

This homework runs from Thursday 26 September 2024 until 12 noon on Thursday 3 October 2024. Submission is to Gradescope Homework 2.

Questions marked with an asterisk * may be a little harder than others. All are still within the course curriculum, though, and can be done using the methods taught in the study guides and textbook.

You should aim to write out solutions that someone who does not already know the answer could follow and understand.

Please remember the good scholarly practice requirements of the University regarding work for credit. You can find guidance at the School page <https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct>. This also has links to the relevant University pages.

Question 1

- (a) For which positive integers n is $3^n < n!$?
 (b) Prove by induction your statement in (a).

[3 marks]

Question 2

Suppose a sequence of integers $a_0, a_1, a_2, \dots$ is defined recursively as follows:

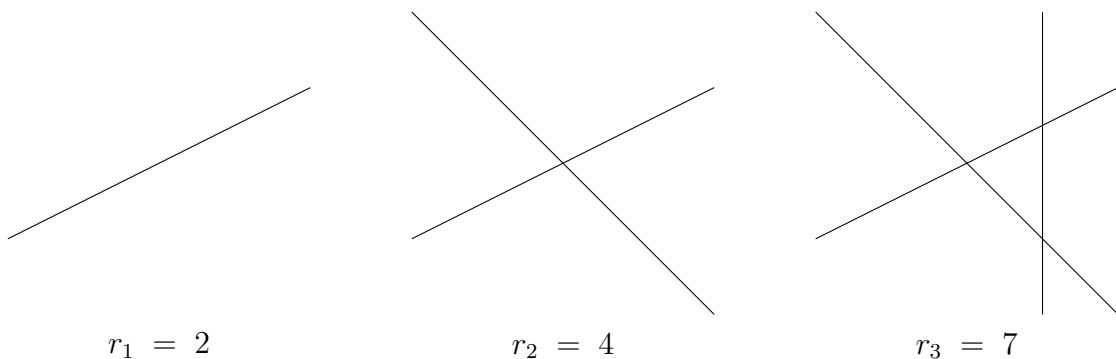
$$a_0 = 0 \quad \text{and} \quad a_n = 3a_{n-1} + 4 \quad \text{for } n \geq 1.$$

Prove by induction that $a_n = 2(3^n - 1)$ for all integers $n \geq 0$.

[3 marks]

* Question 3

The diagrams below show a plane divided into regions by different numbers of straight lines. Suppose r_n is the maximum number of regions a plane can be divided into by n straight lines.



- (a) Write down a recurrence relation expressing r_{n+1} in terms of r_n for $n \geq 0$, with an explanation justifying it.
 (b) Use this to prove by induction that $r_n = \frac{n^2 + n + 2}{2}$.

[4 marks]