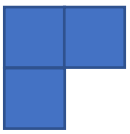
Tutorial 2 in Week 3

1) Discuss your submissions for Homework 1 and where you made mistakes or could improve. See the course web pages for solution notes.

2) Work together as a group on each of the following tasks.

Task A

Consider a 2 x3 and 2 X 6 checkerboard. Draw a covering of the boards by L-shaped trominoes such as this one covering 3 squares.



Now work together using mathematical induction to prove that for each integer $n \geq 1$, any checkerboard with dimensions $2 \times 3n$ can be completely covered with L-shaped trominoes.

Task B

Use mathematical induction to prove that for any integer $n \geq 0$, $7^n - 2^n$ is divisible by 5.

Task C

This is Ex 5.4 Q6 below from the textbook: Is this strong induction? Try to prove as a group.

Suppose that $f_0, f_1, f_2, \ldots$ is a sequence defined as follows:

$f_0 = 5, f_1 = 16, f_k = 7f_{k-1} - 10\, f_{k-2}$ for every integer $k \geq 2$.

Prove by mathematical induction that $f_n = 3 \cdot 2^n + 2 \cdot 5^n$ for each integer $n \geq 0$.

Task D

Suppose that $c_0, c_1, c_2, \ldots$ is a sequence defined as follows:

$$c_0 = 2, \ c_1 = 2, \ c_2 = 6,$$
$$c_k = 3c_{k-3} \quad \text{for every integer } k \geq 3.$$

Prove that c_n is even for each integer $n \geq 0$.

Task E

Compute 9^0, 9^1, 9^2, 9^3, 9^4, and 9^5. Make a conjecture about the units digit of 9^n where n is a positive integer. Use strong mathematical induction to prove your conjecture.