

DMP Class Test

Discrete Mathematics

25 October 2023

There are 30 marks to be earned, and you have 2 hours.

1. Prove that there exist no different positive integers x and y such that $x/y^2 = x^2/y$. [4 marks]
2. Define the sequence s_0, s_1, s_2, \dots by $s_0 = 1$ and $s_k = \sum_{i < k} s_i$ for all integers $k \geq 1$.
Prove that $s_n = 2^{n-1}$ for all $n \geq 3$. [6 marks]
3. Prove the identity $A - (B - C) = (C \cap A) \cup (A - B)$ in two ways. Once by means of the element method, and once algebraically. For the algebraic proof you may use (only) the set identities from Epp Theorem 6.2.2. [4+4 marks]
Also give a Venn diagram showing either side of this equation. [1 marks]
4. Which of the following 3 functions are (i) injective, (ii) surjective, and (iii) bijective? (Note that these are 9 questions.) In each case, explain your answer. [6 marks]
 - (a) $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$.
 - (b) $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ given by $g(x) = \frac{3x}{x-1}$.
 - (c) $h : \mathbb{R} \rightarrow \mathbb{R}$ given by $h(x) = \begin{cases} 3 & \text{if } x = 1 \\ \frac{3x}{x-1} & \text{if } x \neq 1. \end{cases}$
5. Let a “word” be a finite string of letters from the alphabet $\{a, b, c, \dots, z\}$ of 26 English letters. Show that set of all such words is countable. [5 marks]