This homework runs from Thursday 16 November 2023 until 12 noon on Thursday 23 November 2023. Submission is to Gradescope Homework 6.

Question 1

A small commercial passenger aircraft has fuel efficiency that varies under different conditions, such as weather or loading weight. Over time it is observed that in regular use the fuel efficiency X, measured in *miles per UK gallon* (mpg) is distributed with the following PDF and CDF.

$$f_X(x) = 4(x - x^3) \quad 0 \leq x \leq 1 \qquad f_X(x) = 0 \text{ otherwise}$$
$$F_X(x) = 2x^2 - x^4 \quad 0 \leq x \leq 1 \qquad F_X(x) = 0 \text{ if } x < 0 \text{ and } 1 \text{ if } x \geq 1$$

(a) Calculate the expected value of X.

(b) Calculate the probability that fuel efficiency is between 0.5 and 0.75 miles per gallon.

Random variable Y is an approximate measure of fuel efficiency using the alternate metric unit of *litres of fuel per kilometre*.

$$Y = \frac{2\sqrt{2}}{X}$$

(c) Random variable X always takes values between 0 and 1. What is the range of possible values for Y?

(d) Calculate $P(Y > 5)$.

(e) Calculate the PDF for Y.

Include your working for each part.

[*7 marks*]

Question 2

Two continuous random variables X and Y range between 0 and 1 with the following joint probability distribution.

$$f(x, y) = \begin{cases} \frac{4x}{3} + y^2 & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate the marginal probability distribution functions $f_X(x)$ and $f_Y(y)$.

(b) Calculate $f_X(1)$, $f_Y(1)$, and $f(1, 1)$. Use these to show that X and Y are not independent.

(c) Use the following information about X and Y to estimate the expected value and variance of $(X - Y)$ to three decimal places.

$$\mathrm{E}(X) = 0.611 \quad \mathrm{Var}(X) = 0.071 \quad \mathrm{E}(Y) = 0.583 \quad \mathrm{Var}(Y) = 0.082 \quad \mathrm{Cov}(X, Y) = -0.009$$

Include your working for each part.

[*3 marks*]