Discrete Mathematics and Probability **Induction and Recursion**
Session 2023/24, Semester 1 Week 3 Homework 2

This homework runs from 3pm Thursday 28 September 2023 until 12 noon on Thursday 5 October 2023. Submission is to Gradescope Homework 2.

Question 1

Assume n is a positive integer with $n \geq 1$. Prove by mathematical induction that

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}.$$

[*7 marks*]

Question 2

Suppose a sequence of integers $a_1, a_2, a_3, \ldots$ is defined recursively as follows:

$$a_1 = 4 \quad \text{and} \quad a_{n+1} = a_n + 18n + 3 \quad \text{for } n \geq 1.$$

Prove by induction that $a_n = (3n-1)^2$ for all $n \geq 1$. [*3 marks*]