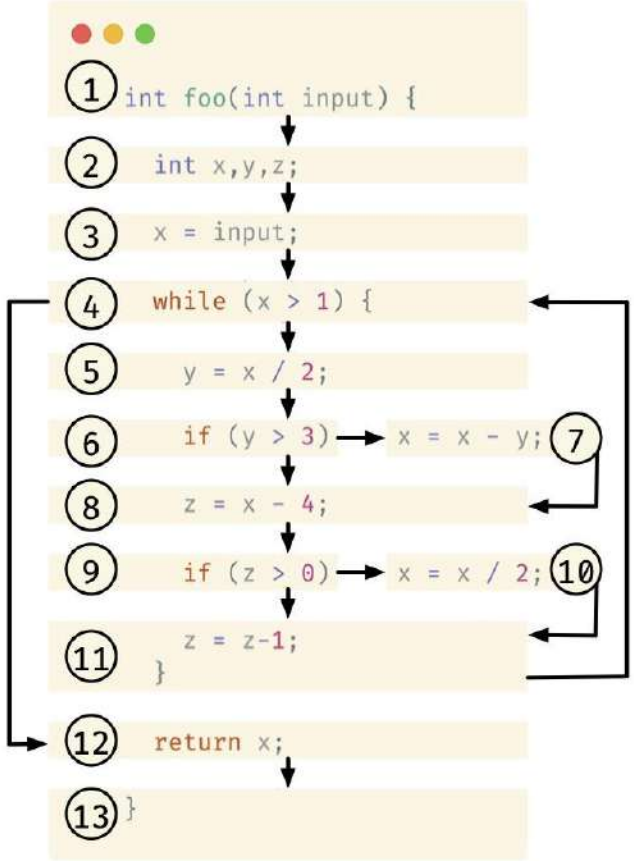
$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s]};$$

$$\texttt{live[n]} = use_{var}(n) \cup (\texttt{candidates[n]} - def_{var}(n));$$



Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	1st candidate[n]	live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{}
11	{z}	{z}	{}	{}
10	{x}	{x}	{}	{}
9	{z}	{}	{}	{}
8	{x}	{z}	{}	{}
7	{x, y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

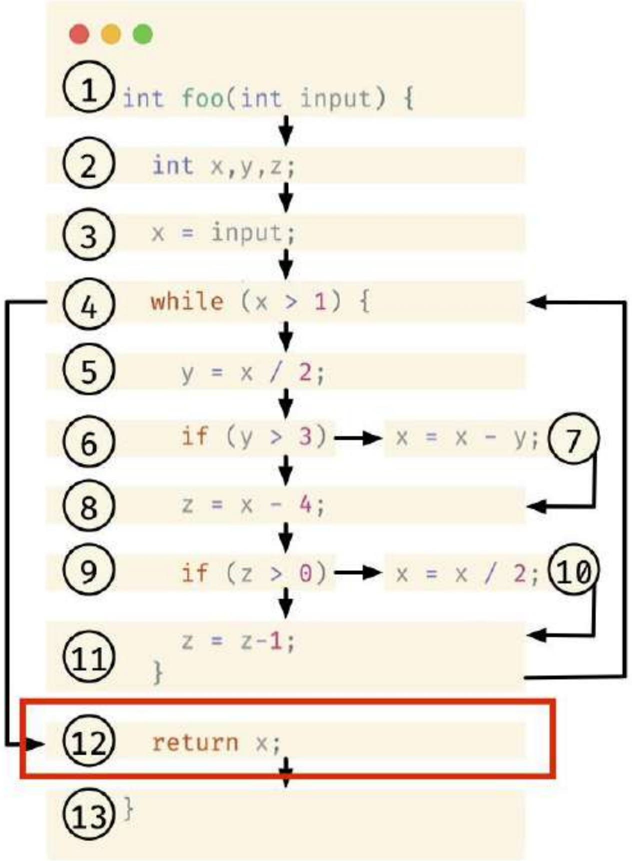


$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s]};$$

$$\texttt{live[n]} = use_{var}(n) \cup (\texttt{candidates[n]} - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n]
13	{ }	{ }	{ }	{ }
12	{x}	{ }	**{ }**	{ }
11	{z}	{z}	{ }	{ }
10	{x}	{x}	{ }	{ }
9	{z}	{ }	{ }	{ }
8	{x}	{z}	{ }	{ }
7	{x, y}	{x}	{ }	{ }
6	{y}	{ }	{ }	{ }
5	{x}	{y}	{ }	{ }
4	{x}	{ }	{ }	{ }
3	{ }	{x}	{ }	{ }
2	{ }	{ }	{ }	{ }
1	{ }	{ }	{ }	{ }

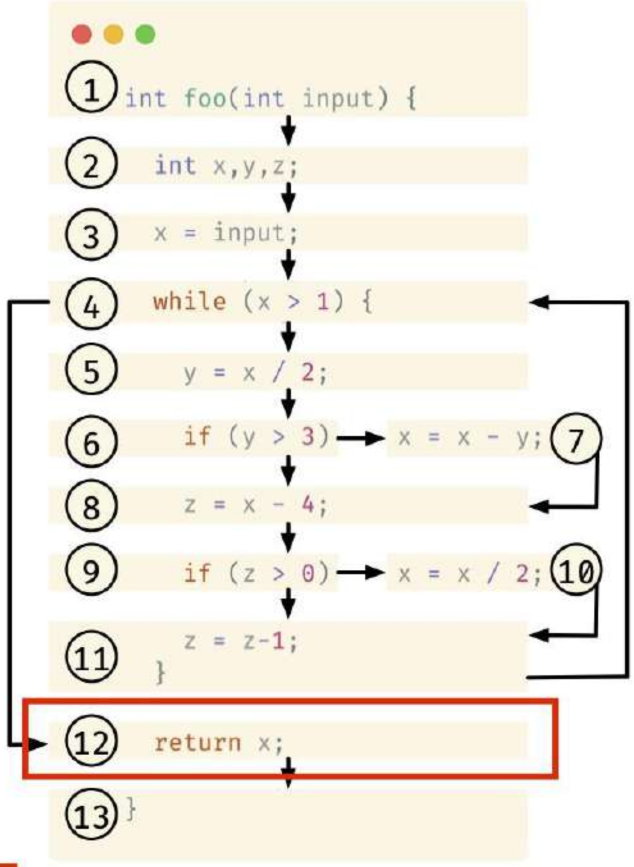


$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

$$\text{live}[n] = use_{var}(n) \cup (\text{candidates}[n] - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n]
13	{}	{}	{}	{}
12	{x}	{}	**{}**	**{x}**
11	{z}	{z}	{}	{}
10	{x}	{x}	{}	{}
9	{z}	{}	{}	{}
8	{x}	{z}	{}	{}
7	{x, y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

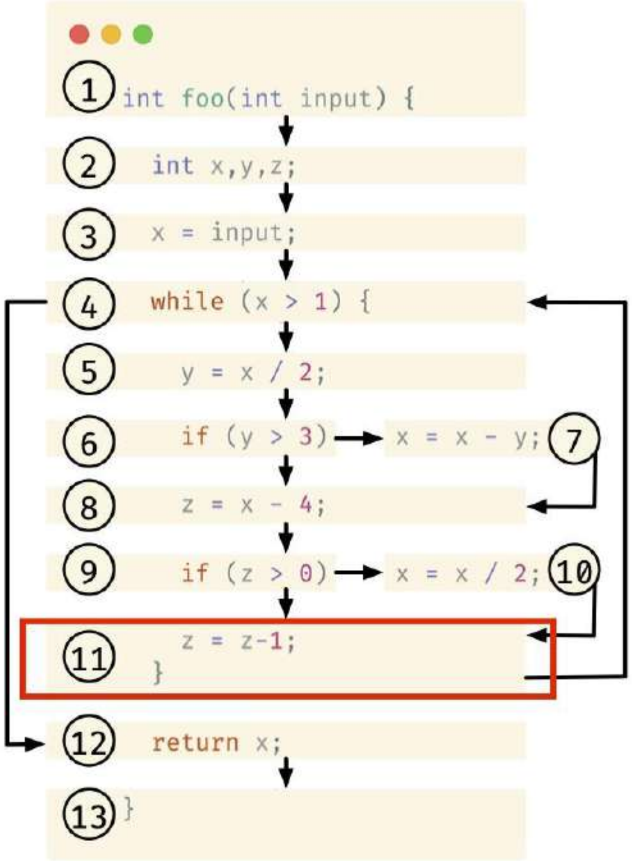


$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s];}$$

$$\texttt{live[n]} = use_{var}(n) \cup (\texttt{candidates[n]} - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{}
10	{x}	{x}	{}	{}
9	{z}	{}	{}	{}
8	{x}	{z}	{}	{}
7	{x, y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}



$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

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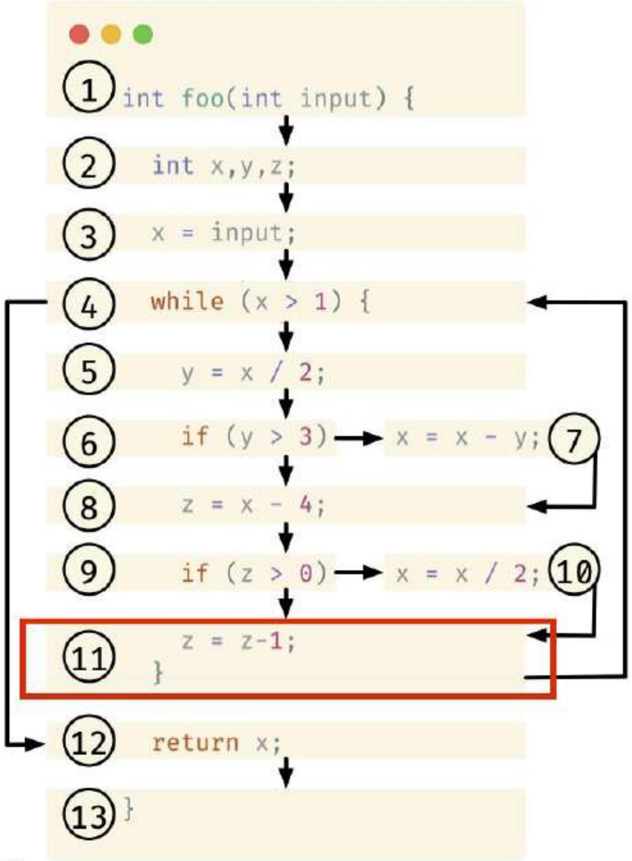
Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	**{}**	**{z}**
10	{x}	{x}	{}	{}
9	{z}	{}	{}	{}
8	{x}	{z}	{}	{}
7	{x,y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}



$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

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Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	**{z}**	{}
9	{z}	{}	{}	{}
8	{x}	{z}	{}	{}
7	{x, y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s];}$$

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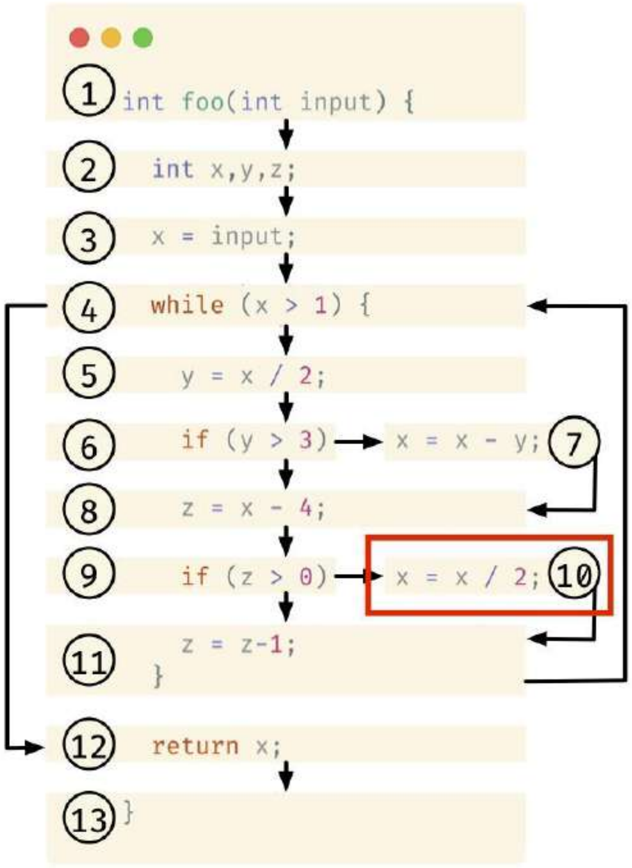

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	1st candidate[n]	live[n]
13	{ }	{ }	{ }	{ }
12	{x}	{ }	{ }	{x}
11	{z}	{z}	{ }	{z}
10	{x}	{x}	**{z}**	**{x,z}**
9	{z}	{ }	{ }	{ }
8	{x}	{z}	{ }	{ }
7	{x,y}	{x}	{ }	{ }
6	{y}	{ }	{ }	{ }
5	{x}	{y}	{ }	{ }
4	{x}	{ }	{ }	{ }
3	{ }	{x}	{ }	{ }
2	{ }	{ }	{ }	{ }
1	{ }	{ }	{ }	{ }

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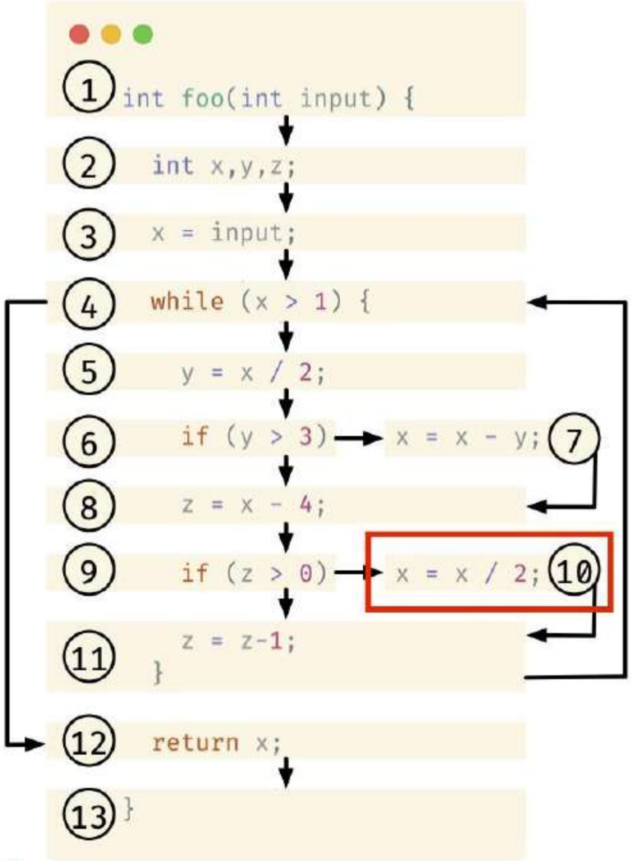

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	1st candidate[n]	live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	**{x,z}**	{}
8	{x}	{z}	{}	{}
7	{x,y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

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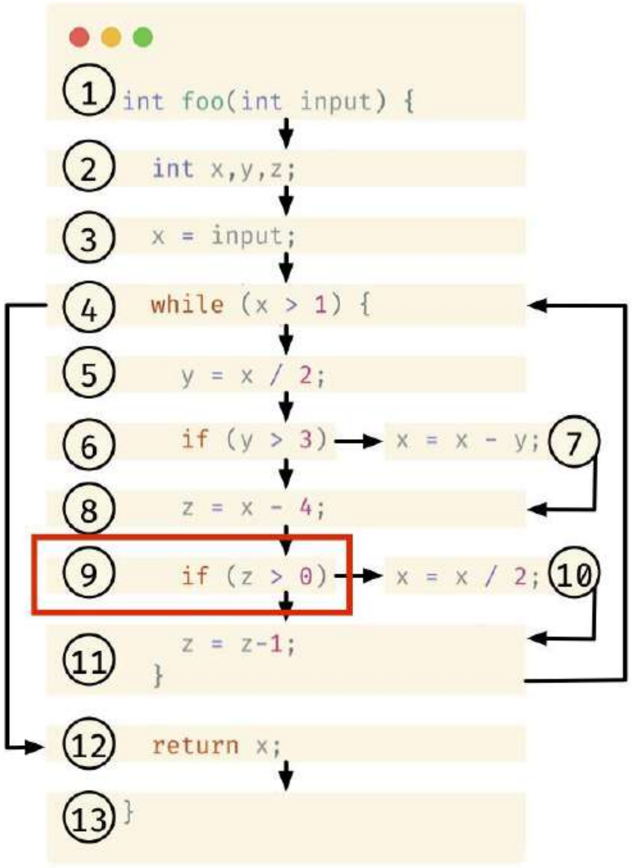
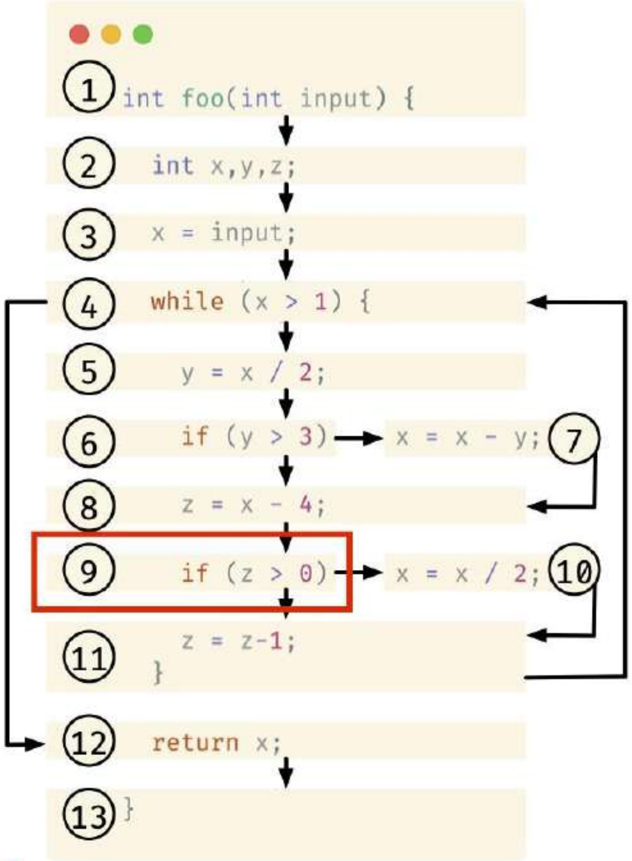


Liveness Analysis Example

1st

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n]	live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	**{x,z}**	**{x,z}**
8	{x}	{z}	{}	{}
7	{x,y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}



$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s]};$$

$$\texttt{live[n]} = use_{var}(n) \cup (\texttt{candidates[n]} - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x, z}
9	{z}	{}	{x, z}	{x, z}
8	{x}	{z}	**{x, z}**	{}
7	{x, y}	{x}	{}	{}
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

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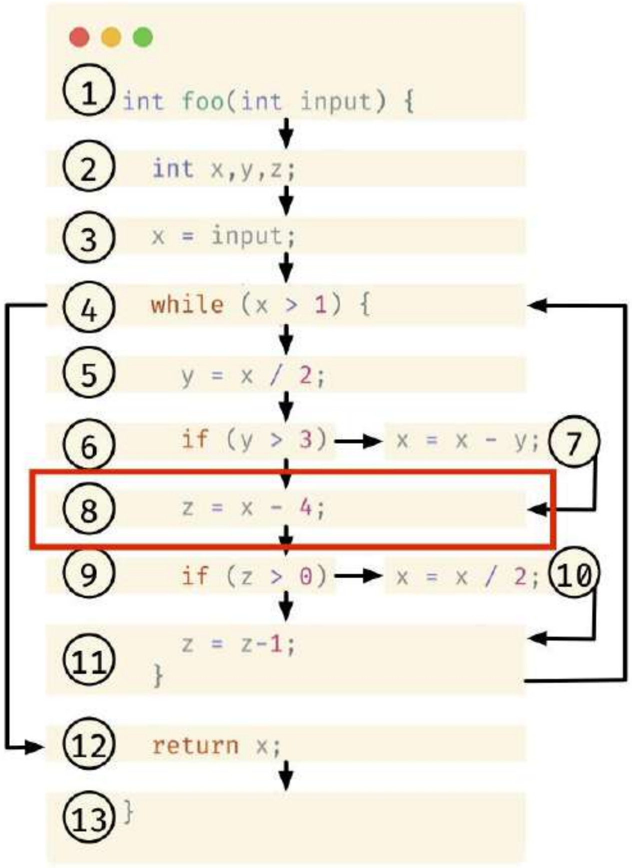
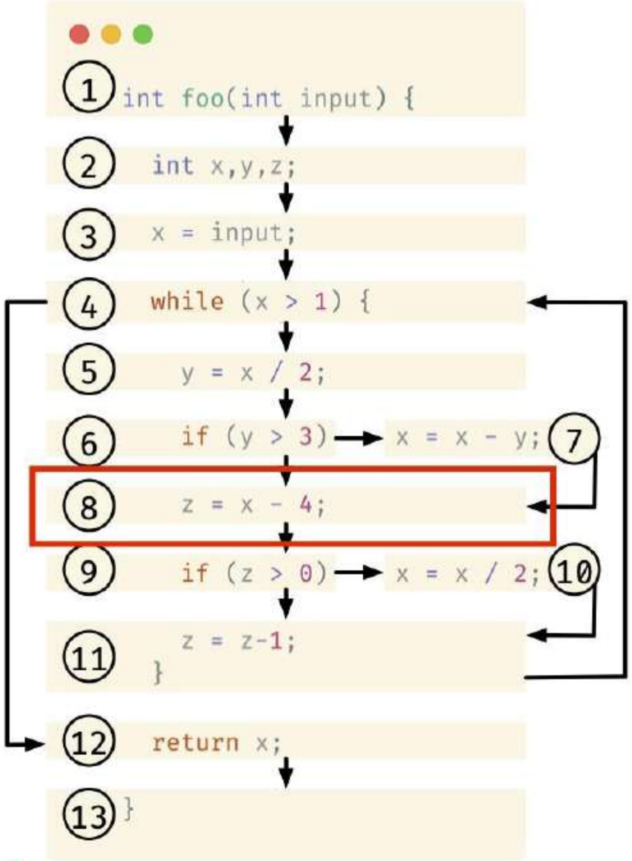


Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n]
13	{ }	{ }	{ }	{ }
12	{x}	{ }	{ }	{x}
11	{z}	{z}	{ }	{z}
10	{x}	{x}	{z}	{x, z}
9	{z}	{ }	{x, z}	{x, z}
8	{x}	{z}	**{x, z}**	**{x}**
7	{x, y}	{x}	{ }	{ }
6	{y}	{ }	{ }	{ }
5	{x}	{y}	{ }	{ }
4	{x}	{ }	{ }	{ }
3	{ }	{x}	{ }	{ }
2	{ }	{ }	{ }	{ }
1	{ }	{ }	{ }	{ }



$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

$$\text{live}[n] = use_{var}(n) \cup (\text{candidates}[n] - def_{var}(n));$$

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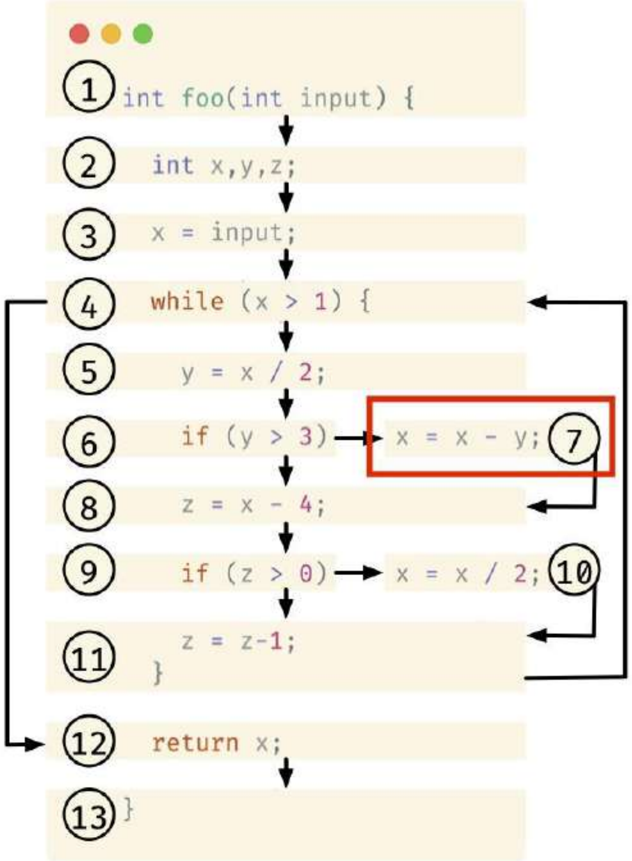
Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}
7	{x,y}	{x}	**{x}**	**{x,y}**
6	{y}	{}	{}	{}
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

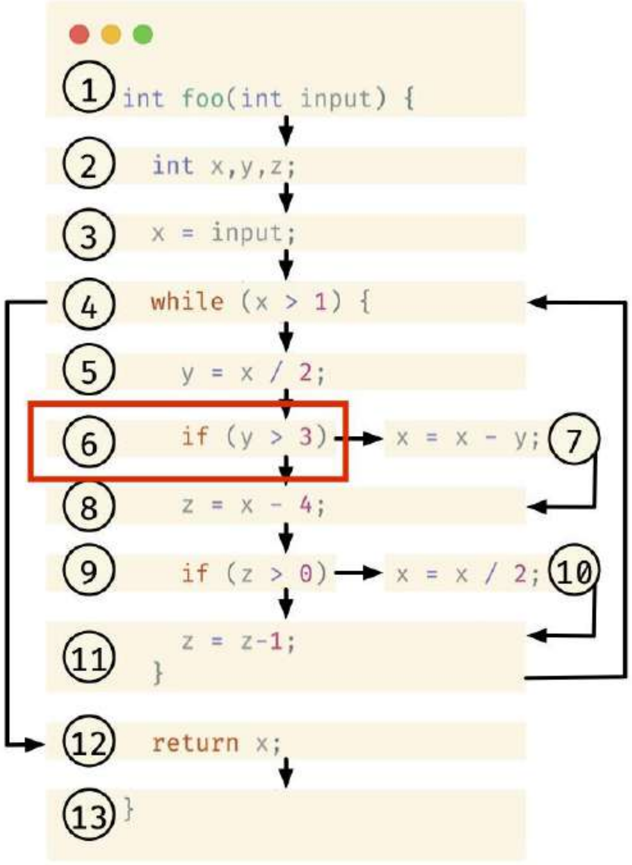


$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

$$\text{live}[n] = use_{var}(n) \cup (\text{candidates}[n] - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}
7	{x,y}	{x}	{x}	{x,y}
6	{y}	{}	**{x,y}**	**{x,y}**
5	{x}	{y}	{}	{}
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}



$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

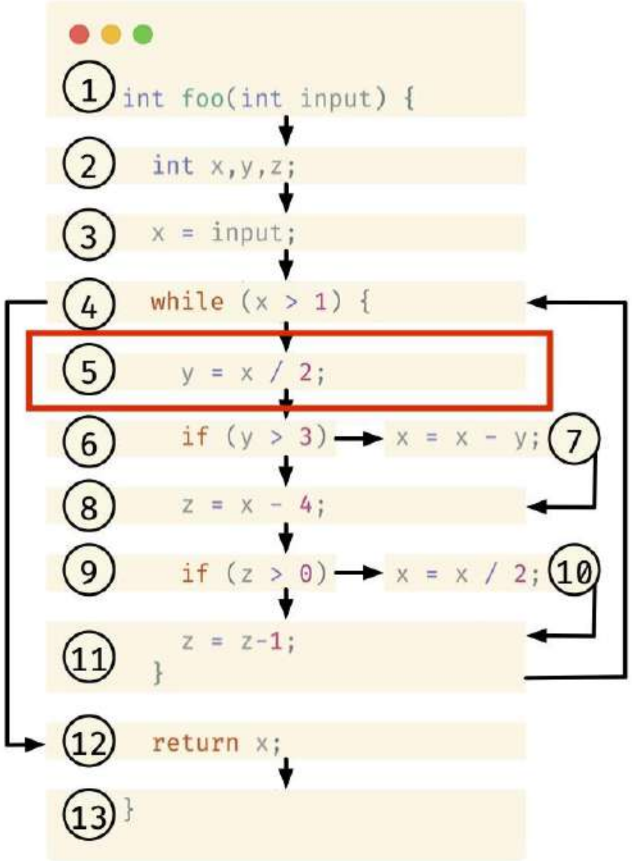
$$\text{live}[n] = use_{var}(n) \bigcup (\text{candidates}[n] - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	1st candidate[n]	1st live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}
7	{x,y}	{x}	{x}	{x,y}
6	{y}	{}	{x,y}	{x,y}
5	{x}	{y}	**{x,y}**	**{x}**
4	{x}	{}	{}	{}
3	{}	{x}	{}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

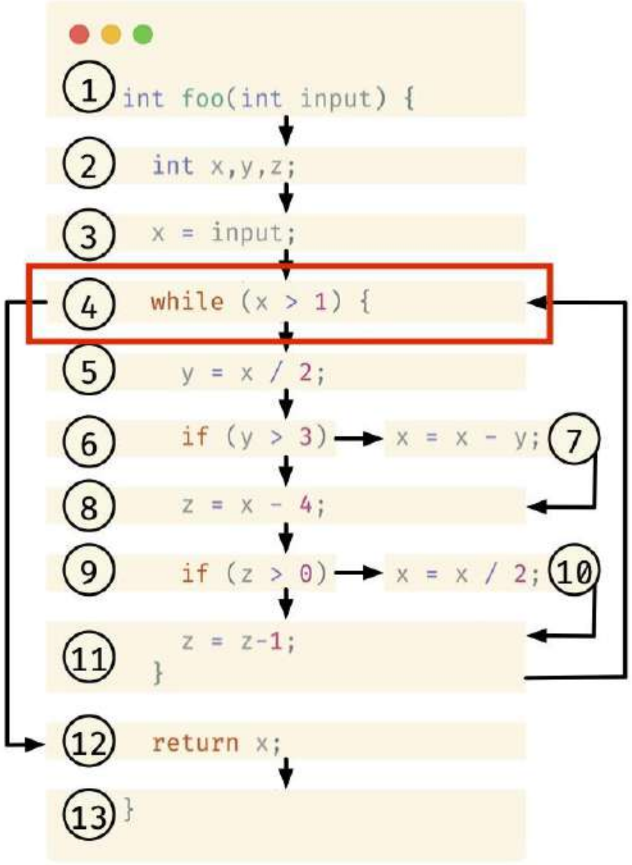
$$\texttt{candidates[n]} = \bigcup_{s \in succ(n)} \texttt{live[s]};$$

$$\texttt{live[n]} = use_{var}(n) \cup (\texttt{candidates[n]} - def_{var}(n));$$



Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n] (1st)
13	{ }	{ }	{ }	{ }
12	{x}	{ }	{ }	{x}
11	{z}	{z}	{ }	{z}
10	{x}	{x}	{z}	{x, z}
9	{z}	{ }	{x, z}	{x, z}
8	{x}	{z}	{x, z}	{x}
7	{x, y}	{x}	{x}	{x, y}
6	{y}	{ }	{x, y}	{x, y}
5	{x}	{y}	{x, y}	{x}
4	{x}	{ }	**{x}**	**{x}**
3	{ }	{x}	{ }	{ }
2	{ }	{ }	{ }	{ }
1	{ }	{ }	{ }	{ }

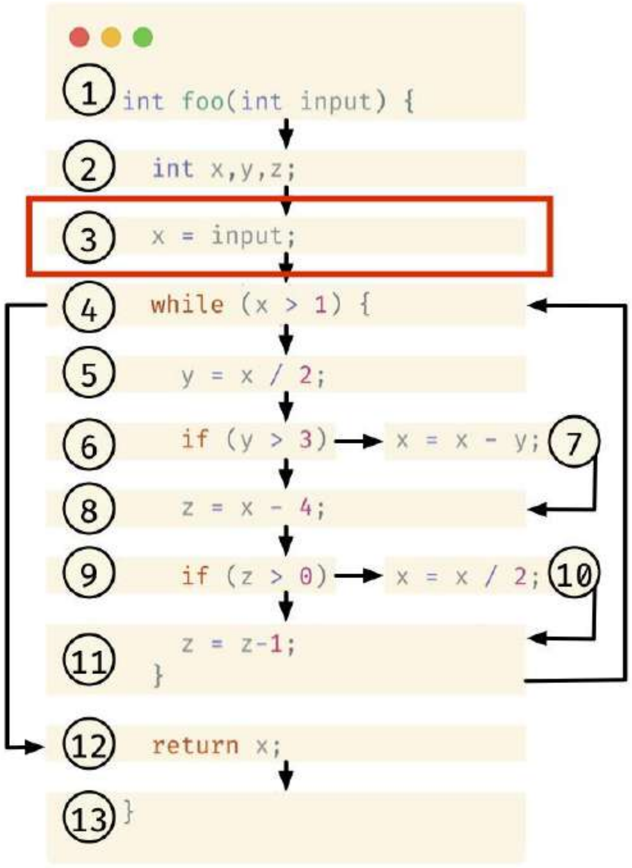


$$\text{candidates}[n] = \bigcup_{s \in succ(n)} \text{live}[s];$$

$$\text{live}[n] = use_{var}(n) \cup (\text{candidates}[n] - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n] (1st)	live[n]
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x,z}
9	{z}	{}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}
7	{x,y}	{x}	{x}	{x,y}
6	{y}	{}	{x,y}	{x,y}
5	{x}	{y}	{x,y}	{x}
4	{x}	{}	{x}	{x}
3	{}	{x}	**{x}**	**{}**
2	{}	{}	{}	{}
1	{}	{}	{}	{}



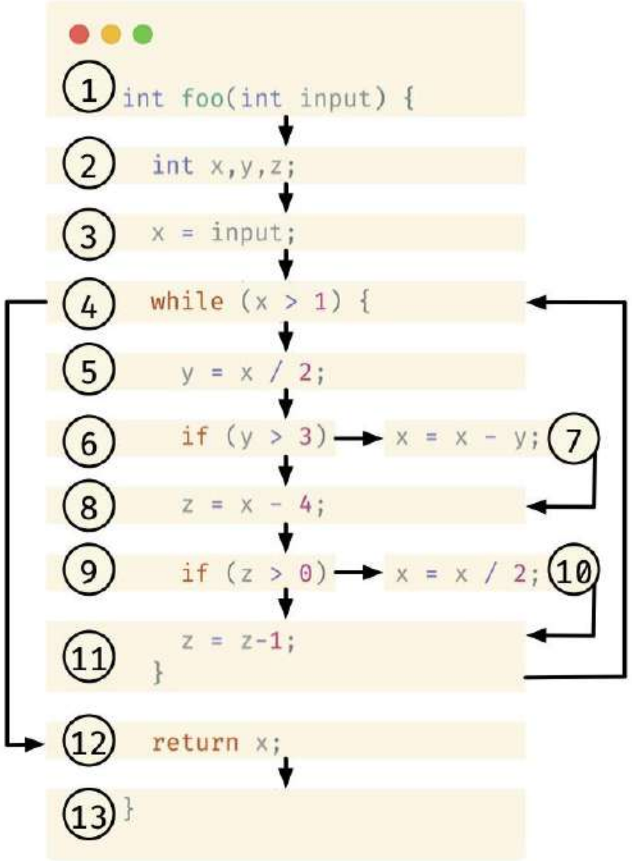
$$candidates[n] = \bigcup_{s \in succ(n)} live[s];$$

$$live[n] = use_{var}(n) \bigcup (candidates[n] - def_{var}(n));$$

Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n]	live[n]
			1st	
13	{}	{}	{}	{}
12	{x}	{}	{}	{x}
11	{z}	{z}	{}	{z}
10	{x}	{x}	{z}	{x, z}
9	{z}	{}	{x, z}	{x, z}
8	{x}	{z}	{x, z}	{x}
7	{x, y}	{x}	{x}	{x, y}
6	{y}	{}	{x, y}	{x, y}
5	{x}	{y}	{x, y}	{x}
4	{x}	{}	{x}	{x}
3	{}	{x}	{x}	{}
2	{}	{}	{}	{}
1	{}	{}	{}	{}

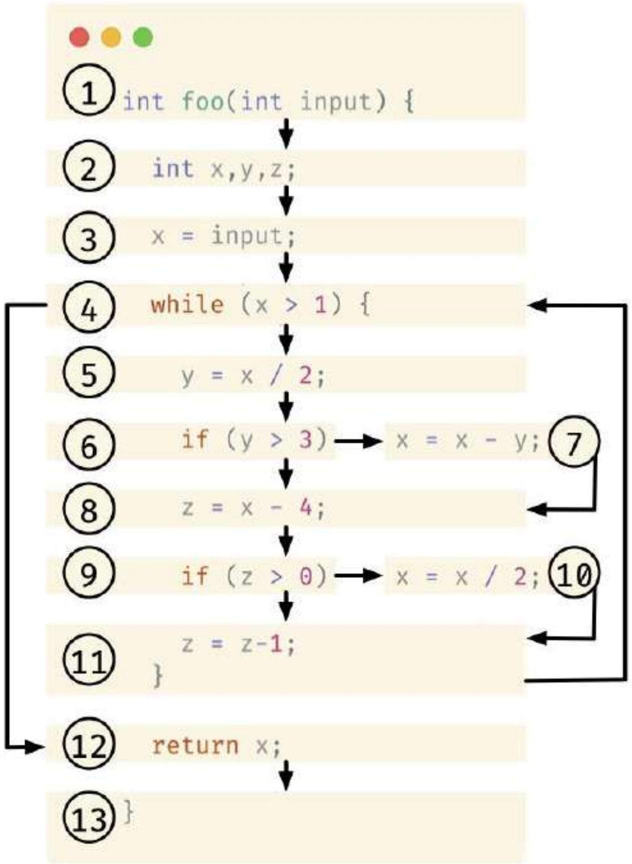
Completed fist iteration



Liveness Analysis Example

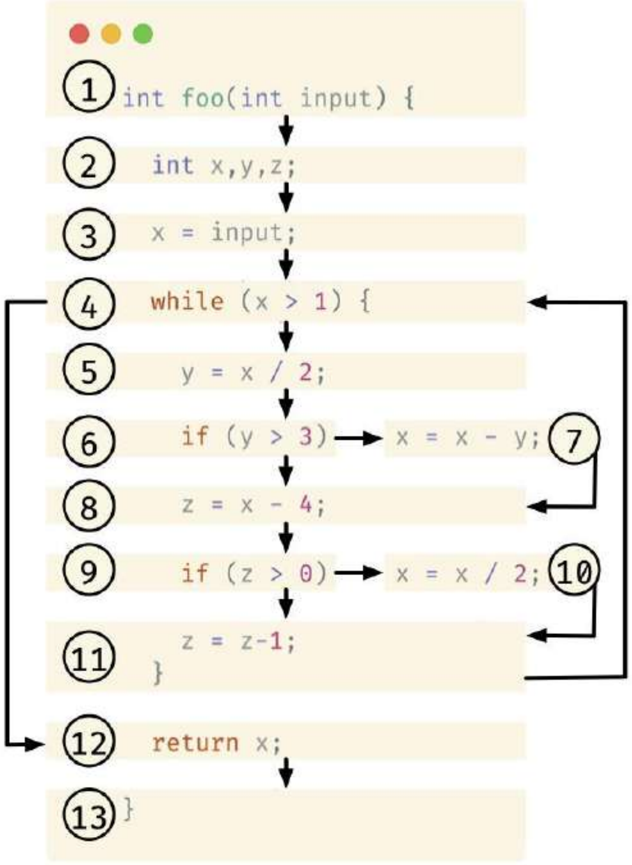
			1st		2nd	
Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n]	live[n]	candidate[n]	live[n]
13	{}	{}	{}	{}	{}	{}
12	{x}	{}	{}	{x}	{}	{x}
11	{z}	{z}	{}	{z}	{x}	{x, z}
10	{x}	{x}	{z}	{x, z}	{x, z}	{x, z}
9	{z}	{}	{x, z}	{x, z}	{x, z}	{x, z}
8	{x}	{z}	{x, z}	{x}	{x, z}	{x}
7	{x, y}	{x}	{x}	{x, y}	{x}	{x, y}
6	{y}	{}	{x, y}	{x, y}	{x, y}	{x, y}
5	{x}	{y}	{x, y}	{x}	{x, y}	{x}
4	{x}	{}	{x}	{x}	{x}	{x}
3	{}	{x}	{x}	{}	{x}	{}
2	{}	{}	{}	{}	{}	{}
1	{}	{}	{}	{}	{}	{}

Completed second iteration



Liveness Analysis Example

Node n	$use_{var}(n)$	$def_{var}(n)$	1st candidate[n]	1st live[n]	2nd candidate[n]	2nd live[n]	3rd candidate[n]	3rd live[n]
13	{}	{}	{}	{}	{}	{}	{}	{}
12	{x}	{}	{}	{x}	{}	{x}	{}	{x}
11	{z}	{z}	{}	{z}	{x}	{x,z}	{x}	{x,z}
10	{x}	{x}	{z}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}
9	{z}	{}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}	{x,z}	{x}	{x,z}	{x}
7	{x,y}	{x}	{x}	{x,y}	{x}	{x,y}	{x}	{x,y}
6	{y}	{}	{x,y}	{x,y}	{x,y}	{x,y}	{x,y}	{x,y}
5	{x}	{y}	{x,y}	{x}	{x,y}	{x}	{x,y}	{x}
4	{x}	{}	{x}	{x}	{x}	{x}	{x}	{x}
3	{}	{x}	{x}	{}	{x}	{}	{x}	{}
2	{}	{}	{}	{}	{}	{}	{}	{}
1	{}	{}	{}	{}	{}	{}	{}	{}



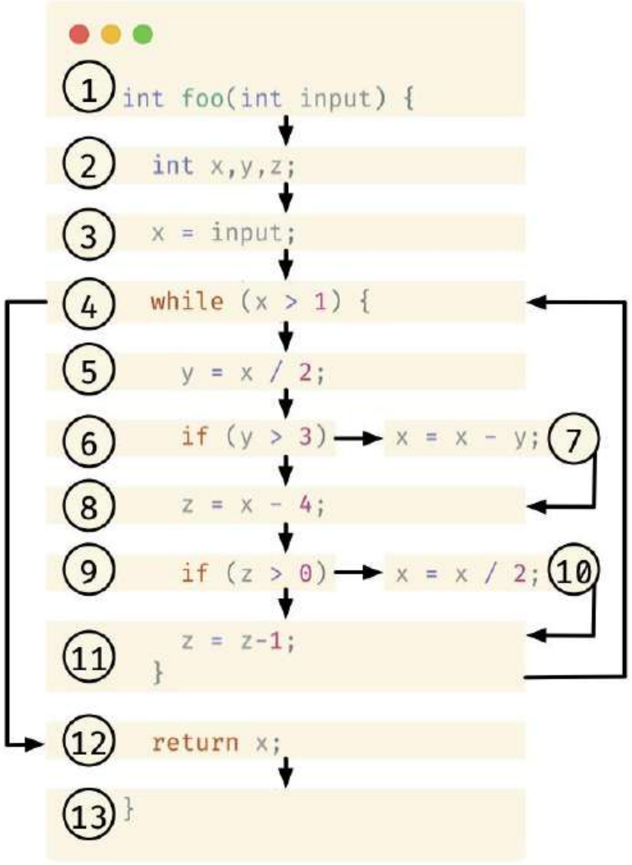
No changes: fixpoint reached

Liveness Analysis Example

			1st		2nd		3rd	
Node n	$use_{var}(n)$	$def_{var}(n)$	candidate[n]	live[n]	candidate[n]	live[n]	candidate[n]	live[n]
13	{}	{}	{}	{}	{}	{}	{}	{}
12	{x}	{}	{}	{x}	{}	{x}	{}	{x}
11	{z}	{z}	{}	{z}	{x}	{x,z}	{x}	{x,z}
10	{x}	{x}	{z}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}
9	{z}	{}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}	{x,z}
8	{x}	{z}	{x,z}	{x}	{x,z}	{x}	{x,z}	{x}
7	{x,y}	{x}	{x}	{x,y}	{x}	{x,y}	{x}	{x,y}
6	{y}	{}	{x,y}	{x,y}	{x,y}	{x,y}	{x,y}	{x,y}
5	{x}	{y}	{x,y}	{x}	{x,y}	{x}	{x,y}	{x}
4	{x}	{}	{x}	{x}	{x}	{x}	{x}	{x}
3	{}	{x}	{x}	{}	{x}	{}	{x}	{}
2	{}	{}	{}	{}	{}	{}	{}	{}
1	{}	{}	{}	{}	{}	{}	{}	{}

No changes: fixpoint reached

y and z are never live together



Data flow Analysis *Limitations*

Data flow analysis has some limitations:

- Static analysis may be (very) conservative
- CFG is only a static approximation of the dynamic control flow
- Pointers introduce aliases:
    - E.g. *x = 10; Does x point to another variable, y or z?
      That would give a definition of y or z. May not know at compile time which …
    - Precise alias analysis still an open problem
- Array access; generally cannot tell which indices are used
- Reasoning across function calls …