Tutorial 2 in Week 3

1) Discuss your submissions for Homework 1 and where you made mistakes or could improve. See the course web pages for solution notes.

2) Work together as a group on each of the following tasks.

Task A
Consider a 2 x3 and 2 X 6 checkerboard. Draw a covering of the boards by L-shaped trominoes such as this one covering 3 squares.

Now work together using mathematical induction to prove that for each integer \( n \geq 1 \), any checkerboard with dimensions \( 2 \times 3n \) can be completely covered with L-shaped trominoes.

Task B
Use mathematical induction to prove that for any integer \( n \geq 0 \), \( 7^n - 2^n \) is divisible by 5.

Task C
This is Ex 5.4 Q6 below from the textbook: Is this strong induction? Try to prove as a group.

Suppose that \( f_0, f_1, f_2, \ldots \) is a sequence defined as follows:
\[
f_0 = 5, f_1 = 16, f_k = 7f_{k-1} - 10f_{k-2} \text{ for every integer } k \geq 2.
\]
Prove by mathematical induction that \( f_n = 3 \cdot 2^n + 2 \cdot 5^n \) for each integer \( n \geq 0 \).
Task D

Suppose that \( c_0, c_1, c_2, \ldots \) is a sequence defined as follows:

\[
\begin{align*}
  c_0 &= 2, \quad c_1 = 2, \quad c_2 = 6, \\
  c_k &= 3c_{k-3} \quad \text{for every integer } k \geq 3.
\end{align*}
\]

Prove that \( c_n \) is even for each integer \( n \geq 0 \).

Task E

Compute \( 9^0, 9^1, 9^2, 9^3, 9^4, \) and \( 9^5 \). Make a conjecture about the units digit of \( 9^n \) where \( n \) is a positive integer. Use strong mathematical induction to prove your conjecture.