The stylised facts of financial markets
Learning outcomes

Understand financial markets from their patterns
Apply stylised facts to ABM validation
Stylised facts

Empirical Regularities
patterns that have been observed so many times they are accepted as truth
returns

Price **cannot be compared** across different stocks
Price time series display **trends**
returns

Solution: relative (%) changes
returns

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Well yes, but actually no
returns

Solution: relative (%) changes
Relative changes also have some “problems”
returns

Instead, we use log-returns (difference of log of price)
returns

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\[ S(t) \]  \hspace{1cm} \text{Price at time } t
returns

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\[ S(t) \]

\[ X(t) = \ln(S(t)) \]

Price at time \( t \)

Log price
returns

Instead, we use log-returns (difference of log of price)

\[ S(t) \quad \text{Price at time } t \]
\[ X(t) = \ln(S(t)) \quad \text{Log price} \]
\[ \Delta t \quad \text{Time unit (minutes, hour, day, etc.)} \]
Instead, we use log-returns (difference of log of price)

\[ S(t) \]

Price at time \( t \)

\[ X(t) = \ln(S(t)) \]

Log price

\( \Delta t \)  
Time unit (minutes, hour, day, etc.)

\[ r(t, \Delta t) = X(t + \Delta t) - X(t) \]

Log return
returns

\[ r(t, \Delta t) = X(t + \Delta t) - X(t) \]

Stationary
Time invariant
Approximate relative returns well when returns are small
Autocorrelation

[Graph showing autocorrelation with time lag (trading days) on the x-axis and correlation on the y-axis, with different functions represented by lines and markers.]
Autocorrelation

The graph illustrates the autocorrelation of a time series. The x-axis represents time lag in trading days, and the y-axis shows the correlation. The red circle highlights the autocorrelation values at different lags, indicating how correlated the series is at different time intervals.
Autocorrelation

\[ C(\tau) = corr[r(t, \Delta t), r(t + \tau, \Delta t)] \]

\[ C(\tau) = 0, \forall \tau \]
Autocorrelation

\[
C(\tau)_\alpha = corr[ | r(t, \Delta t) |^\alpha, | r(t + \tau, \Delta t) |^\alpha ]
\]

\[
C(\tau)_\alpha = A\tau^{-\beta}
\]

\[
\alpha \in \{1,2\} \implies \beta \in [0.2,0.4]
\]
Distribution of returns

Density of 30 minute price changes
S&P 500 index futures

Gaussian
S&P 500
Distribution of returns

\[ \kappa[X] = \mathbb{E} \left[ \left( \frac{X - \mu}{\sigma} \right)^4 \right] \]

\[ \kappa[X] = 0 \quad \text{Normal distribution} \]

\[ \kappa[X] > 0 \quad \text{Long tails} \]
Aggregational gaussianity

\[ \lim_{\Delta t \to \infty} \kappa = 0 \]
## Summary statistics - kurtosis

<table>
<thead>
<tr>
<th>Data</th>
<th>$\mu / \sigma$</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500 futures</td>
<td>0.003</td>
<td>−0.4</td>
<td>15.95</td>
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<tr>
<td>Dollar/ DM futures</td>
<td>0.002</td>
<td>−0.11</td>
<td>74</td>
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<tr>
<td>Dollar/ Swiss</td>
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<tr>
<td>Franc futures</td>
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<td>−0.1</td>
<td>60</td>
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<tr>
<td>IID 95% confidence interval</td>
<td>−</td>
<td>0.018</td>
<td>0.036</td>
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</tbody>
</table>
Gain/loss Asymmetry

Downwards movements are larger but fewer
Upwards movements are more frequent but smaller
This does not apply to forex
Volume

Volume is correlated with all measures of volatility
Calendar effects

Price and volume display regularities based on day of week, week of month, month of year, etc. These are sometimes called price/volume seasonalities
Weekend effect

![Graph showing the Weekend effect]

- **Average Daily Return**
- **Percent of days with negative returns**

- **Day of the Week**
  - Monday
  - Tuesday
  - Wednesday
  - Thursday
  - Friday
January effect

The graph shows the performance of different months, with January having the highest percentage gain at 3.9%. The other months show varying gains and losses, with September experiencing a loss of 0.7%.
Holiday effect

In the 80s and 90s, positive returns the day before a holiday accounted for 50% of all the yearly price increase.

This effect is now smaller but still present (in the three days before any major holiday)
Long memory

Early 1900s by Hurst

Application to finance in the late 60s by Mandelbrot

Many, many papers after that on price, volume, volatility, etc.
Long memory

Correlation

$$\lim_{\tau \to \infty} \frac{C(\tau)}{c_\tau \tau^{-\alpha}} = 1$$

Positive constant \( \alpha \in [0,1] \)
Long memory

\[ H = 1 - \frac{\alpha}{2} \]

\[ \frac{1}{2} < H < 1 \]
Prices were found to have long memory, until they didn’t. This led to a more robust reformulation of this statistic by Lo (1991).

Volume, volatility, bid/ask spread were found to exhibit long memory across several different markets and timescales by a number of studies (albeit with some differences)
Summary

Stylised facts are empirical regularities observed in financial time series.

They provide powerful summary information that condenses knowledge of markets in a few equations/notions.

They can (and should!) be used in abms for validation and estimation.