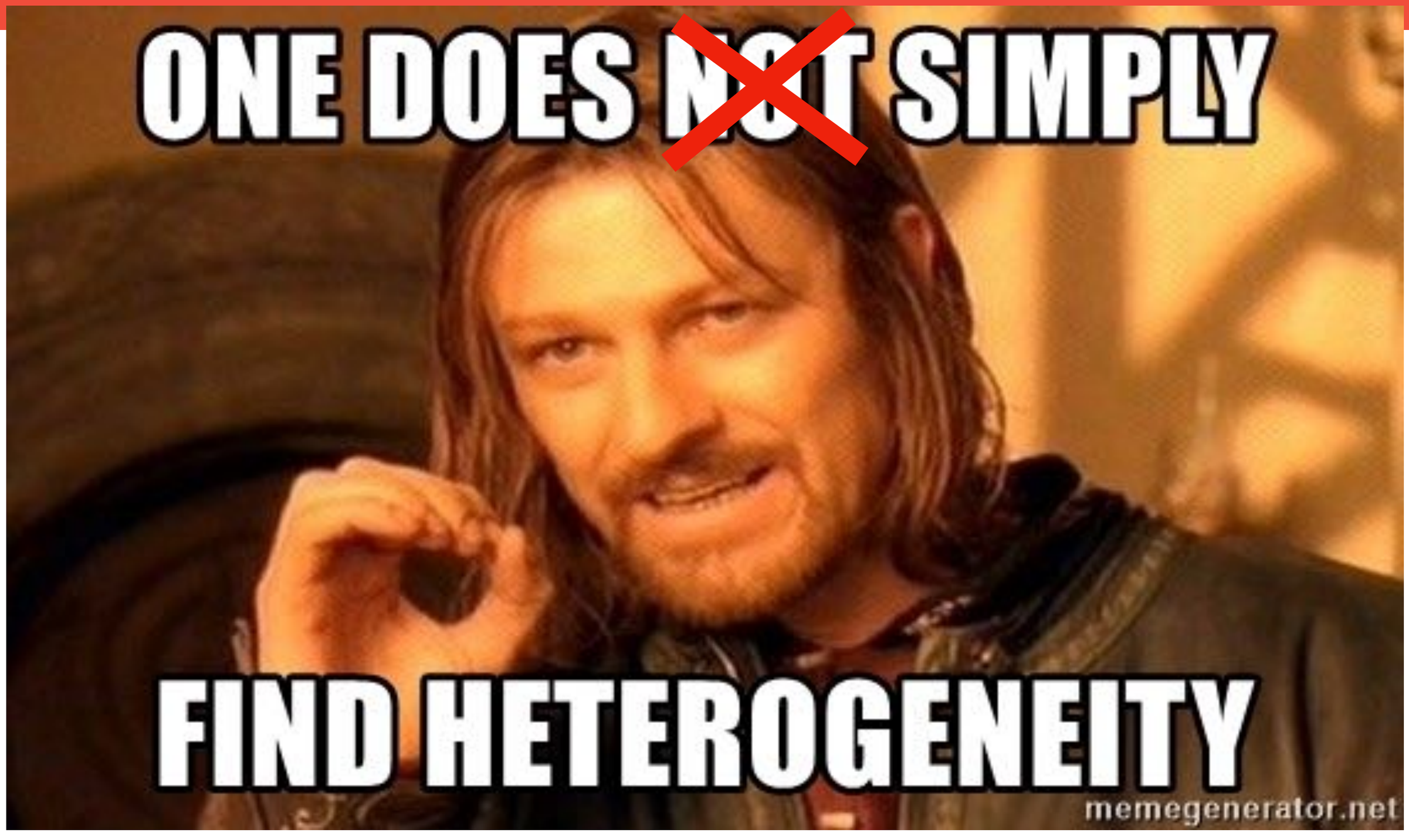
HUBS



LEARNING OUTCOMES

Learn about **network heterogeneity**
Discover how to find **important nodes**
Analyse distributions of degree and centrality

HETEROGENEITY

In real-world networks the importance of nodes is **heterogeneous**

Importance is often measured with centrality

There are **several measures** of centrality

DEGREE CENTRALITY

Trivially, this is the degree of a node

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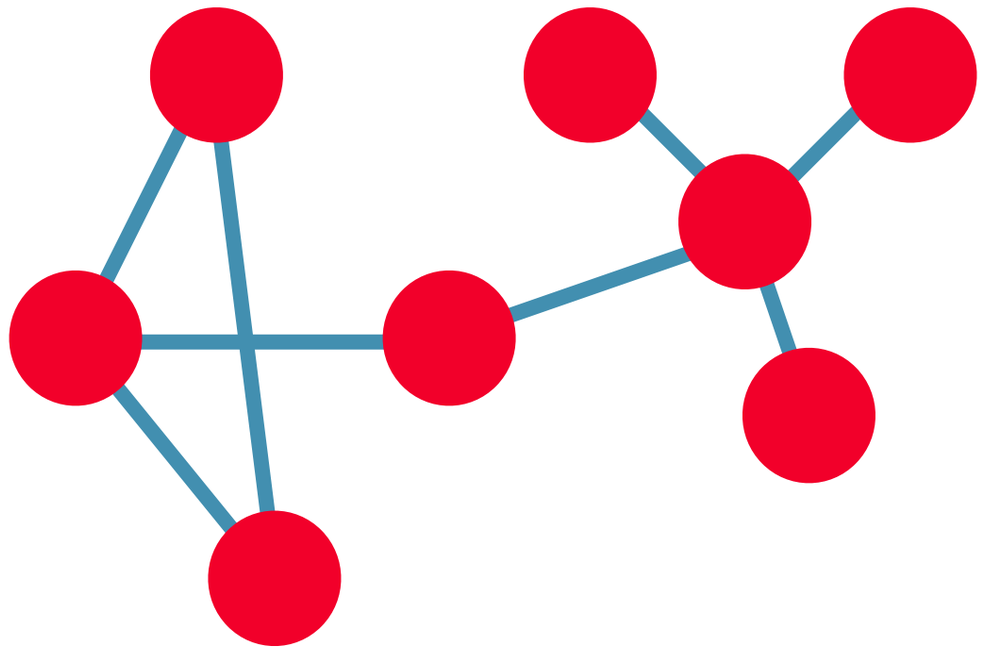

CLOSENESS CENTRALITY

How close a node is to other nodes

$$g_i = \frac{1}{\sum_{i \neq j} \ell_{ij}}$$

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Average distance

BETWEENNESS CENTRALITY

How many shortest paths pass through a node

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

BETWEENNESS CENTRALITY

How many shortest paths pass through a node

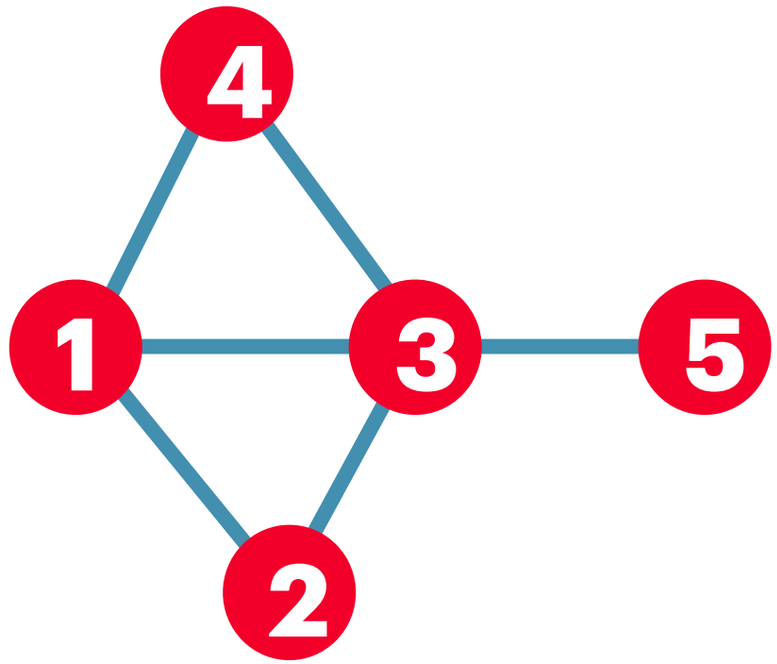
Number of shortest paths between h and j passing through i

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

Number of shortest paths between h and j

EXERCISE

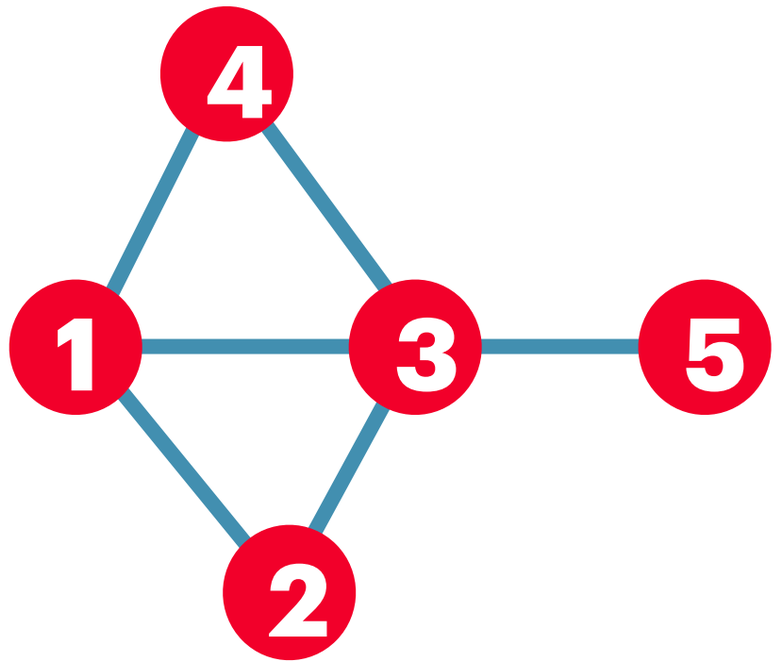
$k_3 = ?$



EXERCISE

$k_3 = 4$

$g_3 = ?$



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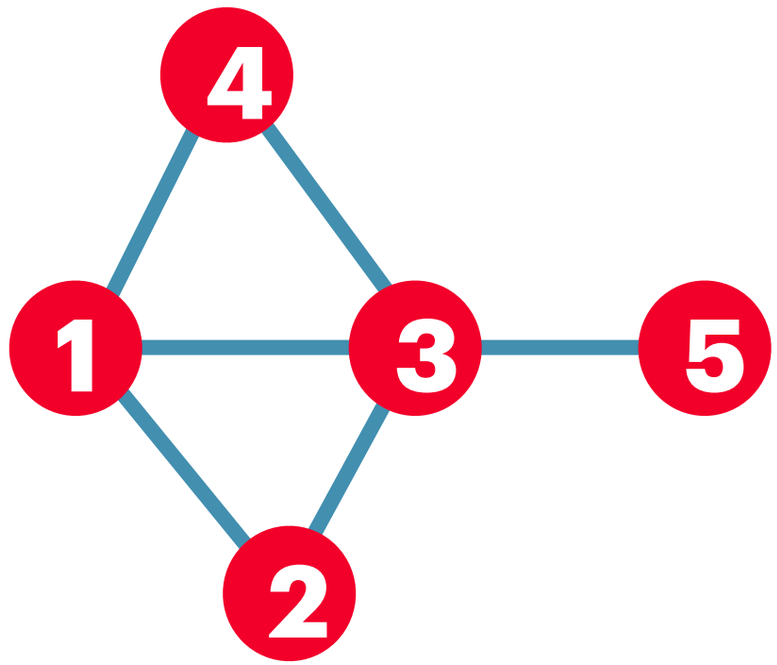


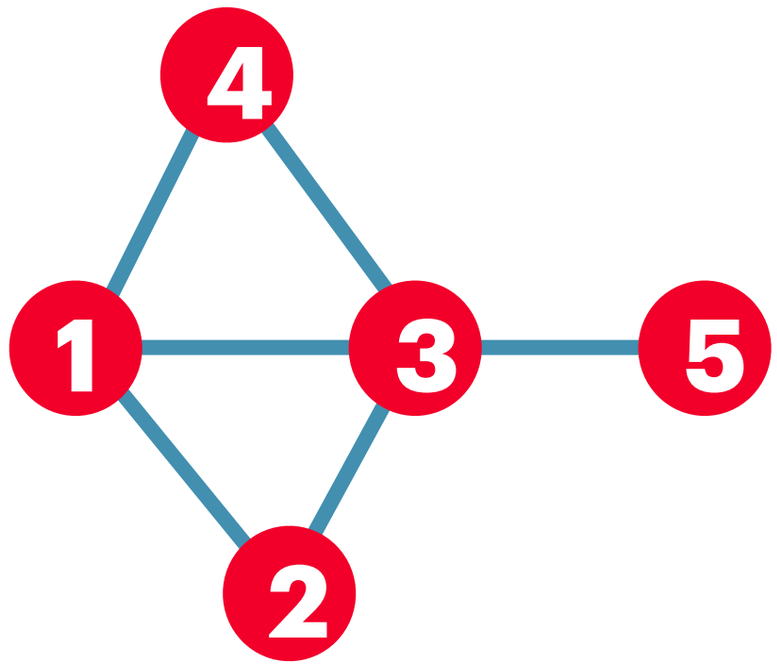
$$g_3 = \frac{1}{\ell_{1,3} + \ell_{2,3} + \ell_{4,3} + \ell_{5,3}} = \frac{1}{1+1+1+1} = \frac{1}{4}$$

EXERCISE

$k_3 = 4$

$g_3 = \frac{1}{4}$

$b_3 = ?$



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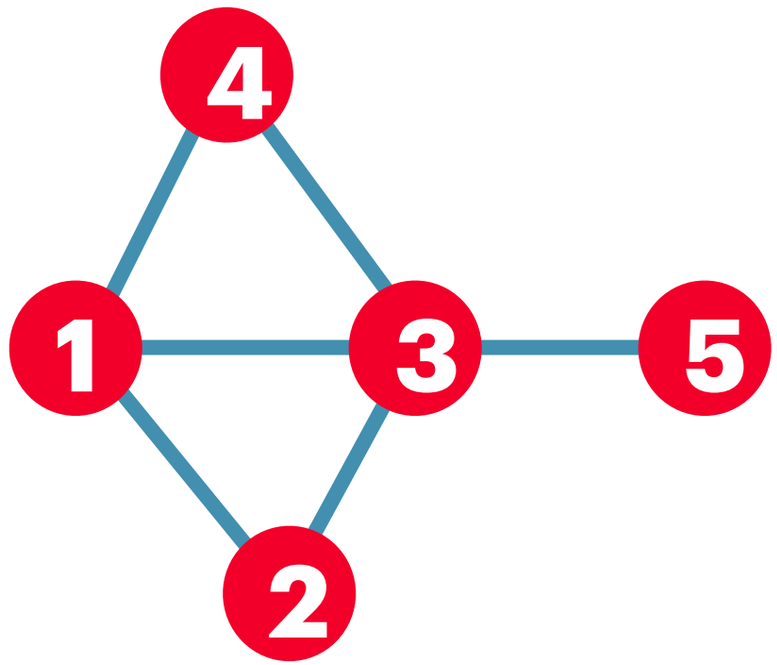

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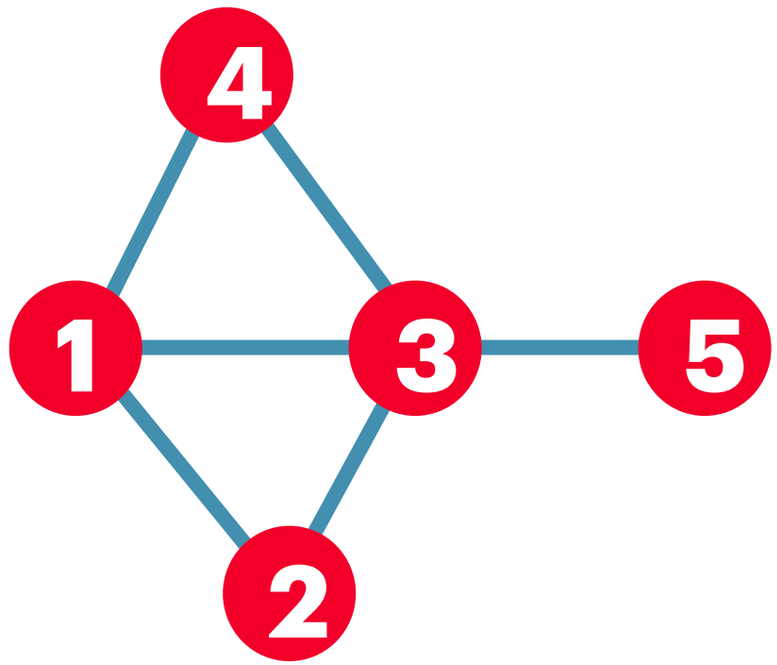

Possible node pairs

1,2 1,3 1,4 1,5
2,3 2,4 2,5
3,4 3,5
4,5

EXERCISE

$$k_3 = 4$$

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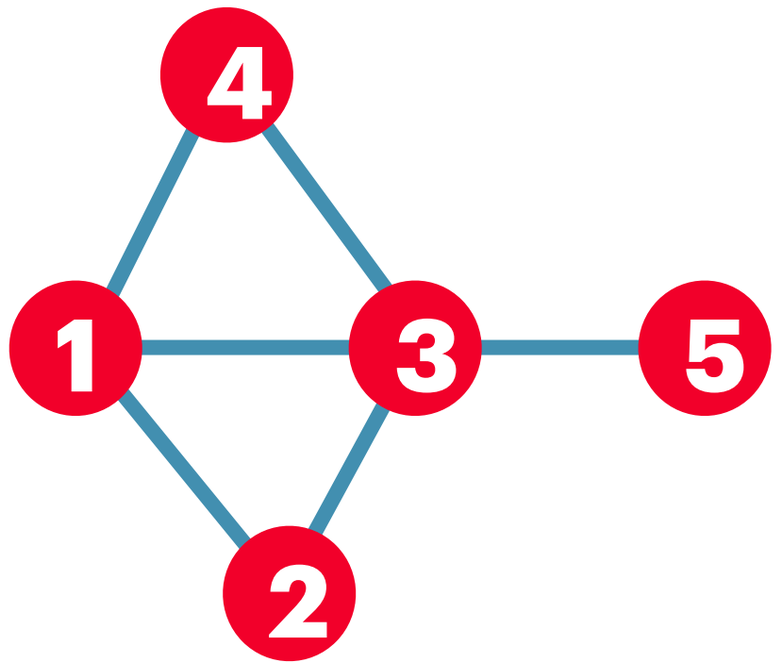

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We need to exclude some pairs

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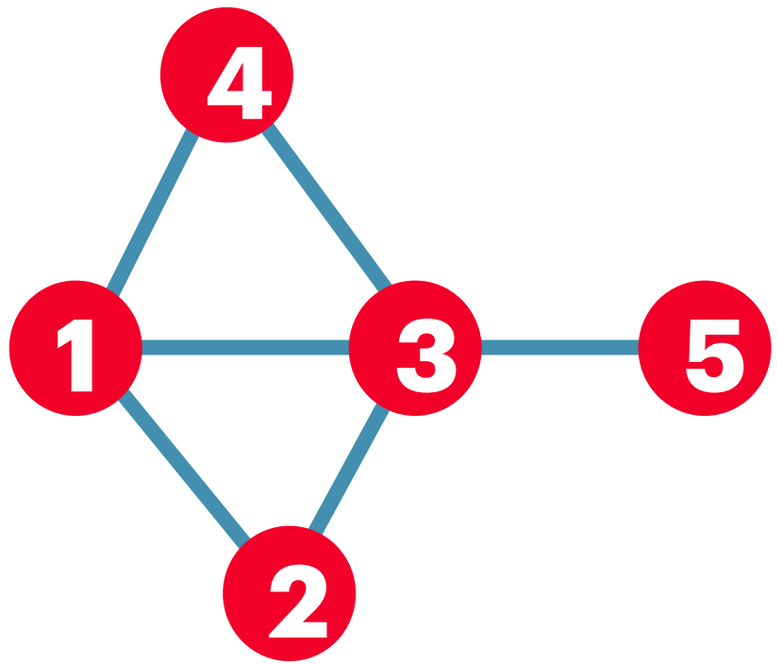

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~~3,4~~ ~~3,5~~

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EXERCISE

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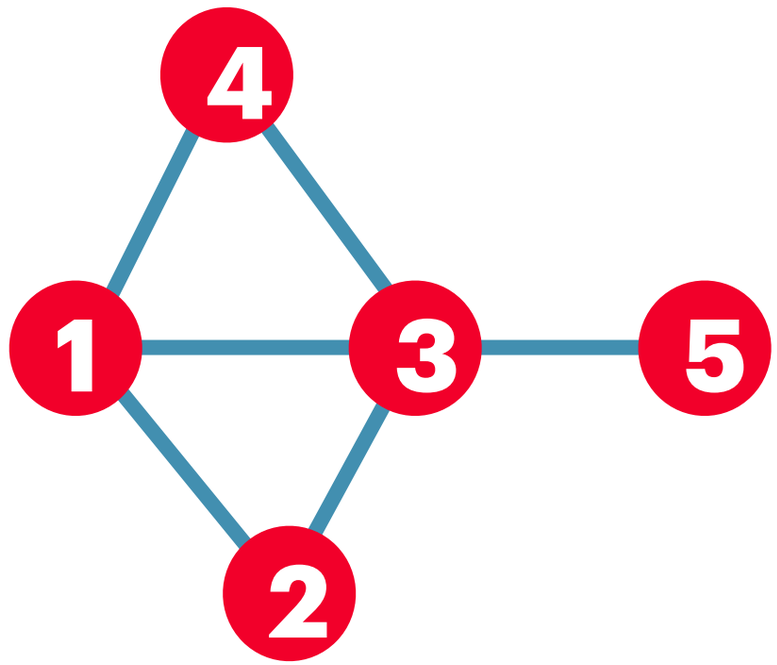

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No s.p. through i = 3

$$b_i = \sum_{h \neq j \neq i} \frac{\sigma_{hj}(i)}{\sigma_{hj}}$$

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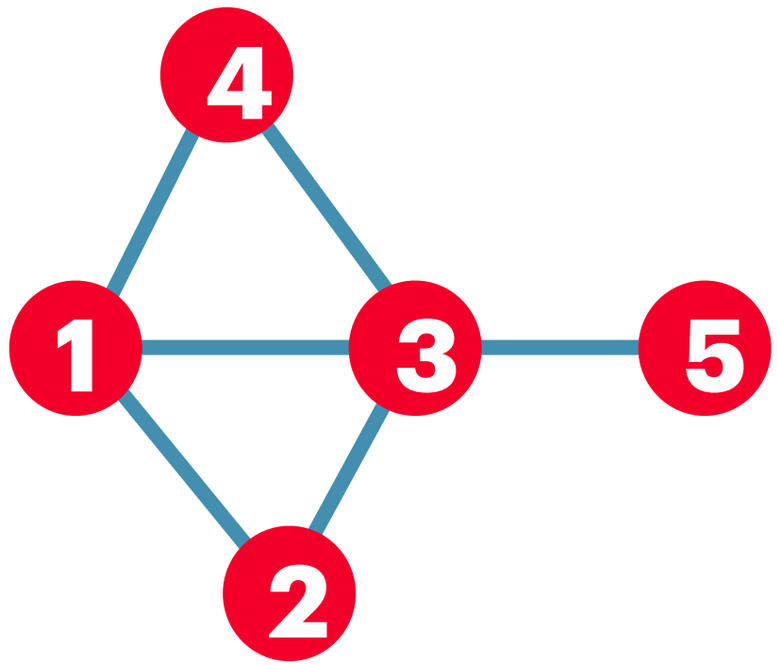

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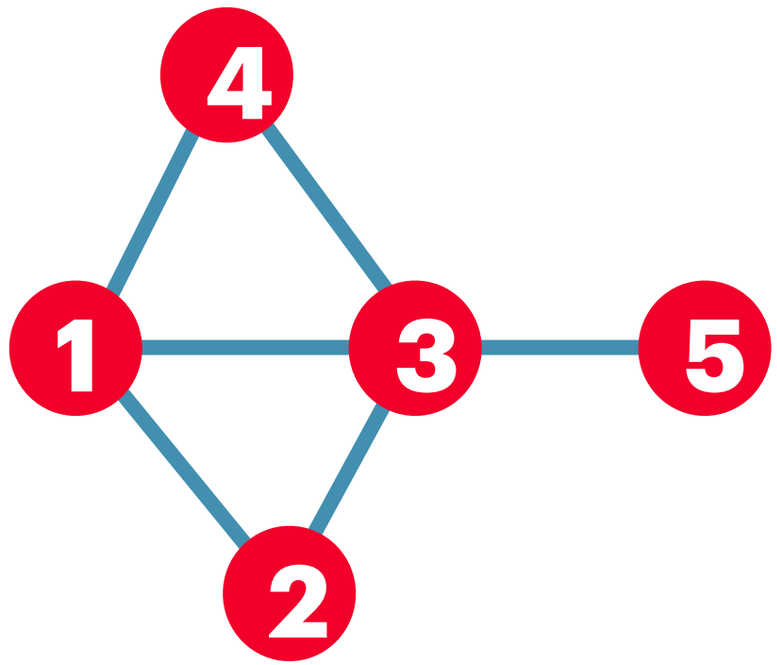

$$b_3 = ?$$

$$b_3 = \frac{\sigma_{1,5}(3)}{\sigma_{1,5}} + \frac{\sigma_{2,4}(3)}{\sigma_{2,4}} + \frac{\sigma_{2,5}(3)}{\sigma_{2,5}} + \frac{\sigma_{4,5}(3)}{\sigma_{4,5}}$$

EXERCISE

$$k_3 = 4$$

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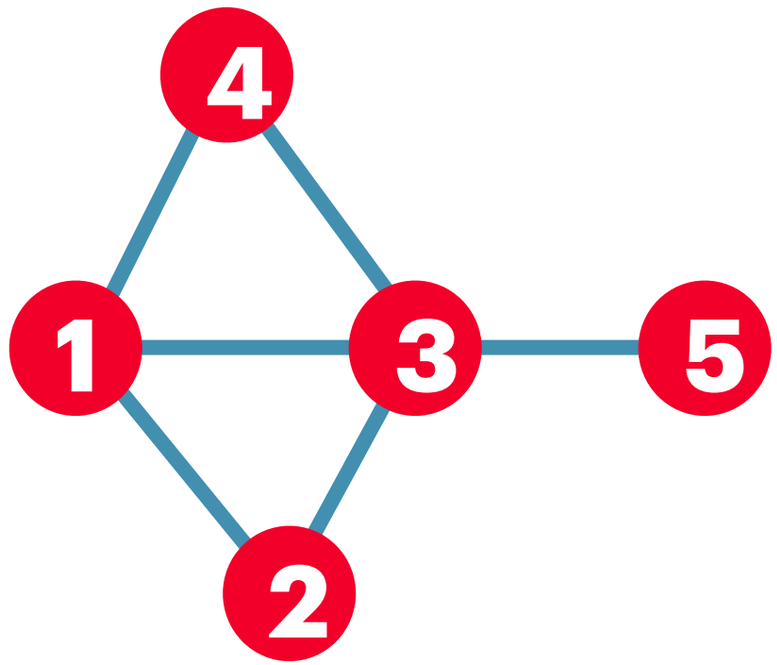
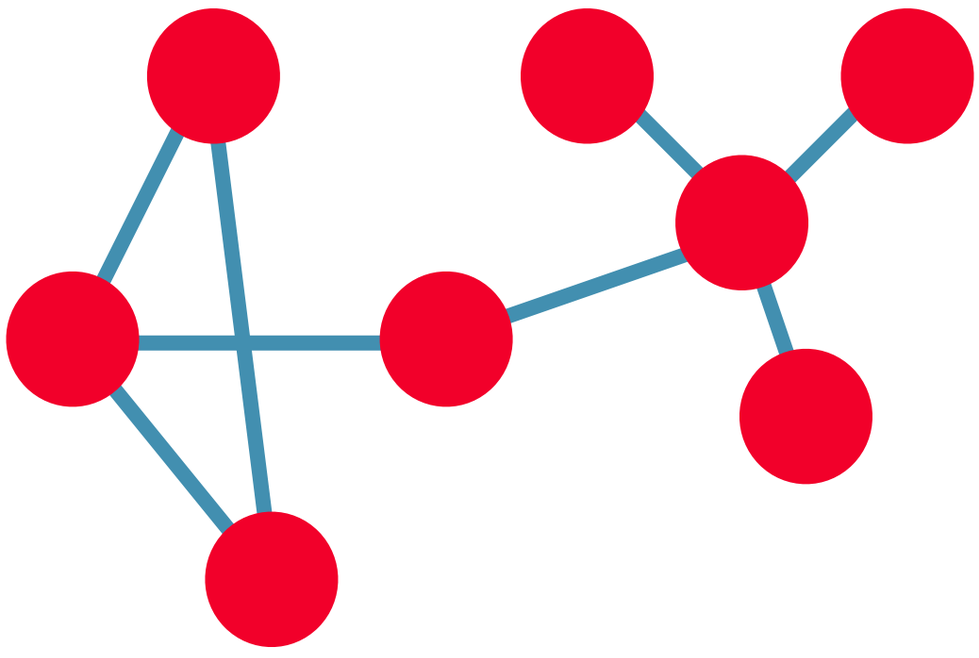


$$b_3 = \frac{\sigma_{1,5}(3)}{\sigma_{1,5}} + \frac{\sigma_{2,4}(3)}{\sigma_{2,4}} + \frac{\sigma_{2,5}(3)}{\sigma_{2,5}} + \frac{\sigma_{4,5}(3)}{\sigma_{4,5}} = 3.5$$

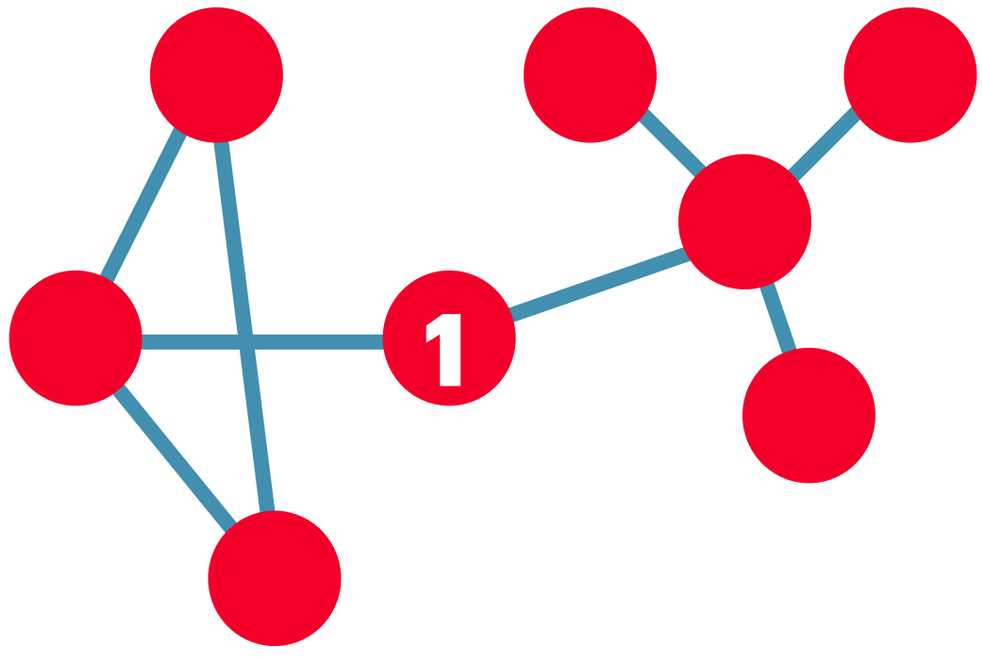
EXERCISE

Node with max betweenness?

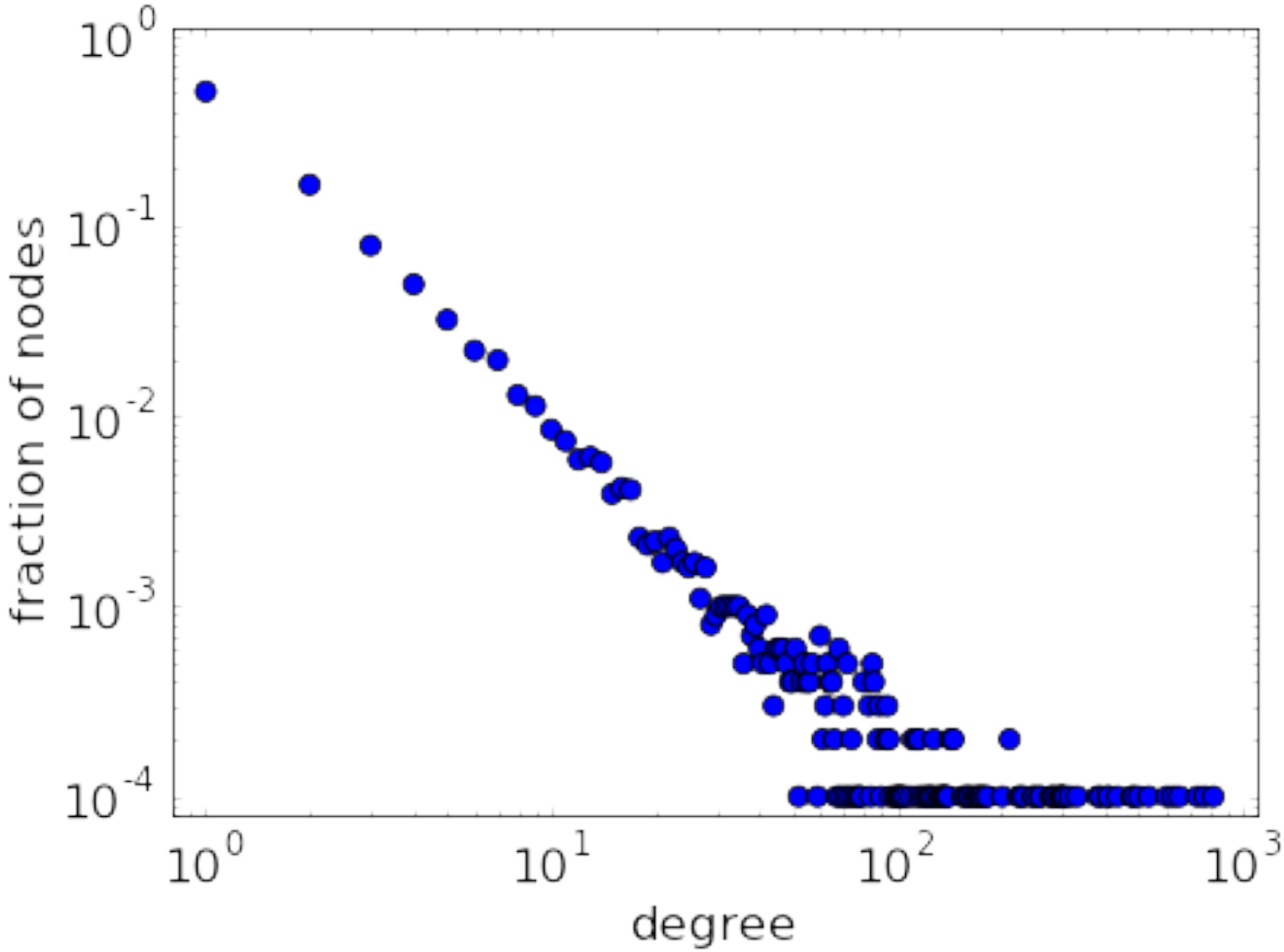


EXERCISE

$b_1 = ?$



DISTRIBUTIONS



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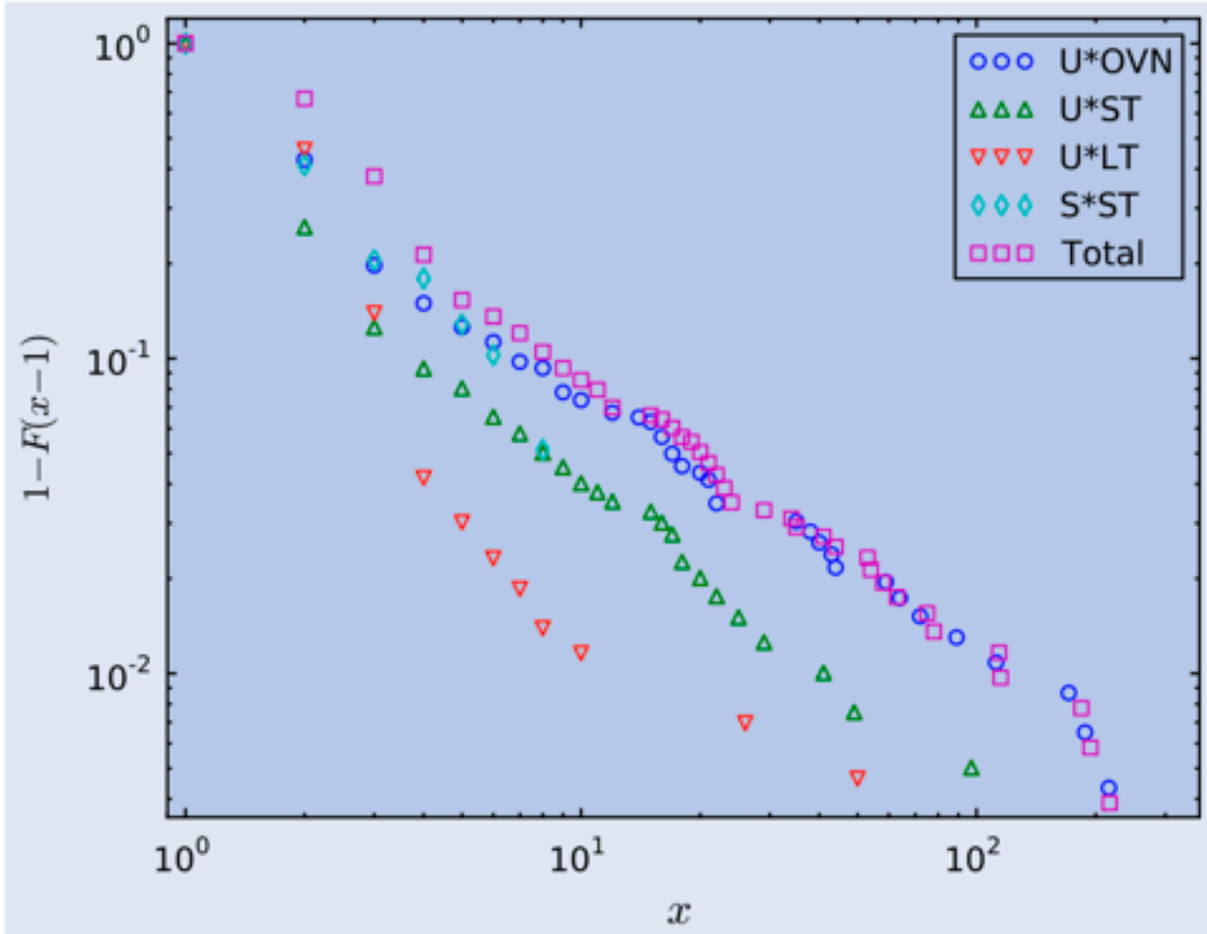
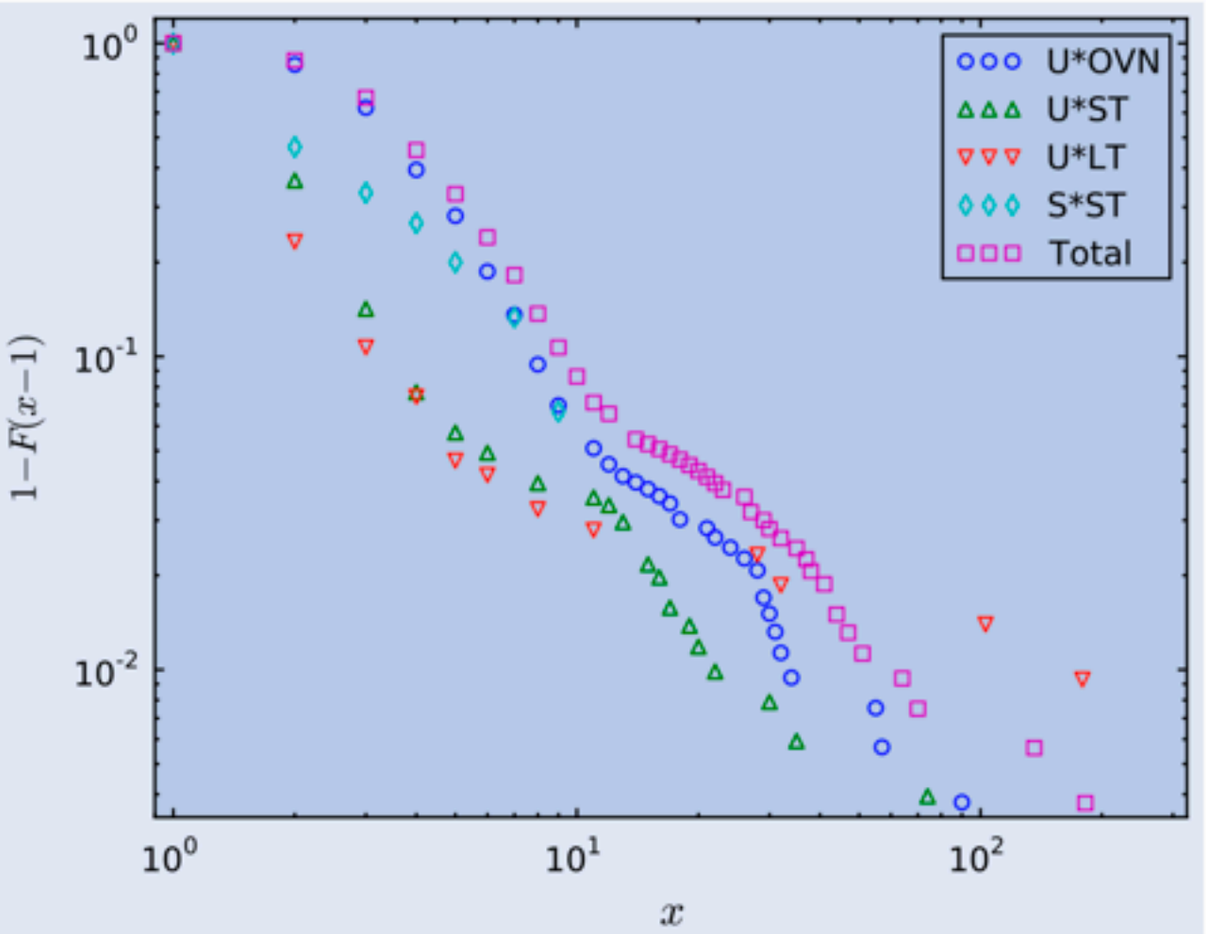


(a) In-degree, 2012



(b) Out-degree, 2012

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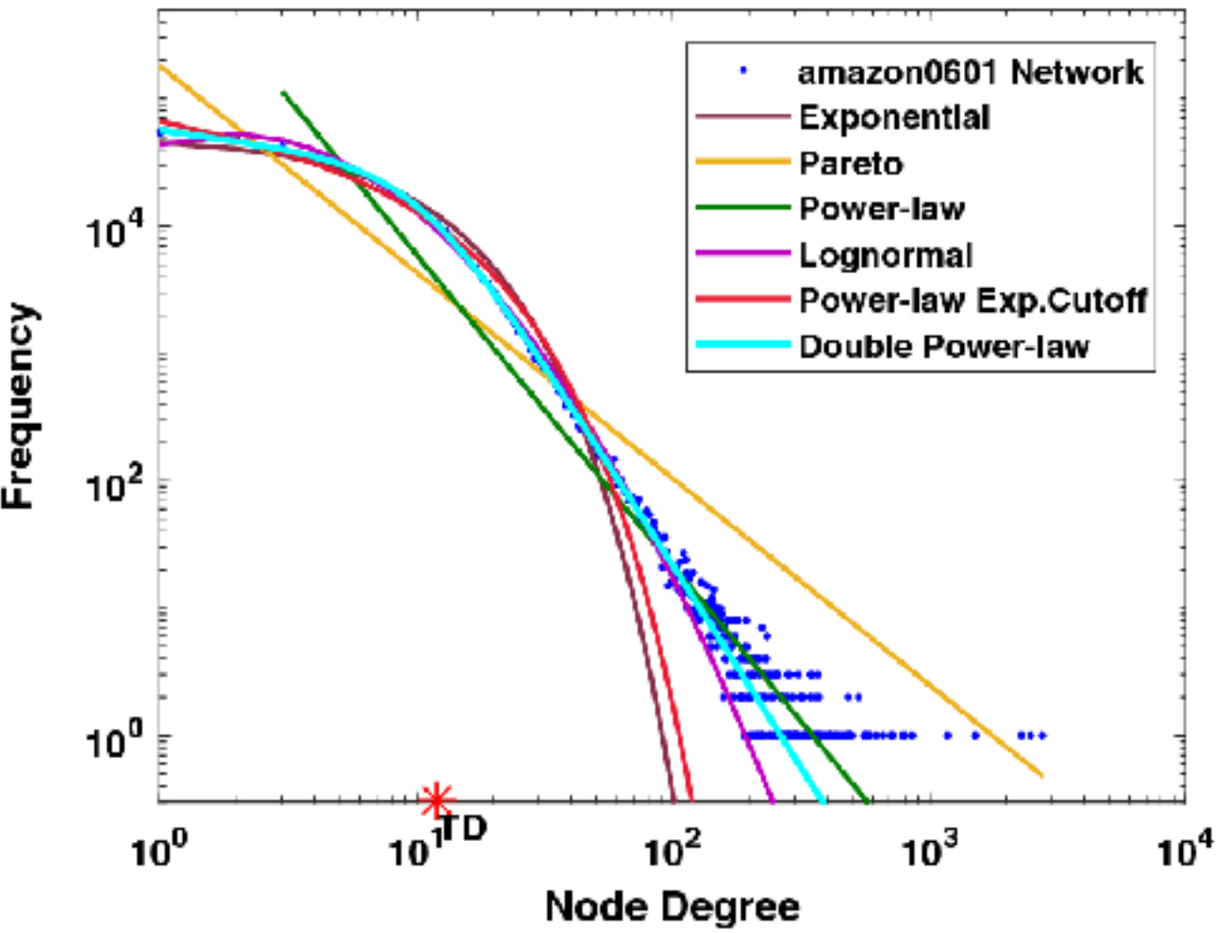

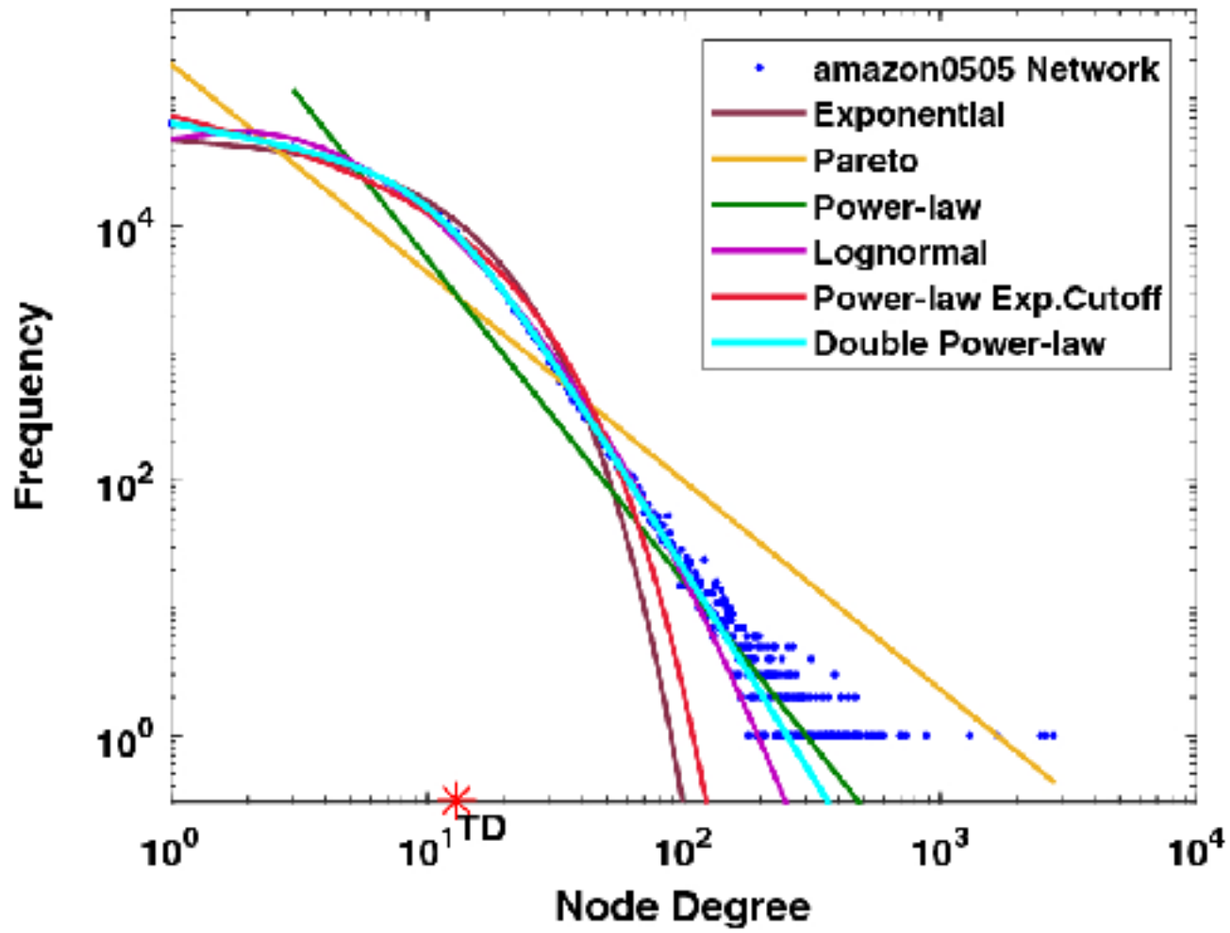

HOW TO MEASURE HETEROGENEITY

$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

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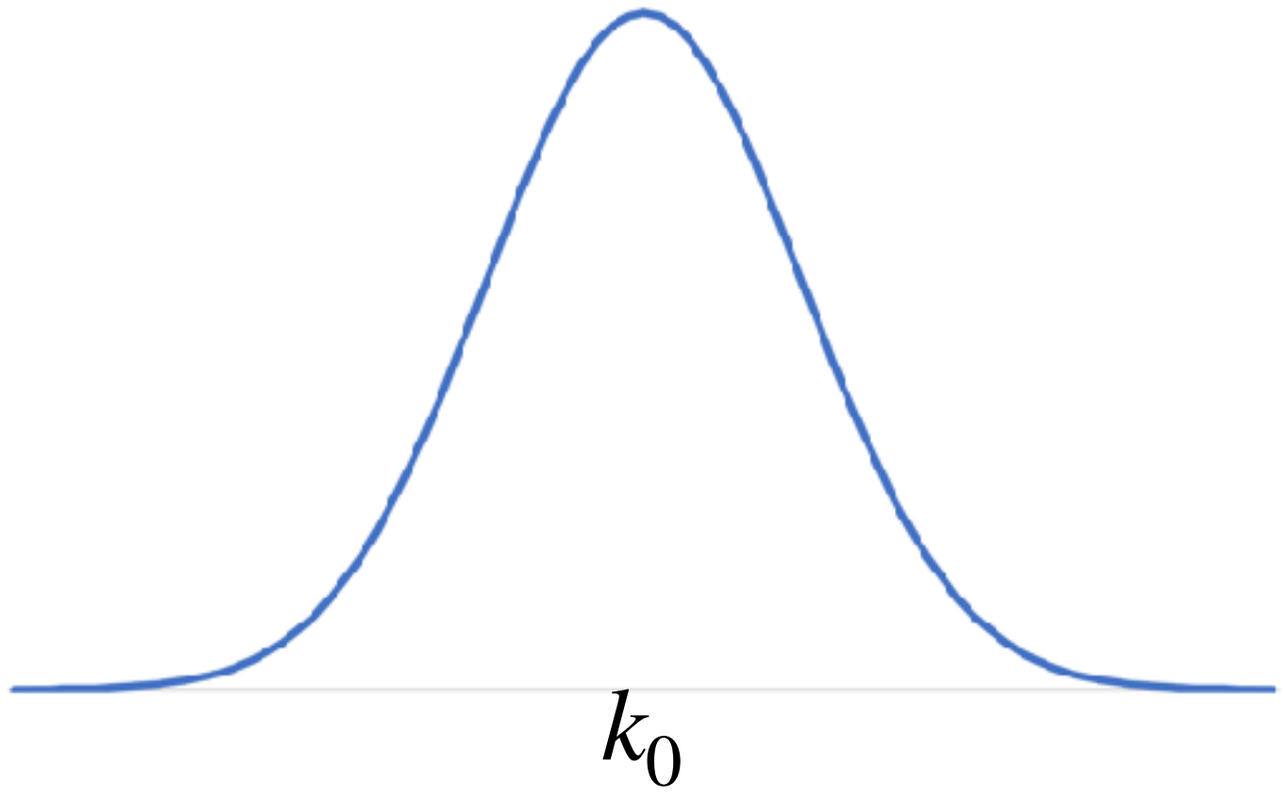
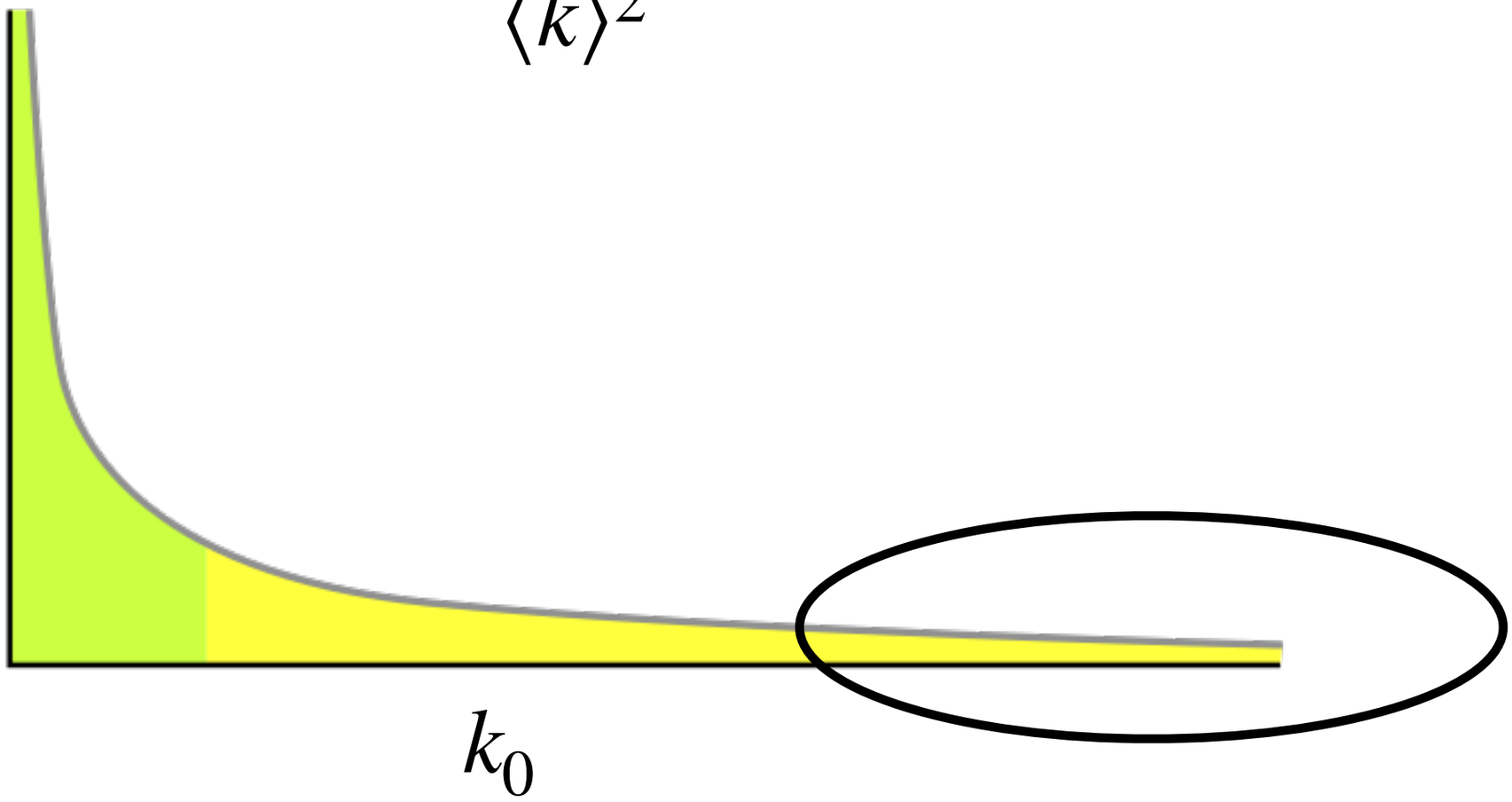


If not heterogeneous $\langle k^2 \rangle \approx \langle k \rangle^2 \approx k_0^2$ $\kappa \approx 1$

HOW TO MEASURE HETEROGENEITY

$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

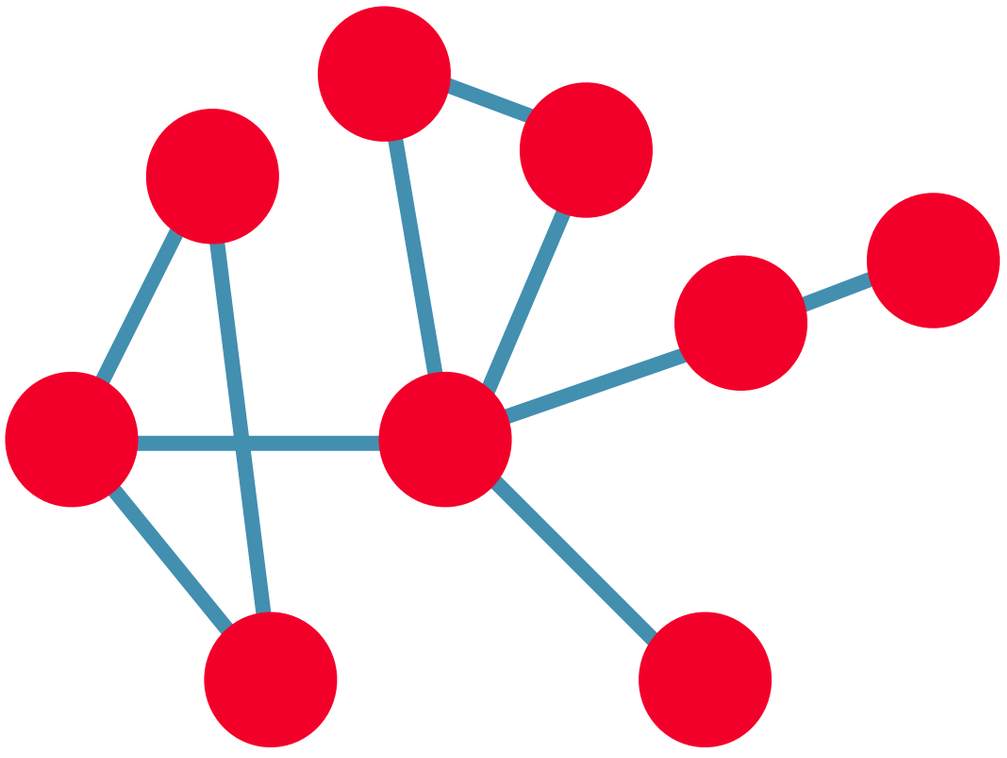


If heterogeneous $\langle k^2 \rangle \gg \langle k \rangle^2$ $\kappa \gg 1$

FRIENDSHIP PARADOX

YOUR FRIENDS HAVE MORE FRIENDS THAN YOU

FRIENDSHIP PARADOX



SUMMARY

Centrality is fundamental to understand the role of nodes

Centrality Distributions represent a great tool to analyse a network

Heterogeneity is a characteristic of **real-world networks**