Tutorial 3 in Week 4

Task 1: go over the homework as last week in smaller groups as necessary.

Task 2: Try these questions

Q1

Find the mistake in the following "proof" that for all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

"**Proof:** Suppose A, B, and C are any sets such that $A \subseteq B$ and $B \subseteq C$. Since $A \subseteq B$, there is an element x such that $x \in A$ and $x \in B$, and since $B \subseteq C$, there is an element x such that $x \in B$ and $x \in C$. Hence there is an element x such that $x \in A$ and $x \in C$ and so $A \subseteq C$."

Q2

Find the mistake in the following "proof."

"Theorem:" For all sets A and B, $A^c \cup B^c \subseteq (A \cup B)^c$.

"**Proof:** Suppose A and B are any sets, and $x \in A^c \cup B^c$. Then $x \in A^c$ or $x \in B^c$ by definition of union. It follows that $x \notin A$ or $x \notin B$ by definition of complement, and so $x \notin A \cup B$ by definition of union. Thus $x \in (A \cup B)^c$ by definition of complement, and hence $A^c \cup B^c \subseteq (A \cup B)^c$."

Q3

Recall that for sets A and B, $|A| = |B|$ if there is a bijection $f : A \to B$, a function f that is both injective (one-to-one) and surjective (onto). Let $E = \{0, 2, 4, \ldots\}$ be the set of non-negative even integers.

(a) Give an example of a function $g : E \to E$ that is injective but not surjective.

(b) Prove that $|\mathbb{Z}| = |E|$ by defining an explicit bijection.

Q4

(a) Determine whether the function $f : (\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}$ is surjective if

i. $f(m, n) = m^2 + n^2$

ii. $f(m, n) = m$

iii. $f(m, n) = |n|$

iv. $f(m, n) = m - n$

(b) Assume functions $g : A \to B$ and $f : B \to C$. Prove or disprove the following statements.

i. If $f \circ g$ and g are injective then f is injective.

ii. If $f \circ g$ and f are injective then g is injective.

Q5

Simplify using an algebraic argument.

$$(A - (A \cap B)) \cap (B - (A \cap B))$$