Discrete Mathematics and Probability **Probability Distributions**
Session 2023/24, Semester 1 Week 9 Homework 5

This homework runs from Thursday 9 November 2023 until 12 noon on Thursday 16 November 2023. Submission is to Gradescope Homework 5.

Question 1

(a) Half of all messages are spam: junk mail that you don't want to read. A spam filter is a piece of software that tries to filter these spam messages before they reach your inbox. Suppose you have a spam filter that correctly detects 95% of spam messages, with only a 1% probability of a *false positive* — a message being marked as spam when it isn't. If a message is flagged by the filter as spam, what is the probability that it is in fact not spam?

[*2 marks*]

(b) The probability that a sports fan supports Sportball Team A is 0.4. Using a binomial distribution find the probability that in a randomly selected sample of 8 fans there are:

(i) exactly 3 who support Team A;

(ii) more than 5 who support Team A. [*3 marks*]

Question 2

A delivery driver travels K_1 kilometres on Tuesday, K_2 km on Wednesday, and K_3 km on Thursday. Here K_1, K_2, and K_3 are independent random variables each having a normal distribution with mean 90 and standard deviation 25.

(a) Calculate the probability that on Tuesday the driver travels more than 100 km.

(b) Calculate $P(80 \leq K_2 \leq 100)$.

(c) Calculate the probability that the driver travels less than 80 km on every one of the three days.

(d) Random variable $T = K_1 + K_2 + K_3$ records the total distance in kilometres travelled over the three days.

Calculate the expectation, variance, and standard deviation of T.

(e) Random variable $A = T/3$ records the average distance in kilometres travelled per day.

Calculate the expectation, variance, and standard deviation of A.

Show your working for each calculation. Look in Appendix A of the course textbook, Carlton and Devore, for Table A.3 on pages 601 and 602 showing the CDF Φ of the Standardized Normal distribution.

[*5 marks*]