1. (a) For \( n \in \mathbb{Z}^+ \) prove by contradiction the statement ‘if \( 5n + 4 \) is even, then \( n \) is even’.

   [marks 3]

   (b) Prove that the simultaneous equations

   \[
   \begin{align*}
   ax + by &= e \\
   cx + dy &= f
   \end{align*}
   \]

   have rational solutions \( x, y \) when \( a, b, c, d, e, f \) are all non-zero integers and \( ad \neq bc \).

   [marks 5]

2. Use the principle of strong induction to show that if \( u_n \) is defined recursively as

   \[
   u_1 = 3, \quad u_2 = 5, \quad u_k = 3u_{k-1} - 2u_{k-2} \quad \text{for} \quad k \in \mathbb{Z}^+, k \geq 3,
   \]

   then the sequence can be represented by \( u_n = 2^n + 1 \) for every integer \( n \geq 1 \).

   [marks 7]

3. We define the symmetric difference of two sets \( A \) and \( B \) as the set

   \[
   A \Delta B = \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}.
   \]

   (a) Write the symmetric difference in set notation using \( - \) and \( \cup \).

   [marks 1]

   (b) Draw a Venn diagram illustrating \( A \Delta B \)

   [marks 1]

   (c) Use the algebraic method to prove

   \[
   A \Delta A \Delta A = A.
   \]
4. (a) Each of the following describes a function where each function has domain and codomain equal to $\mathbb{Z}$. In each case show whether or not the function is one-to-one (injective) or onto (surjective). Also comment on any that are bijective (one-to-one correspondence).

i. $f(n) = 2n + 1$

ii. $g(n) = \begin{cases} 
\frac{n}{2} & \text{if } n \text{ is even} \\
2n & \text{if } n \text{ is odd}
\end{cases}$

iii. $h(n) = \begin{cases} 
n + 1 & \text{if } n \text{ is even} \\
n - 1 & \text{if } n \text{ is odd}
\end{cases}$

(b) Show that the set of all nonnegative integers is countable by showing a bijection between $\mathbb{Z}^+$ and $\mathbb{Z}_{\text{nonneg}}$ using an explicit function.