This tutorial comes with a spreadsheet which you can use as a guide to fill out the results of doing each part of the tasks listed.

**Task 1.** Retrieve your submissions from Homework 5 in Week 9 and compare solutions around the group. For Question 1(b)(ii), how did you calculate the “more than 5” probability? For Question 2(e), how does the standard deviation for \( A \), the mean distance driven over three days, compare to the standard deviation for \( K_1 \), the distance on the first day? What about the standard deviation of the mean distance driven over five days, ten days, or 100 days?

**Task 2.** (This is Question 117 from the textbook [Carlton & Devore 2017].)
If the side of a square \( X \) is random with the pdf \( f_X(x) = x/8 \), \( 0 < x < 4 \), and \( Y \) is the area of the square, find the pdf of \( Y \).

**Task 3.** A farm grows apples for sale to distributors. The price of apples depends on their weight and distributors will only take apples of a certain weight range. This year’s apples from the farm have weights normally distributed with mean 170g and standard deviation 35g. They are sold to distributors in batches of 500.
Distributor A is looking for apples between 120g and 240g in weight: they check for this by taking a random sample of 10 apples from a batch and will accept the batch if at least 8 are in the target weight range.
Distributor B has a different policy. They are interested in any apples weighing over 100g: to test for this they sample 5 and reject the whole batch if even one of those is underweight.

Which distributor is more likely to accept a batch of apples from this farm?

**Task 4.** The Normal distribution can be used as an approximation for calculating probabilities from the Binomial distribution. This task explores how well approximations like this work in a simple case.
If \( X \sim \text{Binom}(n, p) \) is a discrete random variable counting successes in \( n \) trials each with probability \( p \) then we have the following approximation.

\[
P(X \leq x) = B(x; n, p) \approx \Phi \left( \frac{x + 0.5 - np}{\sqrt{npq}} \right) \quad x \in \{0, 1, \ldots, n\}
\]

This is considered a good approximation in practice if \( np \geq 10 \) and \( nq \geq 10 \) where \( q = (1 - p) \).
Here you will look at a situation where both values are much less than 10.

(a) Suppose discrete random variable \( X \sim \text{Binom}(4, 0.5) \) counts the number of heads thrown in four tosses of a fair coin: \( X \in \{0, 1, 2, 3, 4\} \). What are the mean and standard deviation of the random variable \( X \)?

(b) Use the Normal approximation above to estimate the probability that \( X \) takes the value 2.

(c) Complete this to draw up a table of estimates for the probability that \( X \) takes each of the possible values 0 to 4.

(d) Now compute a table of the PMF for \( X \) showing the exact probability that it takes each of these five possible values.

(e) Calculate the table of errors: what percentage variation is there in each estimate compared to the exact value? Which values have the largest error?