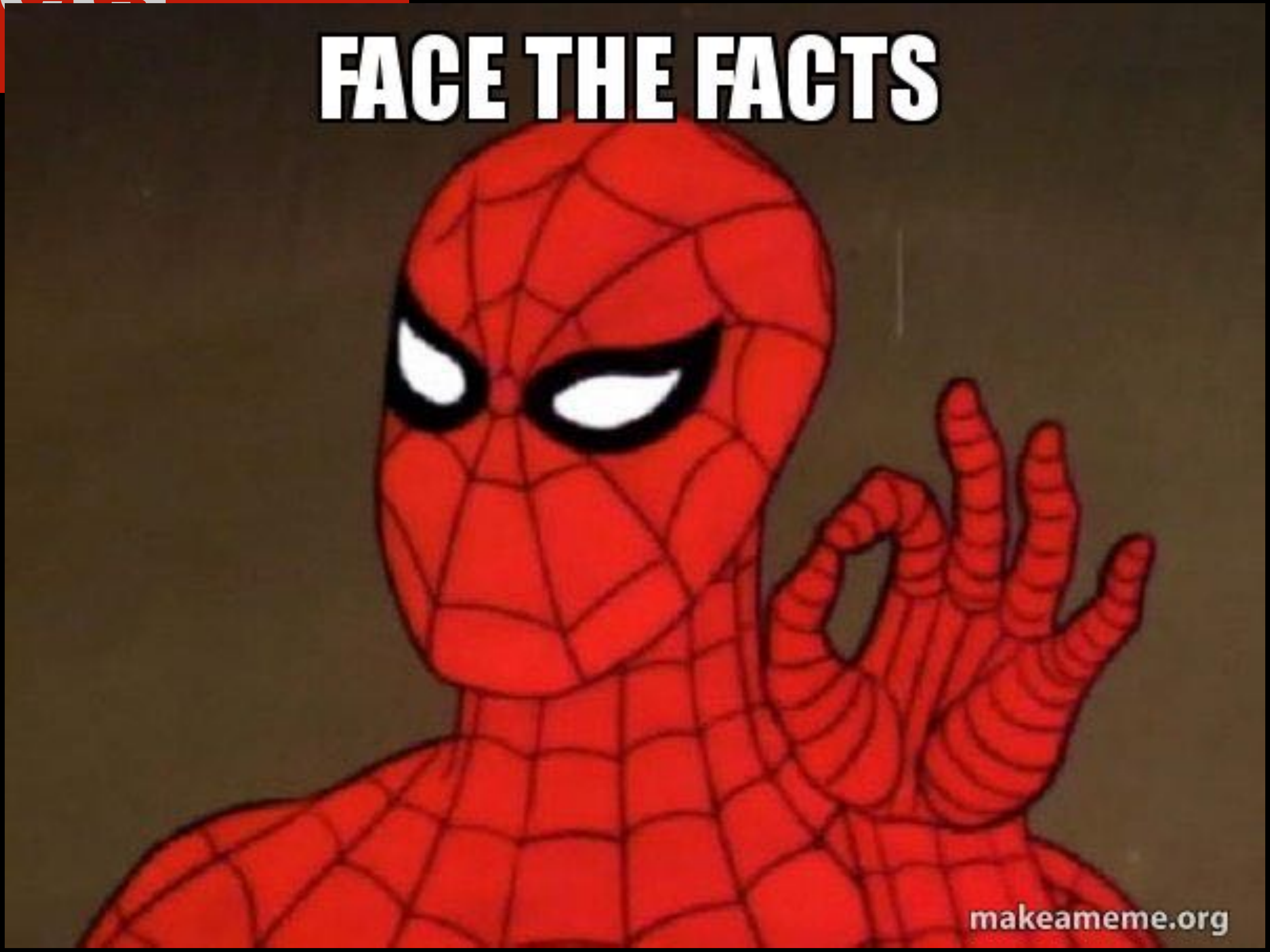
The stylised facts of financial markets



Learning outcomes

Understand financial markets from their patterns
Apply stylised facts to ABM validation

Stylised facts

Empirical Regularities

patterns that have been observed so many times they are accepted as truth

returns

Price **cannot be compared** across different stocks

Price time series display **trends**

returns

Solution: relative (%) changes

returns

returns

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Relative changes also have some "problems"

returns

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Δt **Time unit (minutes, hour, day, etc.)**

$r(t, \Delta t) = X(t + \Delta t) - X(t)$ **Log return**

returns

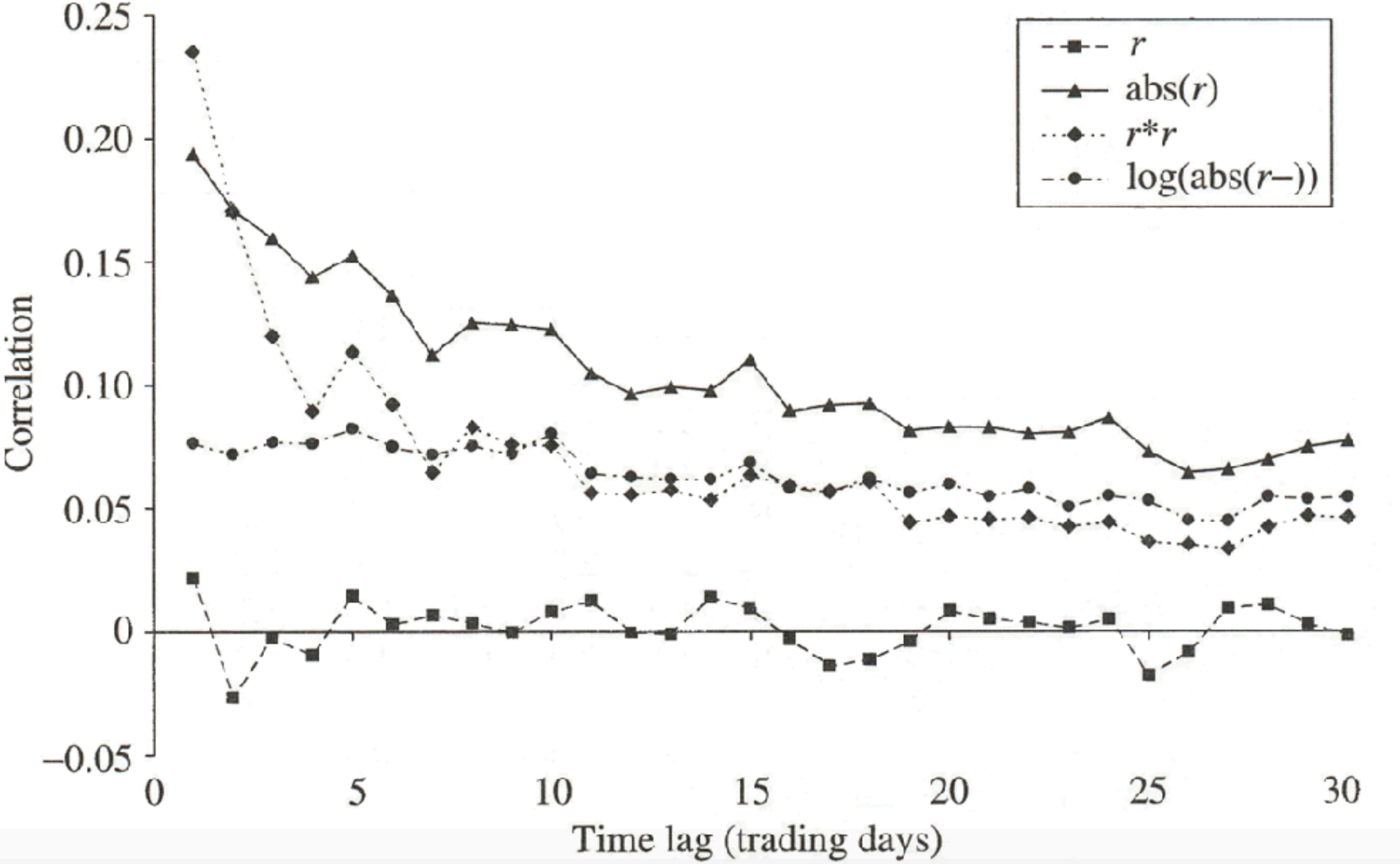
$$r(t, \Delta t) = X(t + \Delta t) - X(t)$$

Stationary

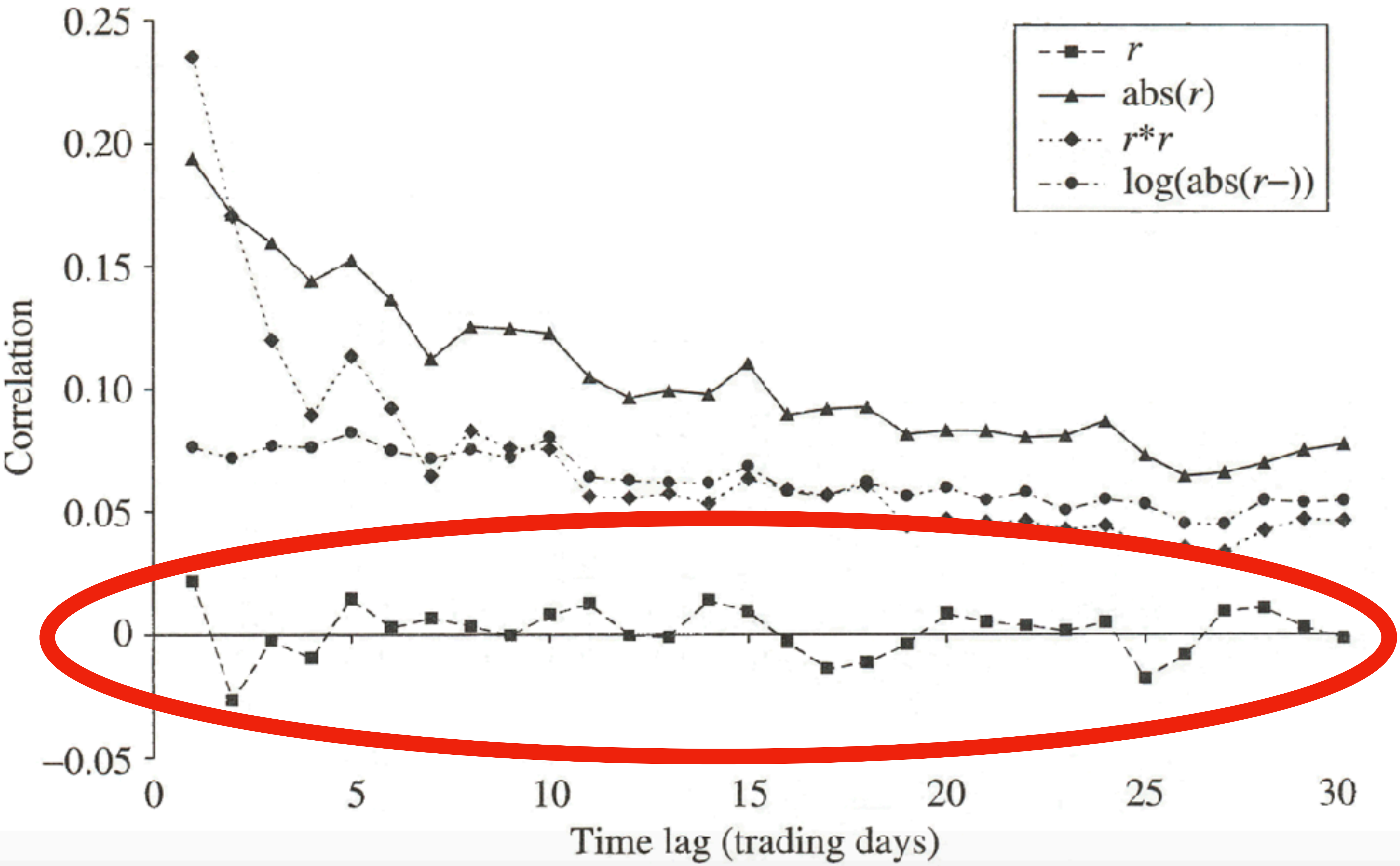
Time invariant

Approximate relative returns well when returns are small

Autocorrelation



Autocorrelation



Autocorrelation

$$C(\tau) = corr[r(t, \Delta t), r(t + \tau, \Delta t)]$$

$$C(\tau) = 0, \forall \tau$$

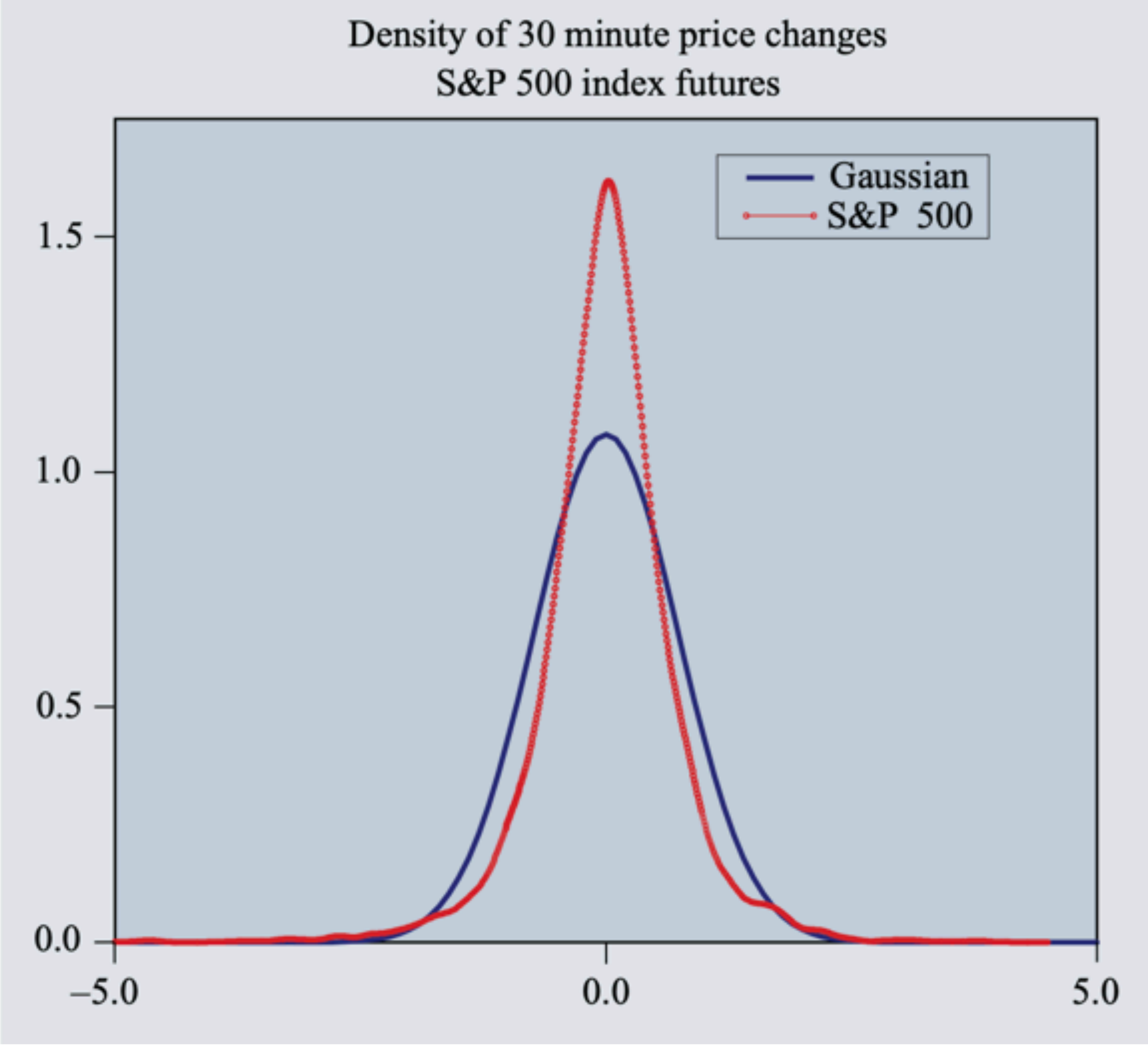
Autocorrelation

$$C(\tau)_\alpha = corr[\,|\,r(t,\Delta t)\,|^\alpha, |\,r(t+\tau,\Delta t)\,|^\alpha\,]$$

$$C(\tau)_\alpha = A\tau^{-\beta}$$

$$\alpha \in \{1,2\} \implies \beta \in [0.2,0.4]$$

Distribution of returns



Distribution of returns

$$\kappa[X] = \mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^4\right]$$

$\kappa[X] = 0$ **Normal distribution**

$\kappa[X] > 0$ **Long tails**

Aggregational gaussianity

$$\lim_{\Delta t \to \infty} \kappa = 0$$

Summary statistics – kurtosis

Data	μ/σ	Skewness	Kurtosis
S&P 500 futures	0.003	−0.4	15.95
Dollar/ DM futures	0.002	−0.11	74
Dollar/ Swiss Franc futures	0.002	−0.1	60
IID 95% confidence interval	–	0.018	0.036

Gain/loss Asymmetry

Downwards movements are larger but fewer

Upwards movements are more frequent but smaller

This does not apply to forex

Volume

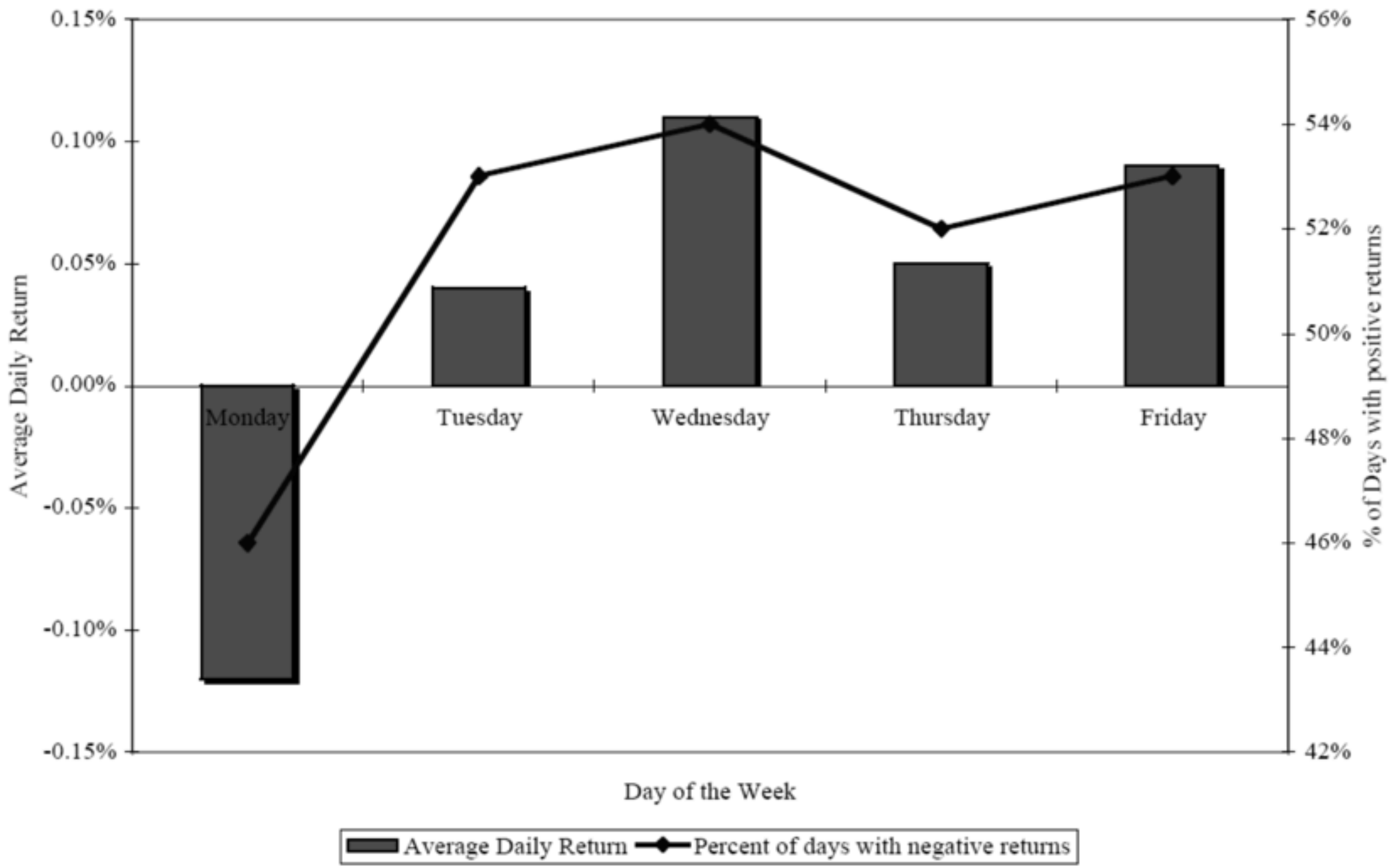
Volume is correlated with all measures of volatility

Calendar effects

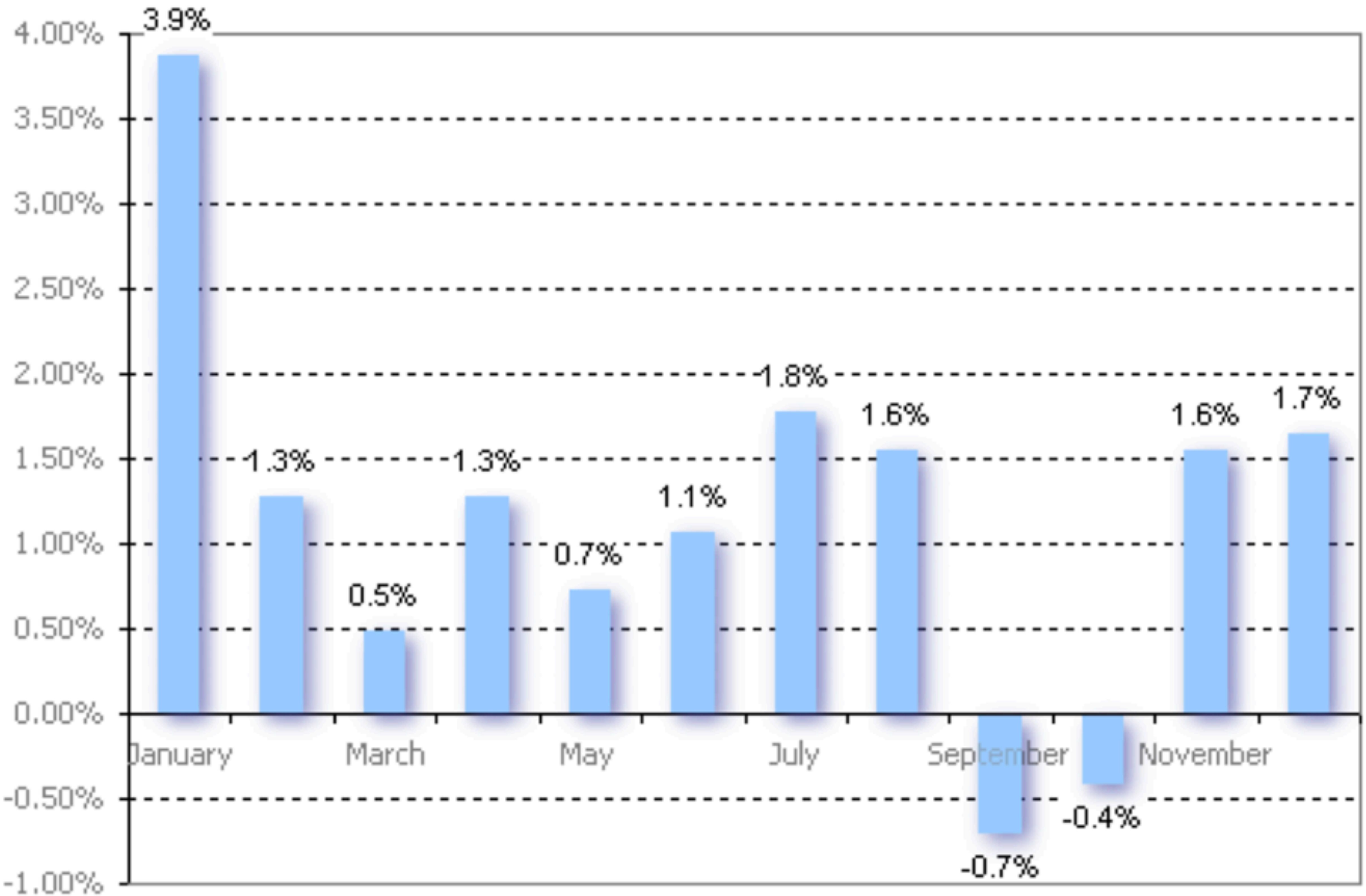
Price and volume display regularities based on day of week, week of month, month of year, etc.

These are sometimes called price/volume seasonalities

Weekend effect



January effect



Holiday effect

In the 80s and 90s, positive returns the day before a holiday accounted for 50% of all the yearly price increase.

This effect is now smaller but still present (in the three days before any major holiday)

Long memory

Early 1900s by Hurst

Application to finance in the late 60s by Mandelbrot

Many, many papers after that on price, volume, volatility, etc.

Long memory

Correlation

$$\lim_{\tau \to \infty} \frac{C(\tau)}{c_\tau \tau^{-\alpha}} = 1$$

Positive costant

$\alpha \in [0,1]$

Long memory

$$H = 1 - \frac{\alpha}{2}$$

$$\frac{1}{2} < H < 1$$

Long memory

Prices were found to have long memory, until they didn't. This led to a more robust reformulation of this statistic by Lo (1991).

Volume, volatility, bid/ask spread were found to exhibit long memory across several different markets and timescales by a number of studies (albeit with some differences)

Summary

Stylised facts are empirical regularities observed in financial time series

They provide powerful summary information that condenses knowledge of markets in a few equations/notions

They can (and should!) be used in abms for validation and estimation