Discrete Mathematics and Probability

Session 2023/24, Semester 1

This homework runs from Thursday 2 November 2023 until 12 noon on Thursday 9 November 2023. Submission is to Gradescope Homework 4.

Question 1

- (a) In how many ways can the integers 1 through 7 be permuted so that no odd integer will be in its natural position?
- (b) If all the possibilities of part (a) are equally likely, what is the probability that exactly two of the seven integers are in their natural position?

[4 marks]

Question 2

A art box has in it 20 pencils: 5 red, 5 blue, and 10 green. You cannot distinguish between pencils of the same colour. You take out three pencils at random, one at a time, without replacement. Suppose X is the random variable 'the total number of red pencils taken out'.

(a) Copy the following table and fill in the probability distribution for X.

$$\frac{x \quad 0 \quad 1 \quad 2 \quad 3}{P(X=x)}$$

- (b) Calculate the expected number of red pencils E(X).
- (c) Calculate the standard deviation of X.

For each part include your working as well as the final answer.

[6 marks]